

STATISTICAL STUDIES OF ROAD ACCIDENTS AND INJURY SEVERITY

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ABSTRACT

This thesis is concerned with the quantitative study of certain characteristics of road traffic accidents, with particular attention being paid to the severity of injury sustained by the victims thereof.

The results fall into two main groups: those dealing with the frequency and severity of accidents to particular categories of road user, and those relating to a new theory of injury severity.

The following is a list of the questions considered in the first of these groups: factors relating to the severity of pedestrian injury (with special attention to design of vehicle), factors affecting the survival time of pedestrians killed in road accidents, relation of model of car to degree of (a) leg, and (b) head injury of its driver, effect of vehicle, age and sex of driver, and locale on (a) the relative numbers of single- and two-car accidents, and (b) the proportion of overturning in single-car accidents. Several sections of the thesis are devoted to developing appropriate statistical procedures for these analyses.

The new theoretical framework for assessing injury severity that is proposed in the penultimate chapter quantifies the correlation between the proportion of casualties who are killed and the proportion who are seriously injured. Finally, this theory is used to explain the positive correlation that is empirically found to occur between the degrees of injury to the two drivers involved in two-vehicle accidents when only a narrow range of mass ratios is considered.

A table of the major results in this thesis, together with some lines for future research which they suggest, is given on pages 357-363.

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- and many other colleagues both within and outside the Traffic Studies Group, and several anonymous referees, for constructive discussions and criticism.

The faults and inadequacies of the work are, of course, the author's own.

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1.1 Outline of thesis

In this thesis, several aspects of road accidents and the severity of injury resulting therefrom are subjected to quantitative study. Most of the data are based upon the information collected by the British police about such accidents: section 1.2.1 describes this "Stats 19" procedure. The remainder of Chapter 1 is devoted to a brief consideration of sources of data providing better information about the nature of injury than is available in the Stats 19 records, and trends shown in such injury data.

Chapters 2 and 3 study the severity of pedestrian accidents, with special attention being paid to the effect of design of vehicle. In Chapter 2 national accident data is used: one of the chief limitations of this is the absence of information about the speed of the accident. In an effort to take account of this, a large number of original police reports of pedestrian accidents in London were examined. Although there was quite a lot of information about speed in the reports of fatal accidents, for injury accidents this was very inadequate, and the original purpose of this study - to determine whether the relation between speed of impact and degree of pedestrian injury was different for different models of car - was not fulfilled. However, several interesting points came out of the study of fatal pedestrian accidents, and these are reported in Chapter 3.

Chapters 4 to 6 are concerned with non-pedestrian accidents. Firstly, in Chapter 4, some advanced statistical techniques applicable to accident data are explained: section 4.1 considers the application

to traffic and accident studies of a program which provides for categorical data the same sort of facilities that general linear model programs do for metric data, and section 4.2 discusses a nonparametric test of a rather general nature that subsumes many of the better-known nonparametric tests (such as those of Spearman, Friedman, and Kruskal-Wallis). It is shown that it is however, somewhat conservative in certain situations, and a FORTRAN program is presented that has the rather novel feature of permitting the direct estimation of significance level by means of randomising ranks within rows. Chapter 5 applies these techniques to the question of whether model of car affects the degree of (a) leg injury, and (b) head injury suffered by the driver, with type of accident being taken into account. (Police records of accidents in London were used to obtain the nature of injury.) It was found that the five models of car (of similar weight) studied did differ in the leg injury incurred, but not in head injury. Chapter 6 turns away from the question of severity of injury, examining instead the relative proportions of different types of accident to different models of car (national accident data was used for this). Two dependent variables were used: the relative numbers of single- and two-car accidents, and the proportion of overturning in single-car accidents; the age and sex of the driver and the locale (urban or rural) of the accident were taken into account as well as the model of car. Comments are made on the relation of this work to that of Thorpe (1964) and Haight (1970, 1973) on induced exposure, and that of Jones (1973, 1975) of which the present is a direct development. Chapter 6 concludes with an Appendix outlining a new statistical test of potential value when correlating a number of mutually-independent variables (perhaps car characteristics) each with a criterion variable (perhaps accident rate).

The most fundamental work in this thesis is reported in Chapters 7 and 8, in which the emphasis once again is on injury severity. One of the difficulties with analysing this is that the relative weights to be assigned to fatal, serious, and slight injuries are unknown. In Chapter 7 a beginning is made at solving this problem. It is shown that there may exist a definition of injury severity such that in all circumstances the distribution of injury severity is exponential, the parameter of this distribution being characteristic of the circumstance. There are then two thresholds on the severity axis, which divide it into fatal, serious, and slight categories, and the proportions of cases falling into each of these are p , $p^m - p$, and $1 - p^m$, where p is characteristic of the conditions being considered and m is related to the definitions of the three degrees of injury. This is essentially a quantification of the correlation to be expected between the proportion of fatalities and the proportion of serious injuries. An alternative theory which supposes that there exists a definition of injury severity such that severity is Normally distributed in all circumstances (though with average level differing according to circumstances) is also considered. A number of applications of this method of considering injury severity are outlined in section 7.3, and in Chapter 8 the theory is used as one of the bases of an investigation of intra-accident correlations of injuries. In Chapter 8 it is shown to be useful to present data on injuries sustained by the drivers involved in two-vehicle accidents in the form of a series of 4×4 tables, with the four columns corresponding to the four degrees of injury (fatal, serious, slight, uninjured) to the driver of the lighter vehicle and the rows to the four degrees of injury to the driver of the heavier vehicle, with each table corresponding to a narrow range of mass ratios of the vehicles involved. It is discovered that in such tables a positive association exists between the injuries to the

two drivers and it is this to which the expression "intra-accident injury correlations" refers. It is suggested that the explanation of this correlation is the relative velocity of the two vehicles at impact, which is the same for the two drivers in any one accident but differs considerably between accidents. A model for the correlation is accordingly constructed which approximates the distribution of impacts speeds by a two-point distribution, each of these two speeds being associated with a particular probability of being killed, and the probabilities of the other degrees of injury are derived using the theory of Chapter 7. The probabilities of death in this model are fitted to the data for each mass ratio separately, and this enables "severity" (probability of being killed) to be plotted against a quantity that is proportional to the velocity change at impact. This has been done for four types of accident (head-on and intersection in urban and rural areas), and has enabled an approximate comparison of the relative means and variances of speeds of these types of accident (head-on rural being found to be the fastest, and intersection urban the slowest), even though no information on speed was included in the data analysed.

General remarks and suggestions for further research are included at several points throughout the thesis, as appropriate, and the most important results together with suggestions for future results are summarised in a table preceding the list of 183 references.

1.2 British national road accident data capture systems

1.2.1 Stats 19 procedure

The details of personal injury road accidents are usually noted by police called to the scene of the accident, and those required by the Stats 19 form (figure 1.1) are collated at local police headquarters and punched onto cards. (Instructions for the completion of this form are given in the Stats 20 booklet (HMSO).) These are sent to a Department of the Environment office at Hemel Hempstead where some details of background information are added and consistency checks carried out. The cards are then forwarded to the Transport and Road Research Laboratory where they are read onto magnetic tape. The data bank of road accidents there now extends for over a decade, though as there have been changes in the Stats 19 form during that time, not all this data is in the same format or contains exactly the same items of information. (See the report by Chapman and James (1973) entitled "The Stats 19 road accident data procedure and its research applications".)

The definitions of the three degrees of injury recognised by the Stats 19 process should now be given:

FATAL: death within 30 days as a result of the accident.

SERIOUS: injury for which a person is detained in hospital as an 'in patient' or any of the following injuries, whether or not he is detained in hospital: fractures, concussion, internal injuries, crushings, severe cuts and lacerations, severe general shock requiring medical treatment.

SLIGHT: injury of a minor character such as a sprain, bruise, or a cut or laceration not judged to be severe.

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Figure 1.1: Stats 19 form used by the police to record details of road accidents

These definitions are commented upon by Chapman and Neilson (1971) as follows:

"Injury categories are a considerable source of variation. The four categories, fatally, seriously, slightly or not injured are barely sufficient for many purposes and the two borderlines between the last three categories are very uncertain. A fatality is precisely classified by the 30 days rule which includes only those dying within 30 days of the accident. But many injured are detained overnight in hospital because they cannot be examined until morning, because they may have suffered concussion or even because they find themselves at a hospital and there is no alternative accommodation for the night. The result is that many of those falling within the definition of seriously injured do not necessarily have any of the injuries listed for serious injury. Again in some Police Forces it is customary for the policeman at the scene to assess injury severity, while in others enquiry is made afterwards at the hospitals. The distinction between slight and no injury determines whether or not a Stats 19 entry is made. Many accidents of a minor nature are not seen by the police but are reported to them afterwards to comply with the legal requirement to do so by those involved. The police opinion then determines the severity rating."

Chapman and Neilson (1971), together with Satterthwaite (1974), should also be consulted for comments on some of the errors encountered in the data. Hakkert (1969, Chapter 2) discusses in some detail requirements for accident reporting systems.

Newby (1969) has examined the proportions of casualties classified

as having fatal, serious, or slight injuries in the (then) 152 British police forces. He shows that those forces with a high ratio of fatalities to fatal plus serious injuries tend to have a low ratio of fatal plus serious casualties to the total, precisely the opposite tendency to what would be expected on theoretical grounds (Chapter 7). The reason for this anomaly is the varying definition of a "serious" injury between police forces: if the boundary between slight and serious injury is at a relatively low level of severity, $\text{fatals}/(\text{fatals} + \text{serious})$ will be too low and $(\text{fatals} + \text{serious})/\text{total}$ will be too high. Newby also gives an example from one particular English town where the numbers of fatal and serious casualties remained consistent over the period (1950-67) examined, but the number of slight casualties suddenly increased in 1959 by 80% and continued to increase from the new level. Thus apparently some accidents that previously would have been classified as non-injury in that town are now regarded as slight injury.

A similar instance is given by Satterthwaite (1975) in which it was apparently the boundary between serious and slight injuries that moved. This is examined further in section 7.6.

Furthermore, the legal requirement to report accidents to the police is very incomplete. Bull and Roberts (1973) summarise it as follows:

"Though it is commonly believed in this country that the Road Traffic Act and Highway Code require all personal injury accidents to be notified to the police, this is by no means strictly true. The only accidents legally required to be notified are those which involve a motor vehicle and cause injury to a person other than the driver and in which exchange of addresses and insurance

information has not occurred. Accidents in which the driver only is injured and no other vehicle is involved need not be notified by law. This applies to drivers of both cars and motor cycles; accidents causing injury to pedal cyclists are only notifiable if a motor vehicle is involved. Other road accidents escape the legal requirements for notification if the prescribed information is exchanged between the parties, thus legally enforced notification applies only to a very limited range of accidents. Another category of accident which, though perhaps technically notifiable, may easily escape, is one in which an injury is overlooked at the time of the accident and the patient may only report later to his doctor or hospital.

In practice the police are often informed for other reasons. In the City of Birmingham and many places elsewhere police and ambulance services share a common information system so that when an ambulance is called to a road accident the police are routinely also alerted. The police are often also informed in cases where someone wishes to make an allegation of an offence such as dangerous driving. This information may come from one of the parties in the accident or from a bystander. In spite of the limitations of the legal requirements mentioned above, there is also the widespread belief that in cases of injury the police should be informed."

Bull and Roberts attempted to trace in police records 1200 patients injured in road accidents and attending Birmingham Accident Hospital. They found that about one-sixth of serious injuries and one-third of slight injuries known to the hospital did not appear in the police records.

Probably the two most important limitations with the Stats 19 data are the lack of information about the speeds of the vehicle(s) involved, either preceding the accident or at impact, and the limited information about degree and nature of injury. The first of these is inherently difficult to obtain, but information about injury may be obtained from the Hospital In-Patient Enquiry (section 1.2.3) and about causes of death from the Registrar-General's Report (section 1.2.2).

1.2.2 Vital statistics collected by the Registrar-General

Based on death certificates and coroners' reports, the Annual Report of the Registrar-General of England and Wales (London: HMSO) includes tabulations of the causes of death of people killed in road accidents, subdivided according to age, sex, and class of casualty (pedestrian, motorcyclist, vehicle occupant, etc). Although invaluable for some purposes, the great limitation of this data is that little information is given about the accident: codes E810-E819 of the International Classification of Diseases (8th Revision, 1965: World Health Organisation, 1967) are used to define the type of motor vehicle traffic accident, the most important of which are E812 (involving collision with another motor vehicle), E814 (involving collision with pedestrian), and E816 (running off roadway or overturning without antecedent collision), together with a fourth digit identifying the injured person into one of the following categories: pedestrian, pedal cyclist, driver (not motorcyclist), passenger (not on motorcycle), motorcyclist, passenger on motorcycle. Thus from the Registrar-General's Report for 1972 we can discover that of the 682 riders and passengers of motorcycles killed in that year, 337 died from fracture of the skull and 187 from internal injury of chest, abdomen, and pelvis. It should be mentioned that all tabulations relying on a single cause of death

have their limitations: the ICD specifies that if more than one kind of injury is mentioned on the death certificate and there is no clear indication as to which caused death, then that to be coded should be selected in accordance with a particular order of preference, at the top of which is fracture of skull; furthermore, the certifying doctor will, in the large number of traffic deaths involving multiple injuries, have a choice of alternatives open to him, and may specify head injury as the primary cause of death when other fatal injuries are also present. In section 1.3 certain trends in causes of mortality from road accidents are examined.

1.2.3 The Hospital In-Patient Enquiry

The Enquiry is based on a 10% sample of in-patient records from National Health Service hospitals in England and Wales. The data relates to discharges and deaths during the year and not to individual patients, i.e. patients discharged more than once during a calendar year may be included in the sample upon each discharge. The following classes of patients are excluded from the Enquiry: psychiatric, mentally subnormal, private, staff being treated for minor ailments, and convalescent. Care is taken to obtain a random sample of the remainder.

The Enquiry has two main purposes:

(i) Administrative use - to provide information about the use of the hospital services in terms of the age, sex and other characteristics of patients, and also of the diseases and operations performed, for the purpose of central planning and to assist regional development and local supervision.

(ii) Epidemiological use - to provide information on a national

and regional basis about illness among hospital patients as a guide to morbidity occurrence in the community.

(Department of Health and Social Security, and Office of Population Censuses and Surveys, 1973.)

The form used to collect data for the Enquiry is illustrated in figure 1.2. It can be seen that victims of road traffic accidents can be identified (item 36, code 1). The brief medical details include the principal condition causing admission and the number of days spent in hospital.

The limitation with this source of data for studying traffic injuries is that no details at all are given about the type of accident or the class of casualty: item 38 is coded according to the nature of injury (N800-N999 of the ICD) not according to the external cause (E800-E999) whereas in the vital statistics discussed in the previous section both codes are used. It should also be noted that casualties dead on arrival at hospital are not included in the Enquiry since they are never a patient.

See Alderson (1974, p.41-47) for a discussion of HIPE. In section 1.3 certain trends in the proportions of different injuries and length of stay in hospital are examined for road accident victims.

1.2.4 Potential for record linkage

By "record linkage" is meant the bringing-together of data from different sources relating to the same event: here the possibility of using either the information about causes of death or that from HIPE

Use typewriter or ball point pen

HMA1 (I.P.) 2

Code no.

6° Unit no.

7° Surname 1 Mr. ☐ 2 Mrs/Miss ☐

8° First names

9° Home address
Tel no. LA

Identification sheet

2° Address from which admitted

3° Family doctor
Dr
Tel no.

4° Referred by (if not family doctor)

5° Next of kin
Relationship Tel no.
Name and address

10° Age (years) 11 Date of birth

12 Place of birth

13 Surname at birth 14 1 Single 4 Divorced 2 Married 5 Separated 3 Widowed 9 Other/NK ☐

15 Occupation of patient

16 Occupation of husband (if married woman) School (if child)

17 NHS no.

18° Religion

19 Category of patient 1 Normal NHS 3 Amenity 5 Conv. 2 Full paying 4 Pre-conv. 6 Staff ☐

20 Source of admission 1 Waiting list 3 Immediate 5 Born in hospital 8 Other-not immed. 2 Booked or planned 4 Transfer from other NHS hospital 9 Not known ☐

Consultant	Specialty/Department	Ward	21 Date on waiting list
22 under whom admitted	23	24	<input type="text"/>

26 to whom transferred	27	28	29 Date transferred
30 to whom transferred	31	32	<input type="text"/>

34 Disposal 1 Transfer to NHS hosp. not conv. 2 Transfer to conv. 6 Died-autopsy 8 Other ☐ 3 Patient's home 7 Died-no autopsy ☐ 35 Date discharged (or died)

36 Accident 1 R.T.A. 3 At work 9 Not applicable ☐ 37 Previous pregnancies (abortion cases only) No. of live births No. of still births No. of abortions

38 Diagnoses (a) Principal diagnosis (b) Underlying cause (c) Other relevant conditions or complications

39 Operation or special investigations 40 No. of visits to theatre 41 No. of days after admission before first operation 42 Date of first operation

Principal operation

Other operations or investigations

43 Research or other local use

Figure 1.2: form used to collect data for the Hospital In-Patient Enquiry

to supplement the Stats 19 data is briefly considered.

The following items of information, collected both on Stats 19 and by HIPE, should enable casualties in one to be matched with patients in the other: date, month, place, age, and sex of casualty. There are about 250 serious casualties per day on the roads. Matching for sex and roughly for age brings this number down to about 10, so if place can be identified to be an area of a million people (remember that for HIPE the place is known only by the hospital of admission, which may have a large catchment area), a high percentage of matches should be possible. A pilot project being carried out by the writer indicates that about 60% of the HIPE records can be matched with a casualty in the Stats 19 data. Many of those not matched are rejected because they are young men, of whom several may be injured in a city on any one day. If the hospital to which the casualty was taken was recorded on Stats 19, such matching would be very much easier.

Using the same items of information, it should be easier to match together Stats 19 and the Registrar-General's data about deaths, since there are fewer of these and also the type of casualty is given in the cause of death.

1.3 Recent trends in traffic injuries

Some basic trends in the numbers of accidents are included in the annual publication "Road accidents in Great Britain". Chart 1 of that for 1972 (Department of the Environment, 1974) shows that accidents are growing at a slower rate than traffic, and Chart 6 shows that car occupant casualties are growing the quickest, pedestrian casualties are steady, and casualties to riders of two-wheeled vehicles are falling. Other changes in accident rates over the years have been examined by Smeed (1972, 1974). These trends in Stats 19 data will not be our concern in this section. Instead, data from the two other sources described in sections 1.2.2 and 1.2.3 will be examined for trends in the nature of injuries received in road accidents. Further graphs are given in Hutchinson (1975c).

1.3.1 Causes of death

The Registrar-General's Reports for the years 1958-1972 have been examined, the proportions of each cause of death calculated for each year, and the trends in causes of death plotted, which in figures 1.3 to 1.9 are split up according to age and class of casualty. In the Registrar-General's Reports, cause of death is classified according to the abbreviated list A of the International Classification of Diseases, in which nature of injury is divided into 13 classes, AN138 to AN150, of which the most important in traffic deaths are the following:

ICD 'A' list number	Corresponding ICD 3-digit number
AN138 Fracture of skull	N800 - N804
AN139 Fracture of spine and trunk	N805 - N809
AN140 Fracture of limbs	N810 - N829
AN143 Intracranial injury, excluding skull fracture	N850 - N854
AN144 Internal injury of chest, abdomen, and pelvis	N860 - N869

However, for comparison with data from the Hospital In-Patient Enquiry, it should be noted that, notwithstanding the above, AN139 is not directly comparable with N805-N809, nor is AN144 directly comparable with N860-N869, since in 1970 (for instance) the following figures were reported:

RTA deaths, AN139: 818.

Estimated RTA hospital discharges, N805-N809: 6130

RTA deaths, AN144: 2010

Estimated RTA hospital discharges, N860-N869: 1170

Clearly some of the injuries that would be coded as N805-N809 in HIPE are coded as AN144 when death is certified. The explanation for this is probably to be found in the rules for classifying cause of death when more than one kind of injury in N800-N959 is mentioned and there is no clear indication as to which caused death. When this is so, there is a prescribed order of preference, in which internal injury (N860-N869) comes before fracture of face bones, spine, and trunk (N802, N805.2-N805.9, N806-N809).

As seen in figures 1.3 to 1.9, the common feature of causes of death to all classes and age groups is the predominance of skull

0-14 YEAR OLDS PEDESTRIAN INJURIES

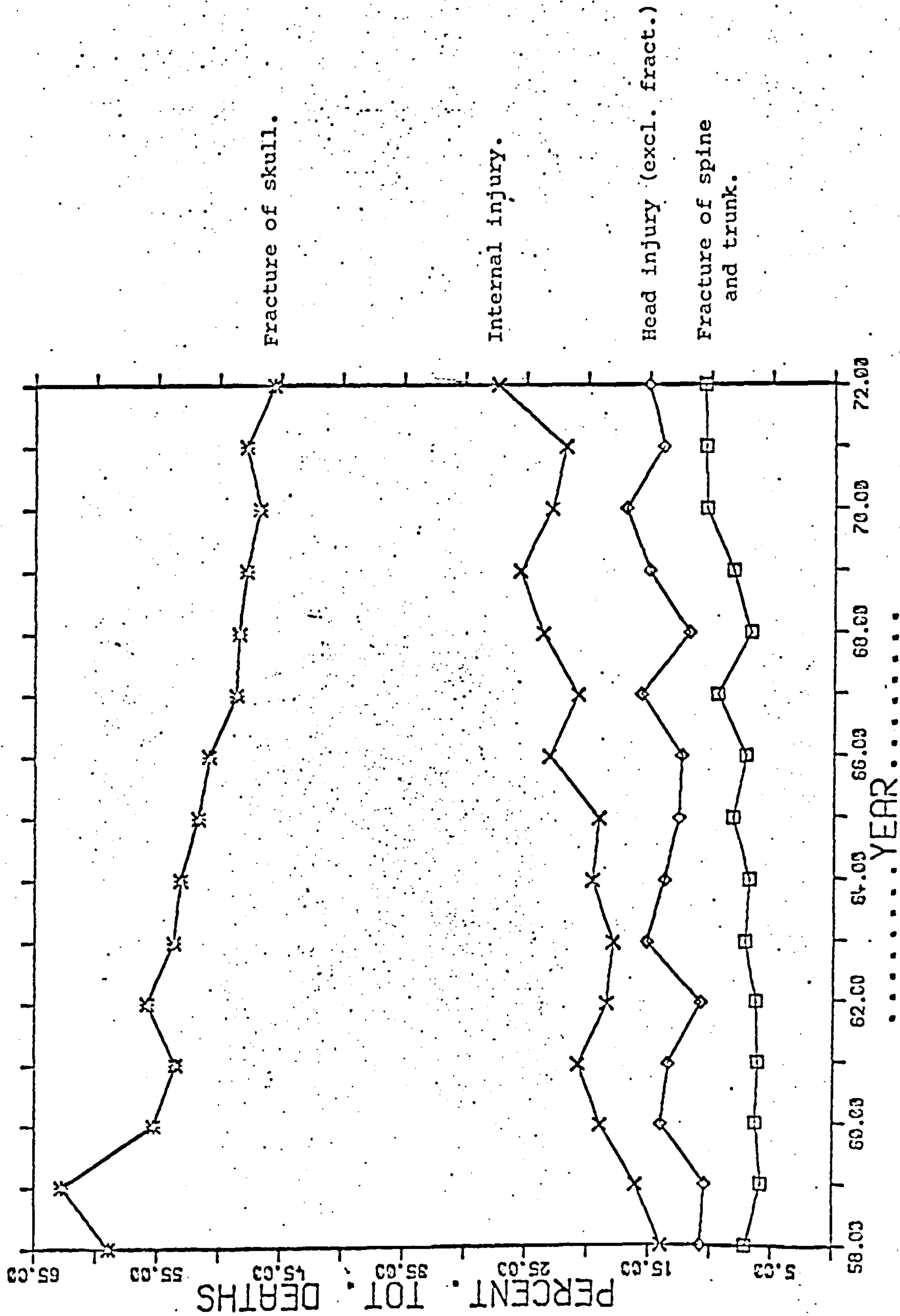


Figure 1.3: trends in cause of death, 1958-72, of child pedestrians killed in road accidents.

15-64 YEAR OLDS PEDESTRIAN INJURIES

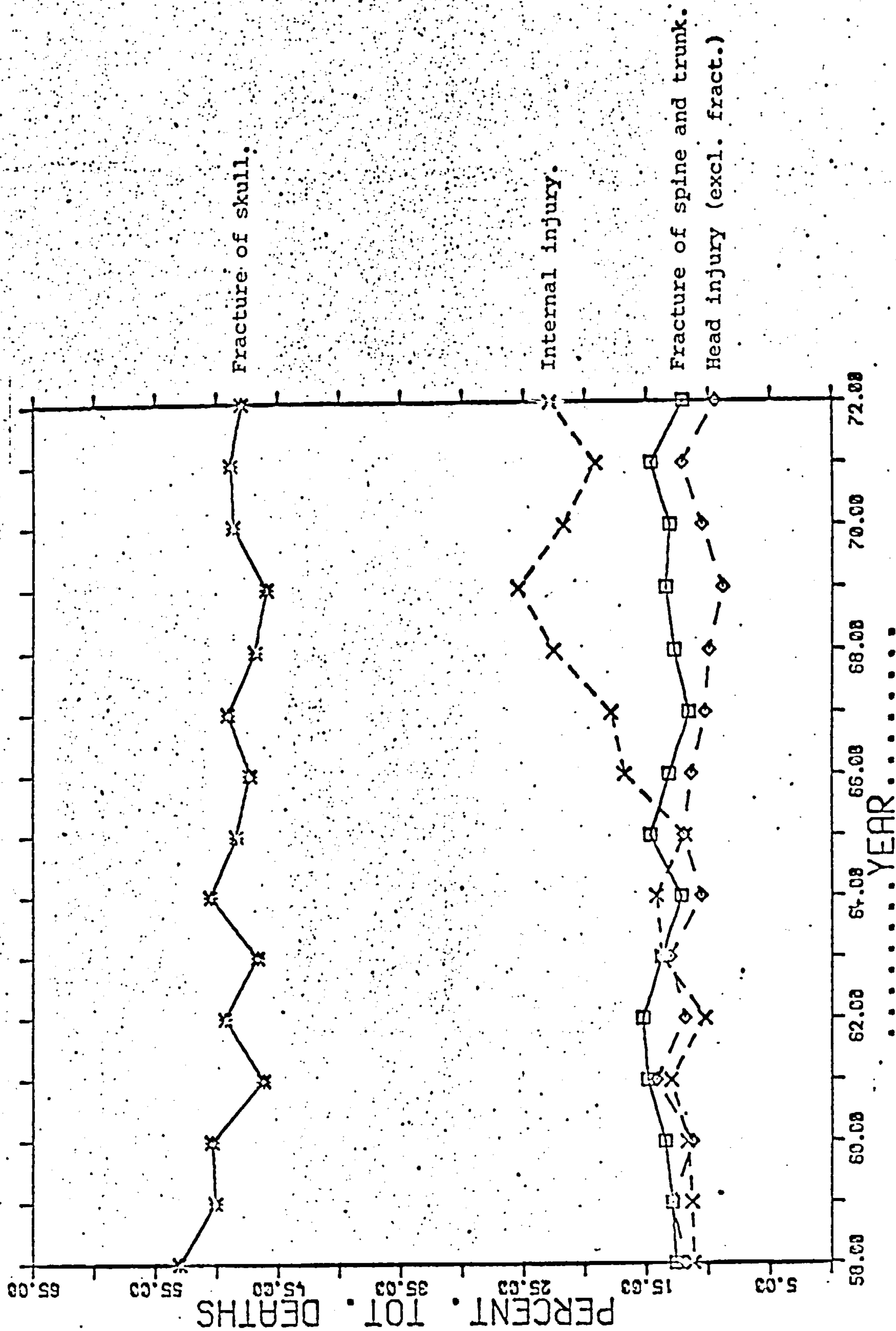


Figure 1.4: trends in causes of death, 1958-72, of adult pedestrians killed in road accidents.

65+ YEAR OLDS PEDESTRIAN INJURIES

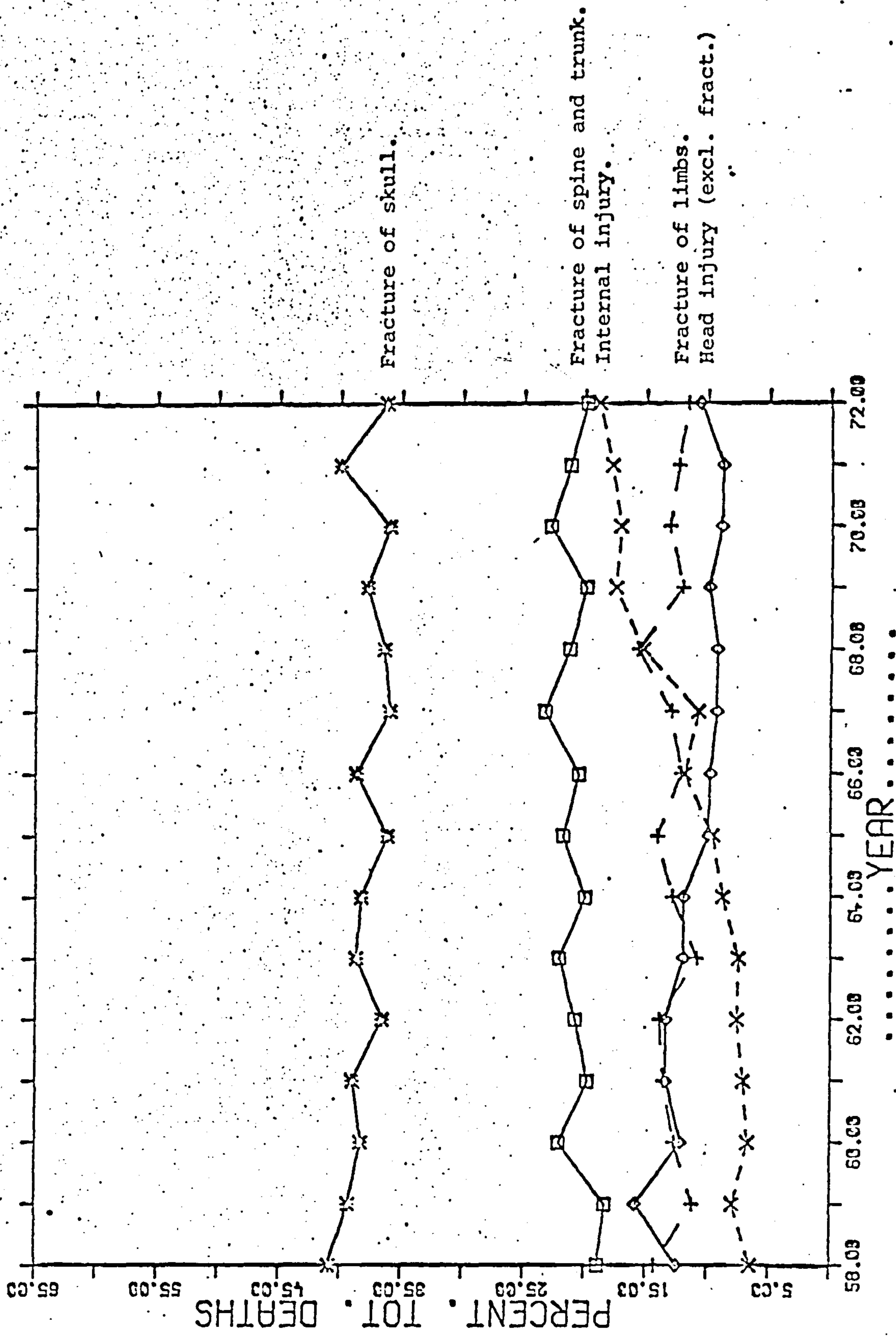


Figure 1.5: trends in causes of death, 1958-72, of elderly pedestrians killed in road accidents.

0-14 YEAR OLDS IN-VEHICLE INJURIES

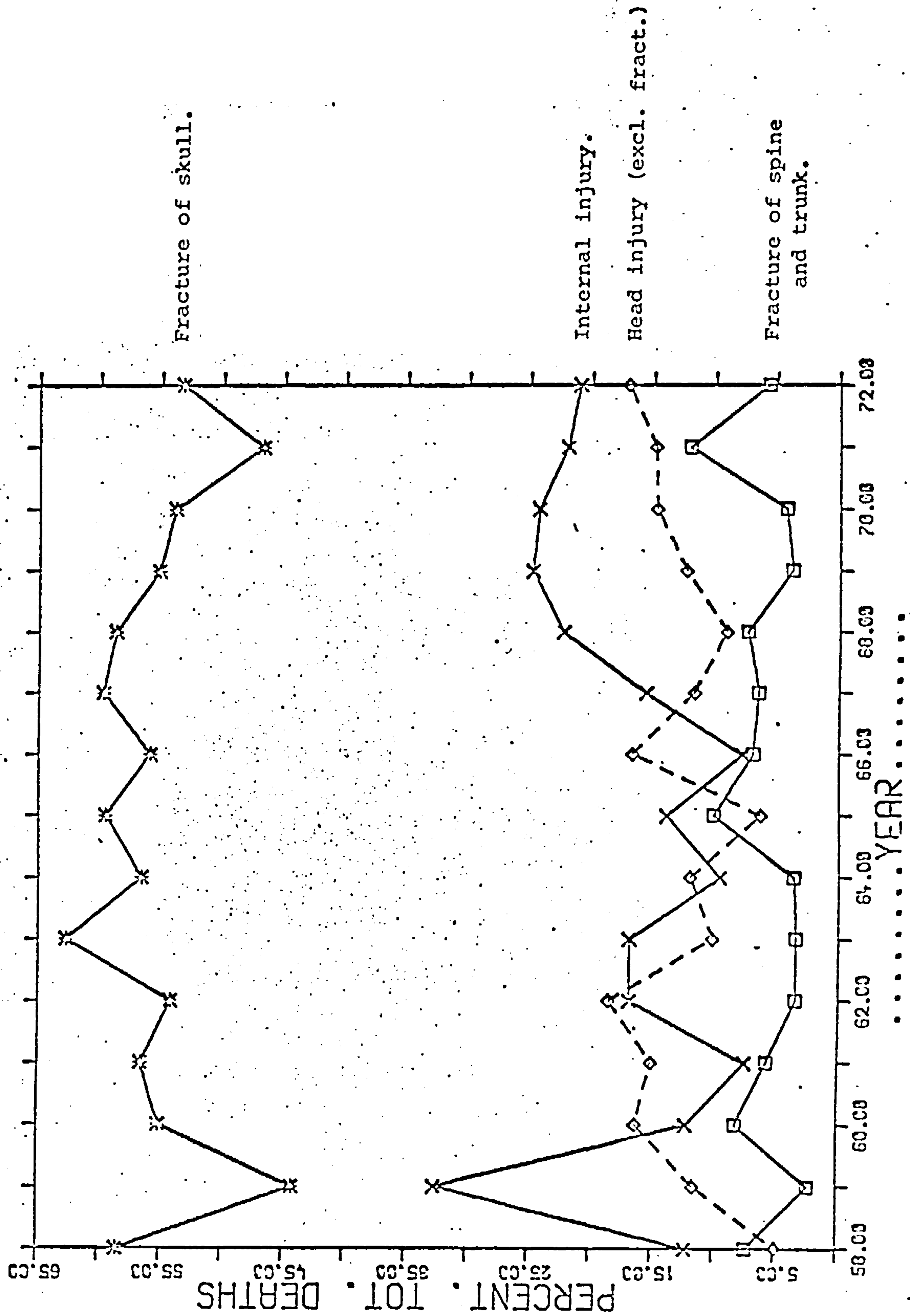


Figure 1.6: trends in cause of death, 1958-72, of child occupants of vehicles in road accidents.

15-64 YEAR OLDS IN-VEHICLE INJURIES

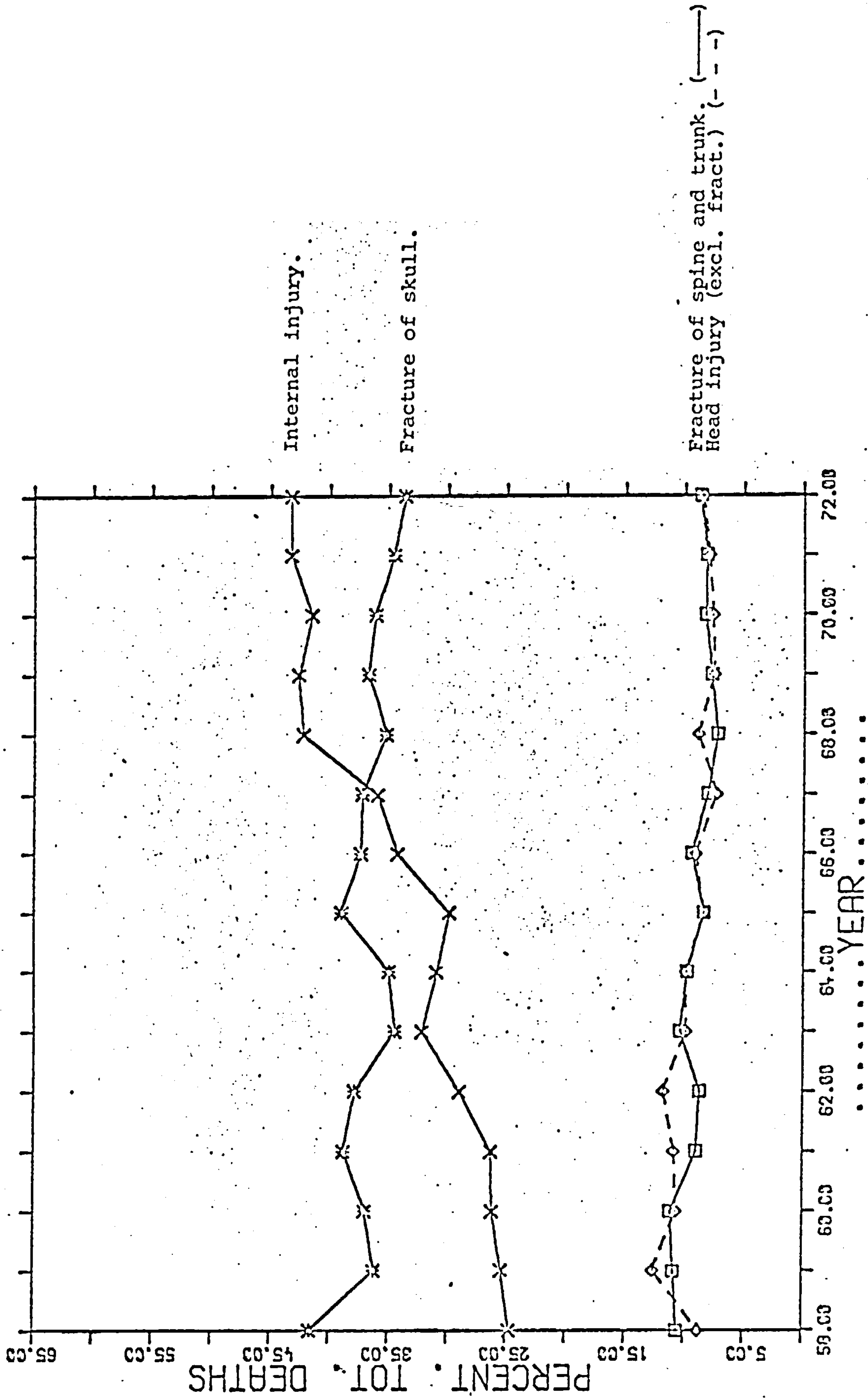


Figure 1.7: trends in causes of death, 1958-72, of adult occupants of vehicles killed in road accidents.

65+ YEAR OLDS IN-VEHICLE INJURIES

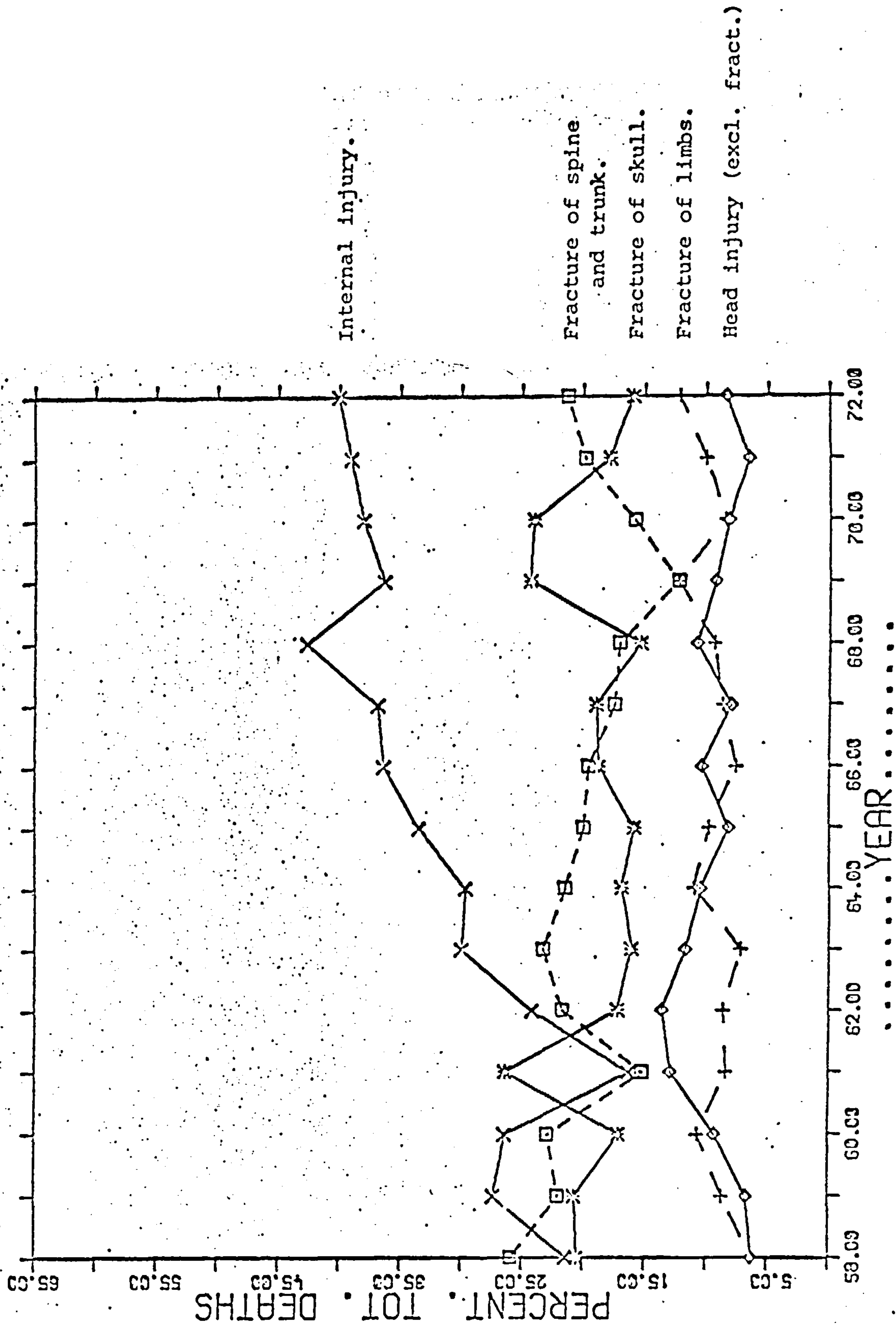


Figure 1.8: trends in causes of death, 1958-72, of elderly occupants of vehicles killed in road accidents.

15-64 YEAR OLDS MOTOR CYCLE INJURIES

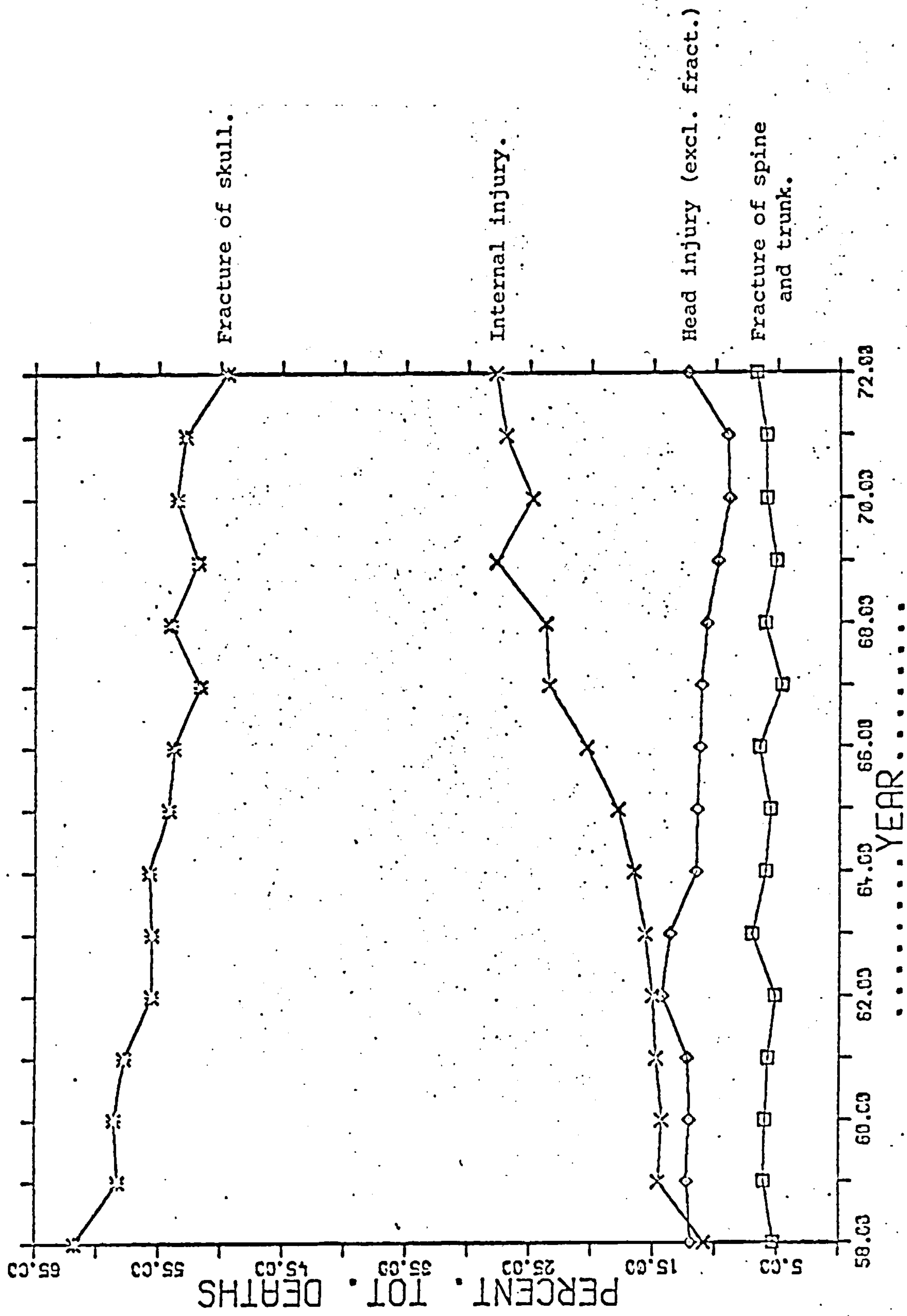


Figure 1.9: trends in causes of death, 1958-72, of adult motorcyclists killed in road accidents.

fracture, followed by internal injuries.

Among children, skull fracture is slightly more important and fracture of spine and trunk slightly less important as causes of death in vehicle occupants than in pedestrians. Among adults in the 15-64 age group, skull fracture is slightly less important and internal injury much more important as causes of death in vehicle occupants than in pedestrians. Motorcyclists are more similar to pedestrians than to vehicle occupants in the causes of their deaths. Among the elderly, as for the younger adults, skull fracture is less important and internal injury is more important in vehicle occupants than in pedestrians.

Comparing children with adults in the 15-64 age group, it may be seen that for pedestrians both internal injuries and non-fracture head injuries are slightly more important in children than in adults, whereas for vehicle occupants internal injuries are less important and both fracture and non-fracture head injuries are more important in children than in adults. For both pedestrians and vehicle occupants, skull fracture is less important, and fracture of the spine and trunk and fracture of the limbs are more important, in the elderly than in younger adults. (It is most unlikely that the elderly are more resistant to skull fracture than the younger: instead they are more likely to succumb to relatively minor injuries which a younger person would probably survive.)

As to trends, there is a consistent increase in the proportion of deaths ascribed to internal injuries in all categories of victim except possibly children in vehicles. Whether this is a real change reflecting the patterns of injury received or the efficacy of treatment, or whether there has been, for some reason, a gradual change in

diagnostic practice, is not known. For child pedestrians and (adult) motorcyclists, and possibly to a lesser extent for some other categories, there has been a decline in the proportion of deaths ascribed to skull fracture. Following the introduction in 1968 of the 8th Revision of the ICD, there was a fall in the proportion in AN150, "all other and unspecified effects of external causes". Thus whereas in 1967 5% of road accident deaths fell into this category, in 1968 only 1% did so.

1.3.2 Injuries requiring admission to hospital

The Reports of the Hospital In-Patient Enquiry for the years 1964-72 have been examined, and graphs relating to the numbers of discharges and the average length of stay for victims of road accidents have been prepared. This section is concerned with the former. Though of less interest than the causes of death, because the class of casualty (pedestrian, car occupant, etc) is not specified, these figures certainly serve to show the burden on the hospital service resulting from road accidents. Table 18 of the 1971 Enquiry shows there were an estimated 92000 discharges from hospital of victims of road accidents, and the average length of stay was 12 days. Since the total number of discharges for all conditions (excluding maternities) was 4 million, with an average length of stay of 15.5 days, casualties from road accidents accounted for 1.8% of patient-days.

Consideration needs to be given to what dependent variable to plot against time: among the more important variables affecting the number of hospital discharges for a particular injury resulting from a road accident are:

- genuine changes in the patterns of road accident injury, resulting from changes in vehicle design, for instance;
- changes in the total frequency of road accidents;
- changes in the definitions of injuries and diagnostic fashions;
- changes in hospital admission policies.

The effects of changes in road accident frequency may be eliminated by expressing each injury as a percentage of the total of RTA injuries.

If it is true that changes in injury definitions and hospital admission practices apply equally to injuries from other types of accident as to RTAs, these factors may be eliminated by using as our dependent variable the ratio of the proportion of RTAs that result in a particular injury to the proportion of all accidents that result in that injury. The disadvantage of using this ratio of proportions is that it is also affected by changes in the patterns of injury in home and industrial accidents. (Road accident injuries are generally less than one-third of the total. For concussion, as much as 40% of the total are RTAs, and it might have been better to use as the denominator the proportion of other accidents (i.e. excluding RTAs) that resulted in concussion, since even genuine changes in the frequency of concussion would be reduced in magnitude when the denominator is so substantially influenced by the number of RTAs.) Thus if n = number of RTAs with a particular injury, x = total number of RTA hospital discharges, y = number of all accidents with that injury, and z = total number of all hospital discharges for accident injury, the two dependent variables to be used are $\frac{n}{x}$ and $\frac{nz}{xy}$. In order that two injuries whose frequencies are changing at the same proportionate rate should give rise to parallel lines on the graphs, these quantities are plotted on a logarithmic scale.

Figure 1.10 is an example using the first of these dependent variables, n/x , for head injuries. These are by far the biggest group of RTA patients, and those suffering from intracranial injuries including concussion but excluding skull fractures accounted for 38% in 1964 and 42% in 1972. Skull fractures account for another 8% (1970) and face lacerations for 6% (1972). Leg fracture is the next most common group of injuries, being 18% of the total in 1972.

Since the lines lie above unity in figure 1.11, it is also clear that head injuries (especially concussion) are relatively more common in RTAs than in other types of accident. This is also true of internal injuries, but fractures of the neck of the femur, on the other hand, are very much more common in home accidents to elderly women than in RTAs.

Figure 1.12 shows how the distribution of injuries varies with age. The progressively increasing importance of leg fractures and declining importance of head injuries with age is evident. Figures 1.10 and 1.11 are not repeated for each age group separately both in order to save space and because this would be of only limited value in any case in the absence of classification according to type of road user.

Looking at figure 1.10 we see that concussion is declining but "other intracranial" is increasing. Is this a genuine change, or is diagnostic or coding practice changing? Referring to figure 1.11 we see there is a slight increase in both. Conclusion: there is a trend towards greater importance of intracranial injuries in RTAs. On top of this, it is either becoming more common to code such an injury as "other intracranial" rather than as concussion, or it is becoming more common to admit as in-patients victims with an intracranial injury.

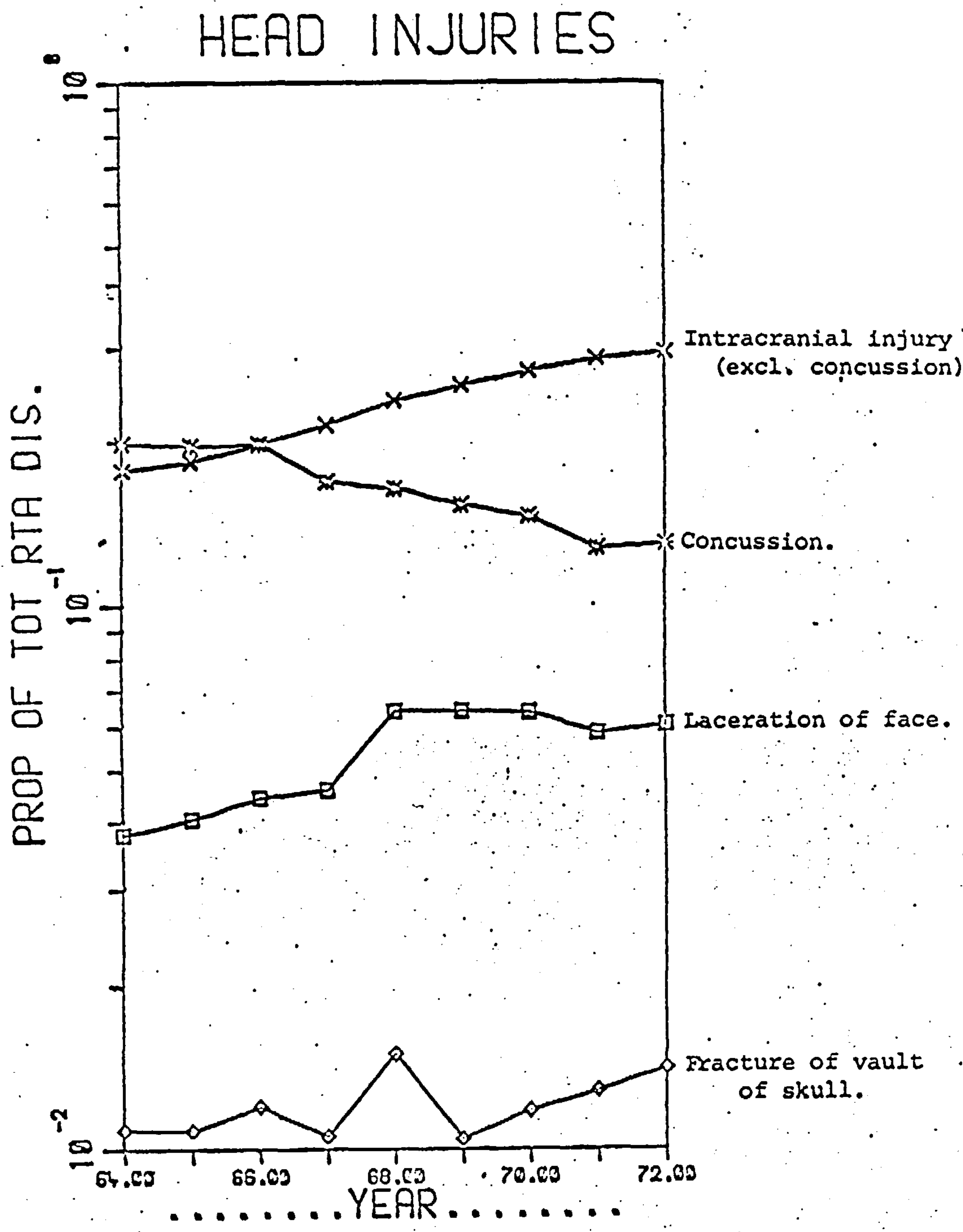


Figure 1.10: hospital admissions, 1964-72; trends in the proportion of RTA victims admitted because of head injuries.

HEAD INJURIES

39

PROP RTAS / PROP ALL

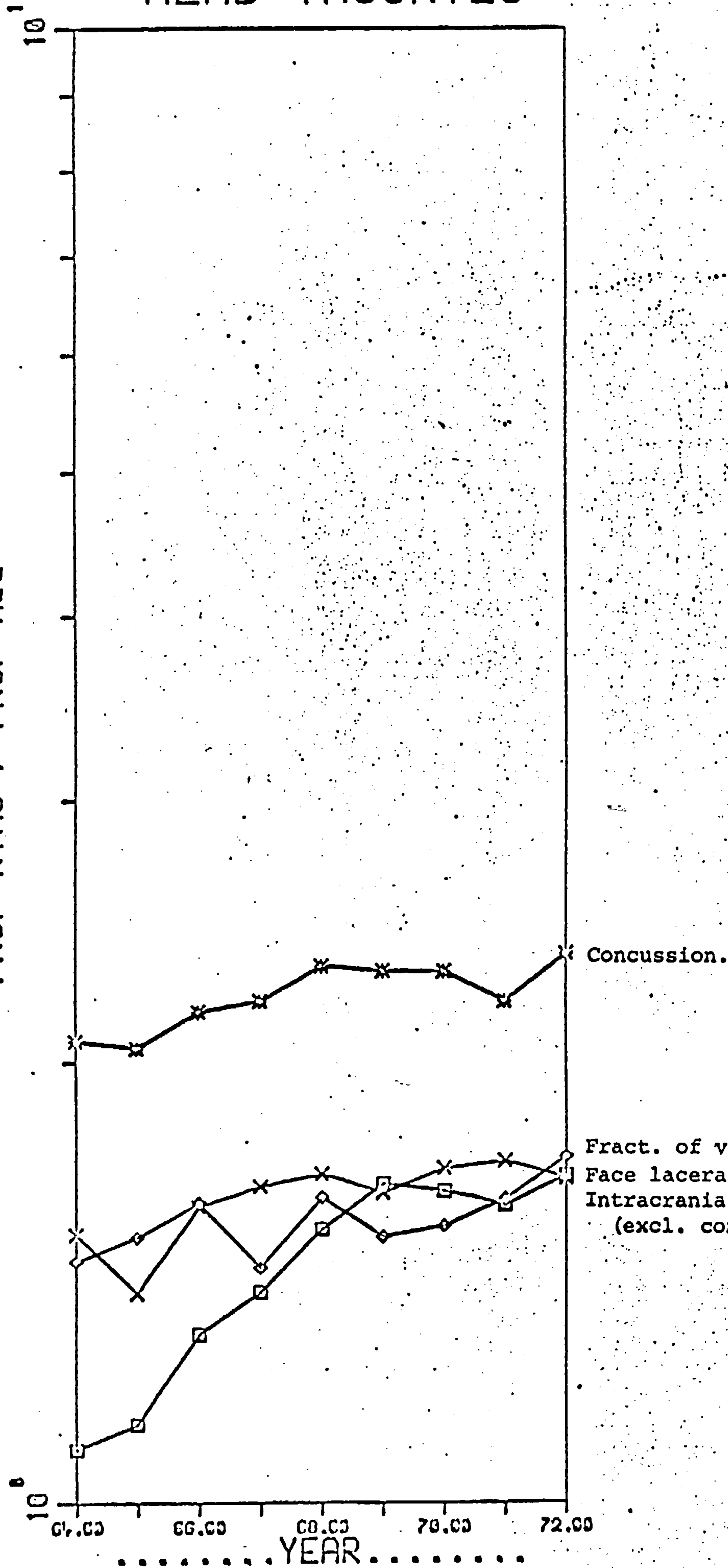


Figure 1.11: as figure 1.10, but expressed as the ratio.
(proportion in RTAs)/(proportion in all accidents).

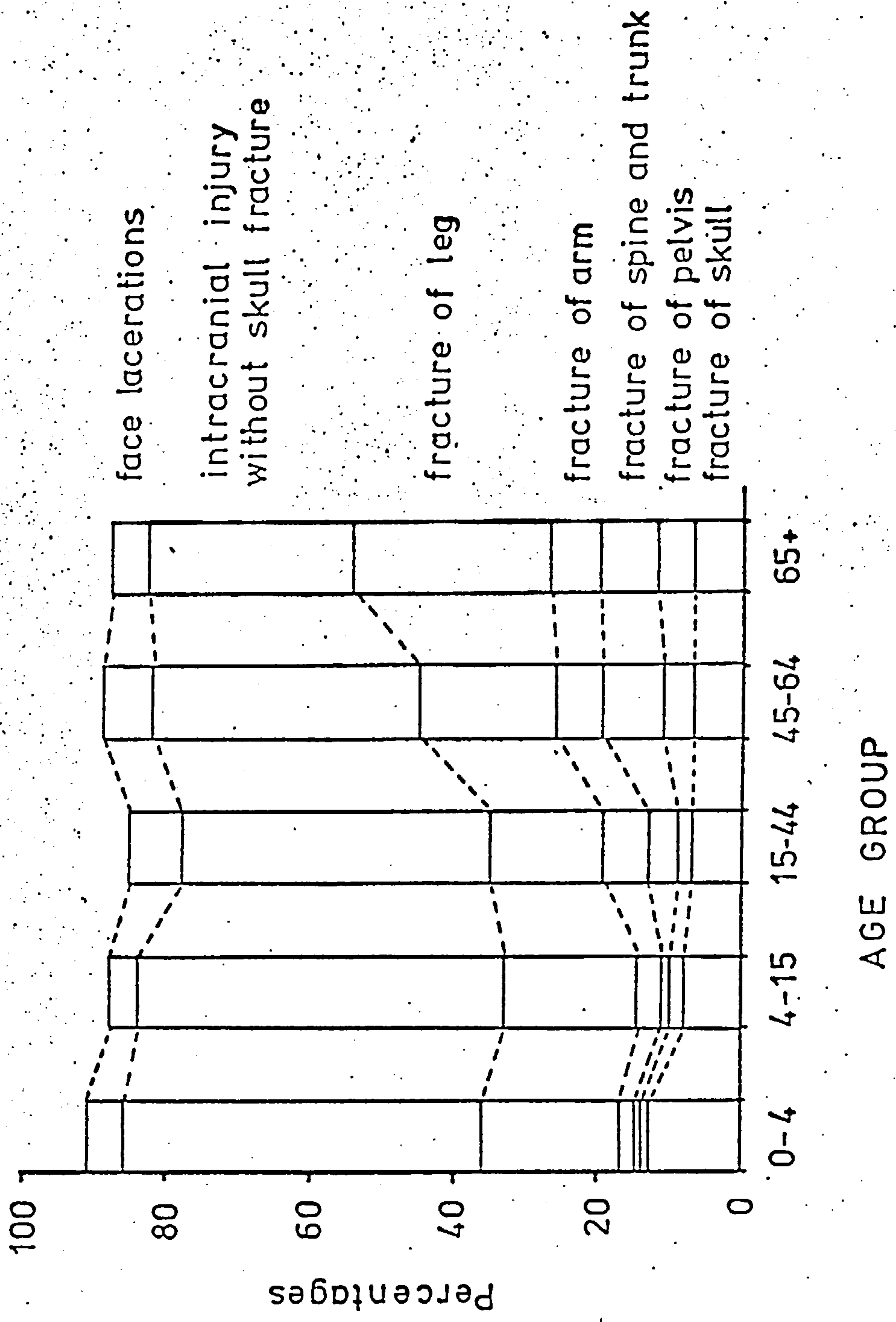


Figure 1.12: comparison of the injury distributions for different age groups (1972 HIPE, RTAs).

not involving concussion.

Is the increase in face lacerations that is seen in figure 1.10 due to changes in diagnosis (especially since much of the increase occurred in 1968, when the 8th Revision of the ICD was introduced)? No, because an upward trend is also seen in figure 1.11. It is noticeable that this trend appears to have levelled off since about 1969.

Hutchinson (1975c) concluded that there was a gradual improvement in the coding of injuries in HIPE (because of a decline in the proportion of injuries classified as "other adverse effects"), and that the following types of injury are becoming relatively more important in RTAs: several types of head injury, internal injuries, and "other and unspecified" fractures of the femur.

1.3.3 Length of stay in hospital

For reasons similar to those discussed in the previous section, as well as using the mean stay in hospital for a particular injury resulting from a road accident, the ratio (mean stay (RTAs))/(mean stay (all accidents)) is also used.

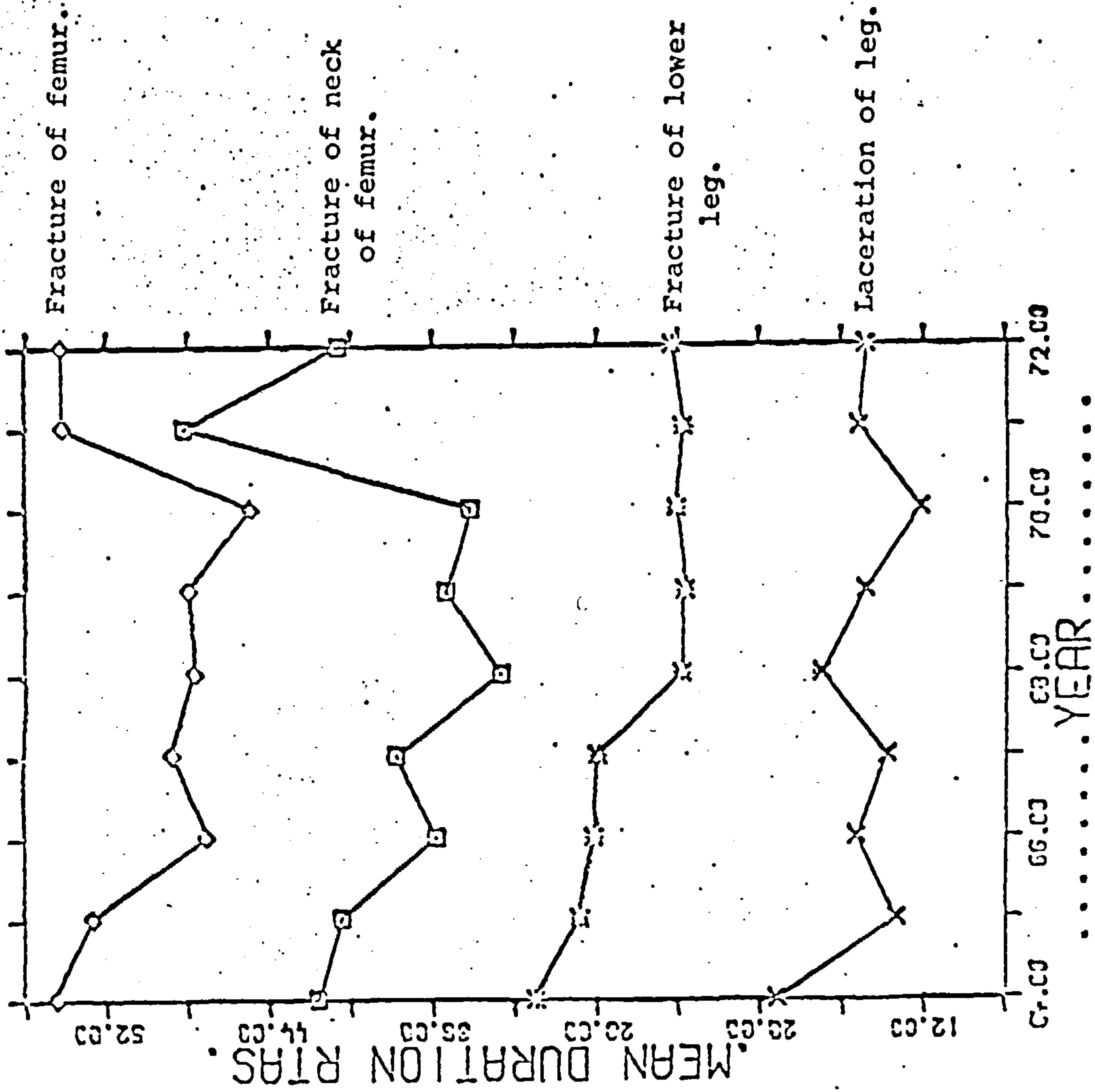
The injuries resulting in the longest time in hospital are fractures of the femur, the average being over six weeks for this injury (figure 1.13). Fracture of the lower leg and fracture of the pelvis come next.

As regards comparison with hospital stay following other types of accident, most types of head injury, fracture of the lower leg, fracture of the forearm, and internal injuries all lead to a longer

hospital stay on average when sustained in road accidents than in other types of accident (figure 1.14).

The large amount of year-to-year variation precludes firm conclusions, but Hutchinson (1975c) suggested that the average stay for fracture of the lower leg, for fracture of the pelvis, and possibly for internal injuries are all decreasing. The picture is somewhat different when we look at the behaviour of the ratio of average stay (RTAs) to average stay (all accidents). Whilst this is decreasing for fracture of the pelvis, there is some increase for fracture of the lower leg. In addition, there is some evidence of an increase for fractures of the femur, for laceration of the face, and for arm fractures. The reasons for this are obscure at present.

LEG INJURIES



HEAD INJURIES

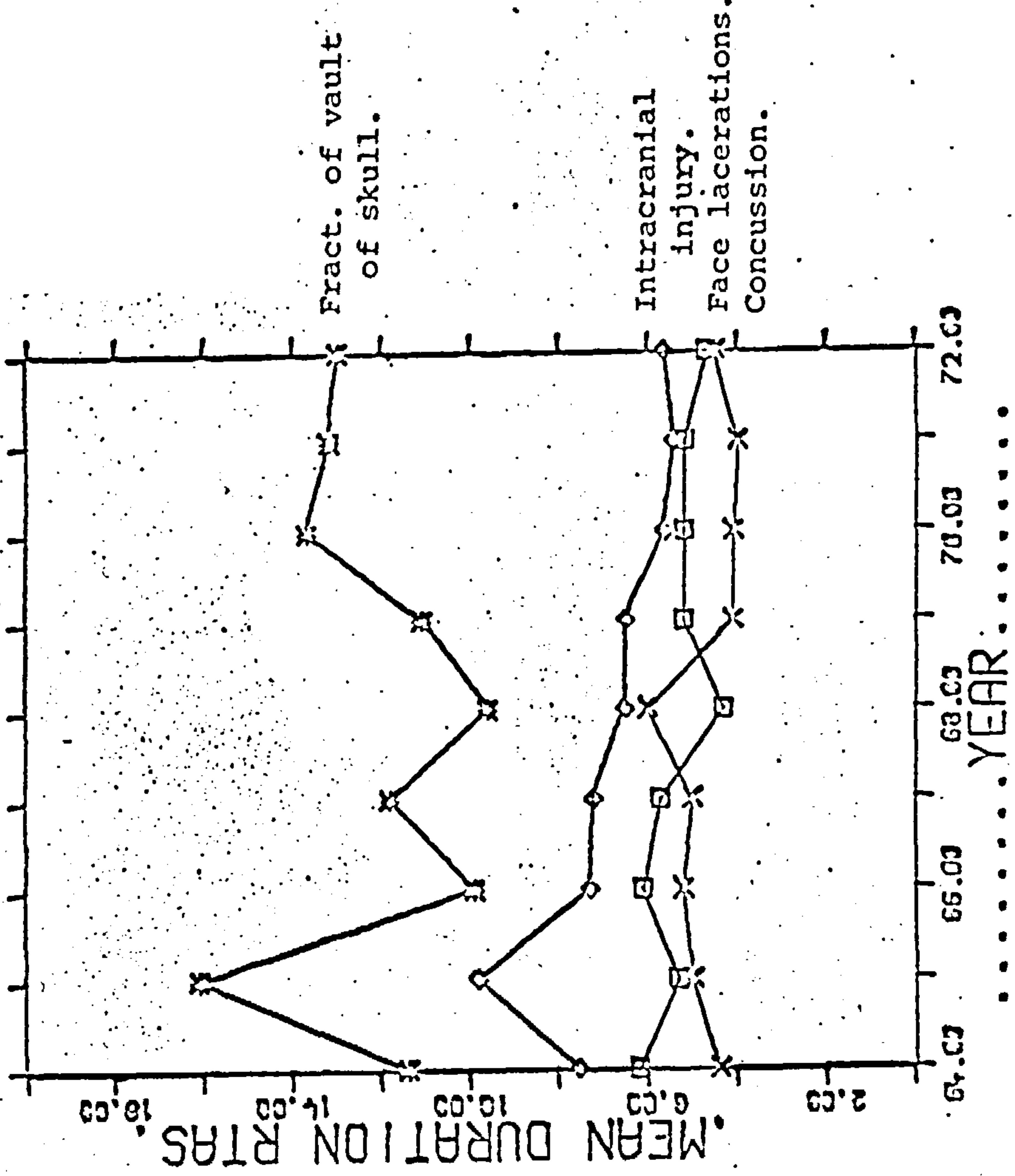
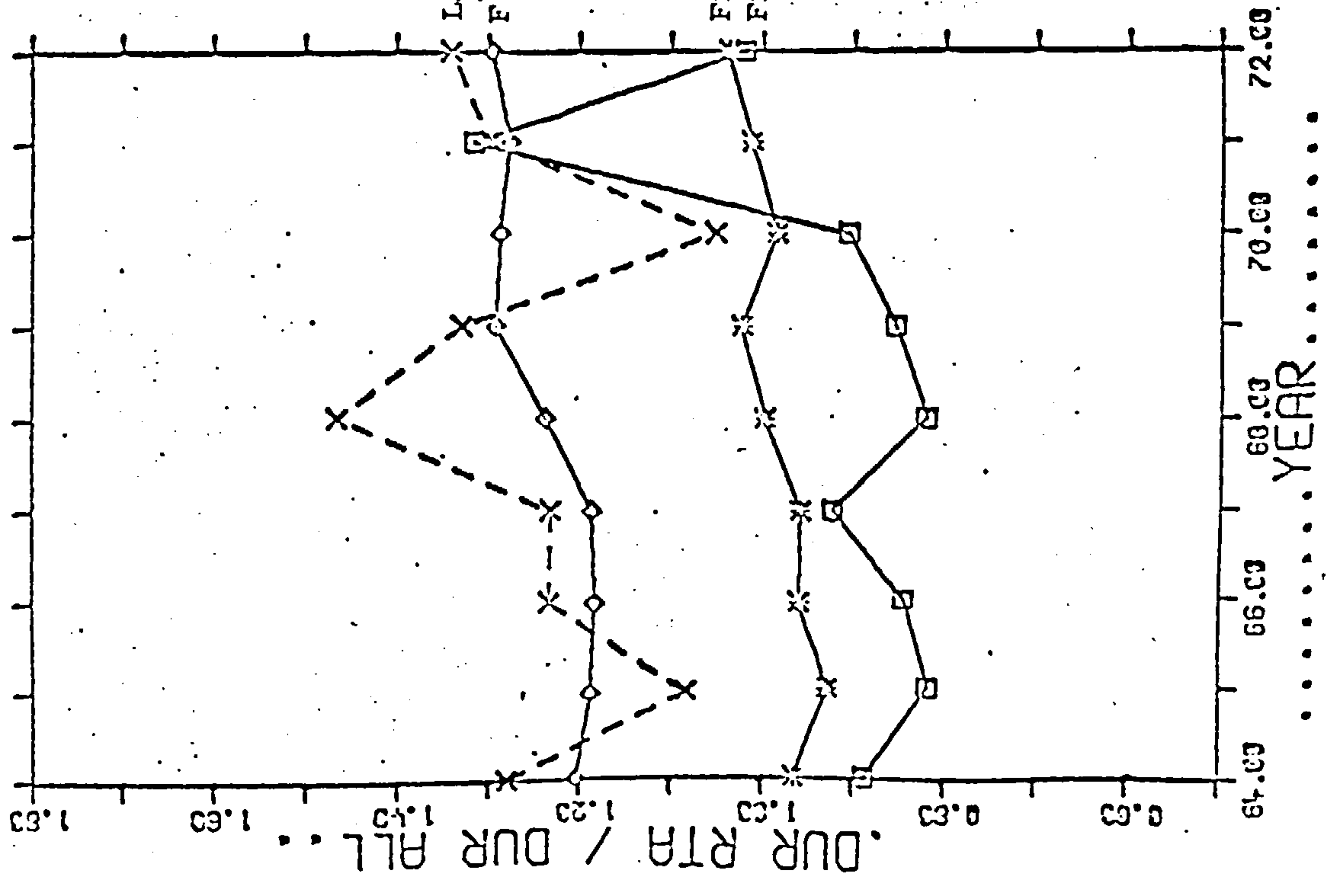


Figure 1.13: trends in the average length of stay in hospital for victims of RTAs with certain injuries.

LEG INJURIES



HEAD INJURIES

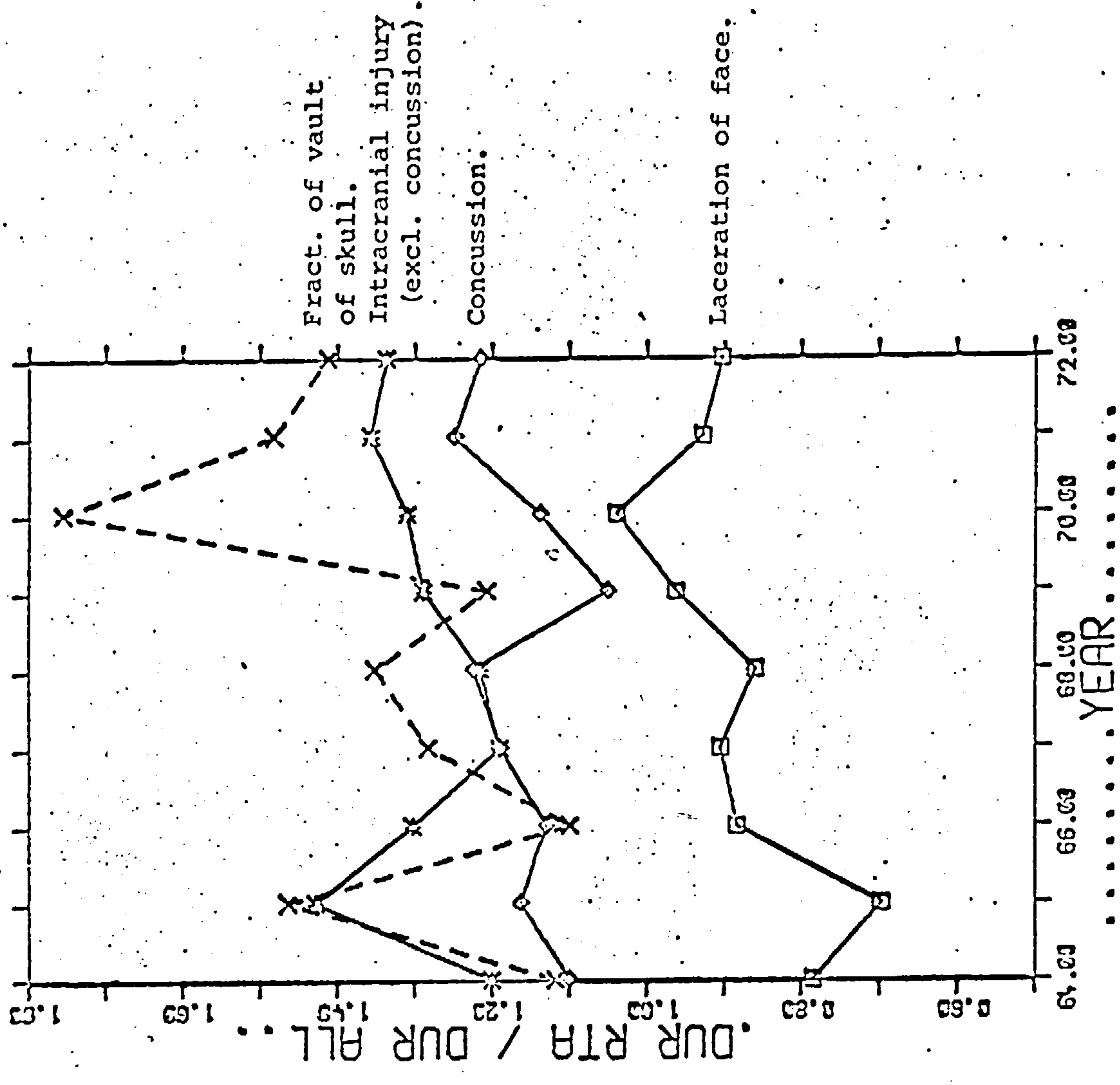


Figure 1.14: as figure 1.13, but using (mean duration (RTAs))/(mean duration (all accidents)) as the dependent variable.

CHAPTER 2: FACTORS AFFECTING THE SEVERITY OF PEDESTRIAN INJURIES

2.1 Introduction

In this Chapter, British accident statistics routinely collected by the police are used to examine factors which affect how severe a pedestrian's injury is when he is struck by a motor vehicle. The measure of severity used is the proportion of casualties classified as fatally or seriously injured. These classes combined will be referred to as 'severe'. Fatalities are not examined separately in order to retain clarity of presentation of results, and because there is theoretical and empirical reason to believe the proportion of fatalities is positively correlated with the proportion of serious injuries (see Chapter 7 of this thesis). Also, the numbers of fatalities in some cells of some of the tables would be very small.

Because of the differences in their circumstances, and the probable differences in the motion of the child and the adult when struck, accidents to children and adults have been analysed separately. The adults have been restricted to those in the age range 15 - 49 because it is well known that the elderly are more seriously injured (see table 2.1) than others, and the aim here is to discover factors which affect severity of injury by means other than through an association with pedestrian age. The data for child injuries is chiefly based on the year 1971, whereas that for adult injuries is for 1969-70. The reason for this discrepancy is that there were technical problems with the computer at the time these analyses were made. The geographical area considered is Great Britain. As has been noted earlier (section 1.2.1), one of the most important limitations with data such as that analysed here is the lack of information about the speed of the crash.

Some of the factors found in this Chapter to affect injury severity, however, very probably arise due to their association with speed.

<u>Age group</u>	<u>% Severely injured</u>
0 - 4	31
5 - 9	28
10 - 14	29
15 - 19	31
20 - 29	28
30 - 39	28
40 - 49	33
50 - 59	38
60 - 69	44
70 - 79	50
80+	55

Table 2.1: Effect of pedestrian's age on injury severity (impacts with cars, 1969-70; differences statistically significant*)

Section 2.2 presents some preliminary information on the relation of various aspects of time to injury severity - time of year, day of week, and time of day, section 2.3 examines some aspects of the environment, and 2.4 turns to the vehicle, which is probably the most hopeful area for injury countermeasures. A discussion and review section concludes the Chapter.

* Except where otherwise stated, statistical testing in this Chapter is by means of the usual χ^2 test for interaction in a 2 x n contingency table. Statistically significant means significant at least at the 5% level.

2.2 Some background data

In this section, only pedestrian accidents involving cars will be considered, as section 2.4 will show that type of vehicle has an important influence on injury severity: thus it is wise to remove this variable which might confound the effects of hours of day and day of week.

Table 2.2 shows that night-time accidents tend to be a good deal more severe than those occurring during the day, and this effect appears to be somewhat stronger for adults than for children. The chief cause of the greater severity at night is presumably higher speed at impact, which in turn will be because of at least three factors - lower density of traffic; higher proportion of drivers affected by alcohol; reduced visibility, hence less time to brake.

Accidents at weekends are more severe than those on weekdays (table 2.3), many of the severe accidents on Sundays occurring in the early hours (i.e. Saturday night). This is presumably again due to higher speeds. Some direct evidence that accidents at night and at weekends occur at higher speeds has been provided by Hutchinson and Satterthwaite (1974), though this relates to all types of accident, not just pedestrian ones. They used police reports of accidents in Oxfordshire to estimate the speeds of accidents, and found that those occurring between 22.30 and 00.30 had an average speed of 38 mph, whereas those between 07.30 and 18.30 had an average speed of 29 mph. The effect of time of day on speed was just statistically significant. As to day of week, they found that those on Sunday averaged 34 mph, whereas those on Monday to Friday averaged 30 mph. This difference, however, was not statistically significant.

As would be expected, month of year has a much smaller effect on accident severity than do hour of day and day of week, but table 2.4 shows there is a tendency (statistically significant at the 5% level) for accidents to adults to be more severe in winter. A possible reason for this is the greater proportion of darkness and bad weather resulting in poor braking.

<u>Hour of day</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
00-06	35	43
07	35	29
08	26	24
09	25	23
10	29	26
11	27	23
12	27	21
13	28	23
14	29	23
15	29	23
16	29	25
17	30	25
18	30	31
19	35	33
20	33	35
21	36	36
22	41	41
23	37	43

Table 2.2: Effect of time of day on injury severity, statistically significant both for children and adults. (Impacts with cars)

<u>Day of week</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
Sunday	32	37
Monday	30	29
Tuesday	29	28
Wednesday	29	29
Thursday	28	29
Friday	29	30
Saturday	30	33

Table 2.3: Effect of day of week on injury severity, statistically significant both for children and adults. (Car impacts)

<u>Month</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
January	29	32
February	29	32
March	30	29
April	30	30
May	31	28
June	30	30
July	29	30
August	28	30
September	29	30
October	29	32
November	29	31
December	30	32

Table 2.4: Effect of season of year on injury severity, statistically significant for adults but not for children. (Car impacts)

2.3 Environment

The tables in this section refer to accidents involving cars and occurring within a 30 mph speed limit zone, except for table 2.10 which compares the different speed limits.

Firstly, the weather and road surface. Tables 2.5 and 2.6 give the percentages severely injured according to weather and road surface respectively, and show there is not a great effect of either. Such differences as there are are not easy to interpret.

Turning now to the road and type of site where the accident happened, table 2.7 shows that dual carriageway roads have a slightly higher proportion of severe injuries than normal, and one-way streets a lower. Table 2.8 shows a (perhaps surprising) lack of effect of class of road on injury severity, and table 2.9 a slight reduction in severity at pedestrian crossings. A variable which might be expected to be rather strongly related to speed at impact and thus to pedestrian injury is the speed limit in force at the place of the accident. Table 2.10 confirms this.

<u>Weather</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
Raining	28	30
Snowing	33	27
Fog	24	32
Other	28	27

Table 2.5: Effect of weather on injury severity, statistically significant for adults but not for children. (Car impacts, 30 mph zone)

<u>Road surface condition</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
Dry	28	26
Wet	29	31
Snow or ice	28	26

Table 2.6: Effect of road surface condition on injury severity, statistically significant for adults but not for children. (Car impacts, 30 mph zone)

<u>Type of road</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
one-way	24	21
dual carriageway	32	33
other	28	28

Table 2.7: Effect of road type on injury severity, statistically significant in both cases. (Car impacts, 30 mph zone)

<u>Class of road</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
A, motorway standard	29	35
A	29	28
B	29	29
C, or unclassified	28	29

Table 2.8: There is no statistically significant effect of class of road on injury severity.
(Car impacts, 30 mph zone)

<u>Type of site</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
Pedestrian crossing, manually controlled	21	25
Pedestrian crossing, light controlled at junction	21	25
Pedestrian crossing, light controlled not at junction	30	22
Pedestrian crossing, uncontrolled	24	25
Elsewhere	28	28

Table 2.9: Showing a tendency for accidents at pedestrian crossings to be less severe than those elsewhere.
(Car impacts, 30 mph zone)

<u>Speed limit</u>	<u>% Severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
30	28	28
40	43	45
50-60	53	62
70	53	55

Table 2.10: Effect of speed limit on injury severity, statistically significant in both cases.
(Car impacts)

2.4 The vehicle

Before coming on to vehicle design, three tables will be given relating to its driver and its manoeuvres at the time of the accident. As can be seen from tables 2.11 and 2.12, young and inexperienced drivers tend to injure pedestrians more severely than do other drivers. Why this should be so is a matter of speculation: recklessness, lack of experience in reacting to a potentially dangerous situation, and an association with older vehicles having poorer brakes are three possibilities. Table 2.13 shows that manoeuvres associated with low speed are also associated with a low severity of accident.

The effect of vehicle design on pedestrian injury is of great interest, since this is a factor which can be controlled. As has already been remarked, one big difficulty with the data analysed here is that there is no indication of the speed of impact. However, it seems plausible that within certain classes of vehicle (e.g. the common models of private car) impact speeds on minor roads with a 30 mph speed limit will not differ. McLean (1972) expresses a similar opinion, and Fisher and Hall (1972) found it was true in their sample of accidents. Whether any particular difference in injury severity can more plausibly be attributed to a difference in speed or to a difference in design will be discussed at the appropriate time. Because of the likely difference in the motion of pedestrians of different sizes struck by vehicles, injuries to children and adults will be discussed separately, and the former will be split into three age groups.

<u>Driver age group</u>	<u>% of pedestrians severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
≤ 19	35	37
20-24	29	32
25-28	30	27
29-34	27	26
35-54	27	25
55-64	26	27
65+	27	28

Table 2.11: Showing a tendency for young drivers to injure pedestrians more severely than do older drivers. (Car impacts, 30 mph zone)

<u>Licence held</u>	<u>% of pedestrians severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
Provisional (i.e. a learner driver)	35	31
Other	28	28

Table 2.12: Influence of driver status on injury severity, statistically significant only for child pedestrians. (Car impacts, 30 mph zone)

<u>Manoeuvres</u>	<u>% severely injured</u>	
	<u>children (0-14)</u>	<u>adults (15-49)</u>
	(1971)	(1969-70)
Starting, stopping, or held up	14	16
Turning left or right or round	16	15
Overtaking (including over- taking held-up traffic)	29	28
Other	29	31

Table 2.13: Showing that vehicle manoeuvres associated with low speed are also associated with low severity of injury. (Car impacts, 30 mph zone)

2.4.1 Children

Table 2.14 shows that type of vehicle has a significant effect on injury; this table applies to children of all ages. Table 2.15 gives the percentages in each of 3 age groups who were severely injured by the different types of vehicles and it appears that older children tend to be injured less severely than do the two younger age groups. But, in passing, it should be mentioned that it is not certain that 'severity' has the same meaning for adolescents as for 5-year olds: for instance, hospital admission policies may be different for different ages, and admission to hospital is one criterion used by police for classifying injury severity.

Table 2.16 and 2.17 go into more detail with respect to size of vehicle. Table 2.16 examines the relation of injury severity to engine capacity for two-wheeled vehicles, and table 2.17 relates injury severity to the unladen weight of goods vehicles. I am indebted to Mr. G. Grime for pointing out that in many cases it seems to be the maximum load that is recorded, instead of the unladen weight. These are highly correlated, though, so the qualitative conclusion stands. It seems more likely that increased severity with increased size of goods vehicle is genuinely due to differences in design and hence different kinematics of impact than that larger vehicles have higher speeds. (Mass per se is unlikely to be the important factor because of the great disparity in size between even the lightest goods vehicle and a child.) But there are other possibilities, such as differences in braking performances.

The statistical significance levels quoted in tables 2.16 and 2.17 apply to the usual χ^2 test for contingency tables, but they are unchanged when a test for the significance of a linear regression of

<u>Type of vehicle</u>	<u>Severe</u>	<u>Slight</u>
Moped	12 (22%)	42
Motor-scooter	78 (30%)	179
Motor-cycle	120 (32%)	251
Car	3176 (28%)	8351
Public Service vehicle	98 (32%)	213
Light goods	505 (29%)	1219
Medium goods	71 (37%)	119
Heavy goods	89 (45%)	108

Table 2.14: Numbers of children injured tabulated according to degree of injury and type of vehicle involved. Statistically significant, $P < .001$. (ages 0-14, minor roads, 30 mph limit)

<u>Type of vehicle</u>	<u>Age group</u>		
	<u>3-6</u>	<u>7-10</u>	<u>11-14</u>
Moped	31	6	20
Motor-scooter	30	35	25
Motor-cycle	33	33	32
Car	28	27	30
Public Service vehicle	32	41	27
Light goods	30	28	29
Medium goods	33	44	22
Heavy goods	50	39	35

Table 2.15: Percentages of children in different age groups severely injured, according to type of vehicle. The numbers in the last column tend to be less than those in the other columns, and, using Friedman's test, this is significant, $P < .01$. (ages 3-14, minor roads, 30 mph limit)

<u>Cylinder Capacity</u>	<u>Severely injured</u>
cc	
≤ 50	22
50-150	31
150-250	35
> 250	30

Table 2.16: Relating engine capacity of two-wheeled vehicles to severity of injury. Marginal statistical significance, P = 0.1. (ages 3-14, two-wheeled vehicles, minor roads, 30 mph limit)

<u>Unladen weight</u>	<u>% Severely injured</u>
≤ 12 cwt	26
12-16 cwt	30
16 cwt-1 ton	33
1 - 1½ tons	37
1½ - 2 tons	25
2 - 3 tons	36
3 - 4½ tons	50
> 4½ tons	46

Table 2.17: Relating size of goods vehicle to severity of injury. Statistically significant, P < .001. (ages 3-14, goods vehicles, minor roads, 30 mph limit)

proportion severely injured on the row variable is made i.e. when the ordered nature of the row variable is taken into account, see Cochran (1954).

We now concentrate on the car (the preponderant involvement of this type of vehicle can be seen from table 2.14) and compare different models with respect to the injury they produce. In this section, accidents to children under three years of age are excluded from consideration because of the likelihood that they differ from other ages in the circumstances and kinematics of impact. In the two years 1970-1971 there were 11 models of car to which more than 500 accidents involving child pedestrians aged 3-14 and occurring on minor roads in a 30 mph speed limit occurred. The numbers of children slightly and severely injured by them are given in table 2.18; all models except two are of fairly conventional design, but model A has an exceptionally short and low (but square) bonnet and model K has a sloping bonnet.

A number of points may be made about table 2.18. Firstly, the proportions of children severely injured do differ significantly between models.

Secondly, the differences are numerically not very great (the range of proportion severely injured is from 25% to 32%) nor is the level of statistical significance attained very high.

Thirdly, the low percentage severely injured with model K cannot be taken entirely at face value. This is because for model K the make only is coded (not the model; a very high proportion of cars of this make are the same model) whereas for the other models normally both the make and model are coded, and in a few cases the model is coded as

unknown. It is likely that the proportion of cases for which the model is unknown will be greater for slight accidents than for severe ones, so for all the above models except K the proportion of accidents which are severe may be slightly inflated owing to the absence of some slight accidents involving that model but for which the model was coded as unknown. It was thought worth checking this by considering table 2.19, which compares the make of car included in the previous table, grouping all models (including cases where the model was unknown) within each make. Model K remains the make with the lowest severity of injury.

Fourthly, it could be argued that different age groups of children should be considered separately, as the different sizes and behaviours of children of different ages may lead to significantly different types of impact. When these eleven models of car were compared for severity of injury to children in the three age groups 3-6, 7-10, and 11-14 separately, the only age group for which a significant difference was found was the 3-6 year olds, for whom the proportion severely injured ranged from 21% for model K and 23% for model F to 31% for models G and I and 32% for model C. This is shown in table 2.20.

Fifthly, it is noteworthy that although it is model K that stands out in tables 2.18 and 2.19 as having a low proportion of severely injured, this make has a relatively low frequency of involvement in accidents; thus omitting it from the tables and testing for a difference in injury severity among the remaining 10 models (in table 2.19) makes no difference to the level of statistical significance attained, namely, the 5% level in the case of table 2.18 and the 10% for table 2.19.

Sixthly, although the desirability of restricting accidents to those occurring in a 30 mph speed limit will be readily recognised, it

<u>Model</u>	<u>Severe</u>	<u>Slight</u>
A	392 (27%)	1052
B	369 (27%)	1001
C	280 (32%)	599
D	315 (27%)	896
E	522 (27%)	1390
F	166 (25%)	505
G	238 (31%)	539
H	185 (27%)	513
I	232 (30%)	552
J	353 (29%)	867
K	129 (25%)	377

Table 2.18: Relating model of car to severity of child pedestrian injury. Statistically significant, $P < .05$. (ages 3-14, minor roads, 30 mph limit, years 1970-71)

<u>Make</u>	<u>Severe</u>	<u>Slight</u>
1	879 (28%)	2256
2	1547 (27%)	4206
3	504 (30%)	1165
4	728 (28%)	1846
5	706 (29%)	1722
6	129 (25%)	377

Table 2.19: Relating make of car to severity of injury. Marginal statistical significance, $P < .1$. (ages 3-14, minor roads, 30 mph limit, years 1970-71) (Make 6 is equivalent to model K in table 2.18.)

is more debatable whether the further restriction to minor roads only is justified. And it happens that the results are very similar whether or not this restriction is imposed: there is a strong positive correlation between the proportion of children severely injured by a particular model on minor roads and on other roads - for the eleven models in table 2.18 this correlation is 0.71 which is all the more impressive because of the small range of variation between models and further confirms the reality of the differences between models. Table 2.21 corresponds to table 2.18 except that it applies to accidents on all roads with a 30 mph speed limit: owing to the larger numbers the level of statistical significance attained is higher, but otherwise the tables are very similar.

Turning now to a comparison of two special classes of private cars - three-wheelers, and sports cars - with the rest, table 2.22 shows, as one might expect, that injuries resulting from being struck by a three-wheeler are less severe, and from a sports car more severe, than usual. The differences in severity are probably due to difference in speed, though of course both three-wheelers and sports cars differ considerably in frontal design from the norm.

A difference in speed is also probably the explanation for increased severity of injury associated with a more powerful and sporty version of model A (table 2.23). Model A is an example of a car of which several versions exist that differ slightly with regard to engine and styling. As well as the common model A, there is a 'souped-up' version of it (model A') and models A_1 and A_2 which differ in styling. Table 2.23 compares these four variants. Testing the sporty versions versus the standard model (rows 1 and 2) a difference significant at the 5% level is found; testing the standard version versus A_1 and A_2

<u>Model</u>	<u>Severe</u>	<u>Slight</u>
A	186 (27%)	512
B	204 (28%)	531
C	145 (32%)	302
D	164 (29%)	400
E	266 (29%)	666
F	77 (23%)	254
G	120 (31%)	266
H	101 (28%)	265
I	124 (31%)	279
J	175 (28%)	447
K	50 (21%)	190

Table 2.20: Relating model of car to severity of injury of young children struck. Statistically significant, $P < .05$. (ages 3-6, minor roads, 30 mph limit, years 1970-71)

<u>Model</u>	<u>Severe</u>	<u>Slight</u>
A	961 (29%)	2403
B	946 (28%)	2399
C	654 (31%)	1424
D	750 (27%)	2008
E	1278 (27%)	3391
F	415 (24%)	1326
G	542 (29%)	1320
H	424 (26%)	1220
I	553 (29%)	1337
J	900 (31%)	1997
K	329 (26%)	950

Table 2.21: As table 2.19, relating model of car to severity of child pedestrian injury, but not restricting the accidents to those occurring on minor roads. Statistically significant, $P < .001$. (ages 3-14, all roads, 30 mph limit, years 1970-71)

<u>Type of car</u>	<u>Severe</u>	<u>Slight</u>
Three-wheeled	18 (19%)	78
Sports	47 (34%)	91
Others	2939 (27%)	7258

Table 2.22: Numbers of children injured, tabulated according to degree of injury by type of car which struck them. Statistically significant, $P < .05$. (ages 3-14, cars, minor roads, 30 mph limit, year 1971)

<u>Variant of model A</u>	<u>Severe</u>	<u>Slight</u>
A	392 (27%)	1052
A'	25 (40%)	38
A ₁	10 (24%)	31
A ₂	9 (24%)	28

Table 2.23: Severity of injury to children struck by different variants of model A. (ages 3-14, model A, minor roads, 30 mph limit, years 1970-71)

<u>Make of car</u>	<u>Severe</u>	<u>Slight</u>
7	9 (19%)	39
8	41 (24%)	129
9	4 (14%)	25
10	136 (31%)	298

Table 2.24: Severity of injury to children struck by four of the more powerful, luxury, makes of car. Statistically significant, $P < .05$. (ages 3-14, minor roads, 30 mph limit, years 1970-71)

(rows 1, 3, and 4) no significant difference is found.

A comparison was also made of four of the more powerful makes of car (table 2.24). As three of these give less than average severity and one greater than average, there would seem to be no strong association between luxury cars and injury. It is interesting, though, that these makes of car differ among themselves in severity produced.

2.4.2 Adults

Table 2.25 compares different types of vehicle with respect to the proportion of adult pedestrians severely injured, and table 2.26 goes into more detail for two-wheeled vehicles. An ordinary χ^2 test on this tables gives $\chi^2 = 4.4$ which, with three degrees of freedom, is not statistically significant; but when the test is strengthened by taking into account the ordered nature of the classification of engine size (Cochran, 1954), the evidence for a linear dependance of proportion severely injured on motorcycle engine size is of marginal statistical significance ($P < .1$). Evidence of increasing injury severity with increasing size of goods vehicle is given in table 2.27. The statistical significance of this table is $P = .01$ for the ordinary χ^2 test; when the test takes into account the ordered nature of vehicle size, the significance level drops to $P = .05$, because of the irregular nature of the correlation.

In the years 1969-72 there were six models of car each of which were involved in at least 500 accidents with pedestrians aged 15-49 on minor roads in the 30 mph limit zone. Results for these six models are given in table 2.28. The differences between them are statistically significant at the 5% level, but are numerically small. And nor is

<u>Type of vehicle</u>	<u>Severe</u>	<u>Slight</u>
Moped	16 (26%)	45
Motor-scooter	126 (33%)	260
Motor-cycle	173 (34%)	335
Car	1601 (29%)	3996
Public Service vehicle	107 (25%)	327
Light goods	202 (23%)	677
Medium goods	26 (17%)	126
Heavy goods	89 (33%)	179

Table 2.25: Numbers of adult pedestrians severely and slightly injured by different types of vehicle. (ages 15-49, minor roads, 30 mph zone)

<u>Size of engine</u>	<u>% Severely injured</u>
≤ 50 cc	29
50-150 cc	34
150-250 cc	35
> 250 cc	43

Table 2.26: Showing some tendency for more powerful two-wheeled vehicles to give rise to more severe pedestrian injuries. (two-wheeled vehicles, minor roads, 30 mph zone, ages 15-49)

<u>Unladen weight</u>	<u>% severely injured</u>
≤ 12 cwt	26
12-16 cwt	23
16 cwt - 1 ton	24
1 - 1½ tons	22
1½ - 2 tons	13
2 - 3 tons	18
3 - 4½ tons	34
> 4½ tons	36

Table 2.27: Showing increasing injury with increasing size of goods vehicle. (goods vehicles, minor roads, 30 mph zone, ages 15-49)

<u>Model of car</u>	<u>Severe</u>	<u>Slight</u>
A	298 (32%)	643
B	171 (27%)	472
C	148 (27%)	398
D	207 (32%)	450
E	267 (27%)	734
J	161 (31%)	357

Table 2.28: Numbers of adult pedestrians severely and slightly injured by the six models of car involved in the most pedestrian accidents. (minor roads, 30 mph limit, years 1969-72, ages 15-49)

model K (the one with the sloping front), which severely injured 32% of the pedestrians it struck, exceptional.

Table 2.29 compares three-wheelers and sports cars with the others, and the same trend as in table 2.22 is evident.

Since we have no information on the speeds of the accidents, the accidents included in table 2.28 were restricted to those occurring in a 30 mph limit zone - the assumption being that in built-up areas differences between models in their speeds will be negligible, thus enabling us to attribute differences in injury severity to differences in model design. For the same reason the accidents were further restricted to those occurring on minor roads. But in view of the finding in table 2.8 that, within the 30 mph limit, severity of pedestrian accidents is not associated with type of road, this further restriction may be thought unduly cautious. Accordingly, table 2.30 compares pedestrian injury severity in accidents involving different models of car, on all classes of road where a 30 mph limit is in force. There were 20 models of car with at least 500 accidents in this category in the years 1969-72. Comparison of the relative numbers of pedestrians severely and slightly injured by all 20 models in table 2.30 gives $\chi^2 = 33.3$, 19 d.f., which is statistically significant at the 5% level. Comparison of the six models involved in most accidents (those six included in table 2.28) gives $\chi^2 = 12.6$, 5 d.f., again significant at the 5% level. These results confirm those of table 2.28, that there are differences between models, but they are of quite small magnitude. Consistent differences between models should imply that those models that are most injurious in one year should also be most injurious the next, etc. And, indeed, correlating the proportions severely injured in 1971-72 gives a correlation coefficient of 0.43, which is of marginal statistical

<u>Type of car</u>	<u>% Severely injured</u>
Three-wheeled	20
Sports	33
Others	28

Table 2.29: Comparison of special types of car with the rest. (minor roads, 30 mph limit, years 1969-72, ages 15-49)

<u>Model</u>	<u>Severe</u>	<u>Slight</u>
A	1110 (30%)	2645
B	762 (28%)	1989
L	163 (30%)	382
C	591 (30%)	1364
M	211 (28%)	546
D	807 (29%)	1961
N	237 (32%)	503
O	181 (32%)	382
E	1191 (27%)	3153
P	287 (29%)	711
Q	385 (29%)	930
R	165 (30%)	388
F	394 (27%)	1078
G	463 (31%)	1023
S	216 (27%)	583
H	421 (29%)	1028
T	307 (27%)	846
I	469 (29%)	1129
J	655 (31%)	1476
K	360 (27%)	964

Table 2.30: Numbers of adult pedestrians severely and slightly injured by those models of cars involved in over 500 accidents of this type. (all roads, 30 mph zone, years 1969-72, ages 15-49)

significance ($P < 10\%$).

Several British Leyland models are produced under two names - Austin and Morris. They are virtually identical and thus should not differ in the injury they produce (and thus their results have been combined in tables 2.28 and 2.30). On the other hand, if some unknown methodological factor is producing the differences between models in those tables, it might be expected to reveal itself in differences between corresponding Austin and Morris models. It is some comfort that no such differences have been found; the Austin and Morris Minis were compared, as were the 1100s, and the 1800s, and the Austin Cambridge with the Morris Oxford.

2.5 Review

Previous papers on the relation between vehicle design and pedestrian injury fall into three main groups - those concerned with describing the collision mathematically using the laws of dynamics; crash testing, driving a vehicle into an anthropomorphic dummy fitted with accelerometers and filming its movements; and those which use data from actual accidents to compare the injury produced by different types of vehicle. A selection of papers from each of these categories will now be briefly summarised, and then it will be shown how the present study complements them.

2.5.1 Simulation

One approach to understanding the factors which influence pedestrian injury is to use the laws of dynamics to construct a model of the behaviour of a pedestrian when impacted. The effects of variations in vehicle design are then predicted by evaluating the results of changing the corresponding parameters of the model.

(i) Culkowski et al (1971) "Research in impact protection for pedestrians and cyclists". Appendix K - "Equations of motion for the two-dimensional pedestrian-vehicle impact simulation" - gives the mathematics of a two-dimensional pedestrian-vehicle simulation. The pedestrian is represented by three articulated rigid body segments (and thus has a total of five degrees of freedom in two-dimensional motion) with elastically deformable elliptic outlines. The vehicle side-view periphery is divided into five sections, each of which may have different properties to represent varying structural characteristics. Impact forces are derived from geometric interference between

the pedestrian and vehicle or pedestrian and ground.

The results are described in section 3.1 and in Segal (1969). They indicate: as expected, a strong influence of impact speed on the accelerations sustained by the pedestrian; an important influence of vehicle stiffness - the softer the vehicle, the more mild the impact; that a vehicle with a sloping front leads to a less severe impact than does one with a square front; and that the presence or absence of vehicle braking has an important influence on the trajectory of the pedestrian. The detailed design of the vehicle front was found to have an even more important influence on the motion of a child pedestrian than it did on that of an adult.

(ii) Glockner (1973) "Simulation of a collision between a motor vehicle and a pedestrian". In this study, the pedestrian has three degrees of freedom (two translational and one rotational). Both vehicle mass and vehicle contours are found to affect the likely injury sustained by the pedestrian, rounded car fronts being less dangerous.

(iii) Jamieson et al (1971) and Schmidt and Nagel (1971) give simple analyses of the motion of a pedestrian when struck. The former study (which neglected road-shoe friction) concluded that contact between the bumper bar and the lower leg of the pedestrian is unimportant in imparting significant motion to the pedestrian, and continues "Gross angulation (of the ends of the broken bones) implies that the bumper bar has literally 'driven through' the limb and hence the body has not significantly rotated away as a result of bumper bar contact". The Cornell workers seem to disagree with this - Segal (1969) says "Because of its large mass and moment of inertia, the motion of the leg has a substantial influence on the kinematics of the entire

pedestrian body and so the initial impact delivered to the leg is important not only in its potential localised injury, but also in its effects on the whole body motion." Culkowski et al (1971, p 123) reiterate this view.

Schmidt and Nagel (1971) use a simple two-dimensional model to reconstruct an impact at over 40 mph. It indicated that the pedestrian received an acceleration of over 170g. His survival is attributed to the blow being inflicted over a large area of a strong portion of his body, the head sustaining hardly any injuries at all.

(iv) Other work. See also Young, Lammert, and Ross (1974), Katayama and Shimada (1970), Bartz and Butler (1972), Tarriere et al (1974), Gagnon et al (1974), and McLaughlin and Daniel (1974).

2.5.2 Collision testing

An alternative approach is to drive a vehicle into an instrumented dummy which measures the accelerations sustained by different parts of the body. Work of this kind has been conducted in California and in Japan. And workers at Cornell and in Texas conducted series of component tests, which will be described first.

(i) Culkowski et al (1971) "Research in impact protection for pedestrians and cyclists" (Section 3.2). Rather than carry out their own program of full-scale testing, these workers impacted simulated pedestrian heads into automobile components; the head was chosen because of the seriousness of pedestrian head injuries, and the vehicle components tested were the top and the front edge of the bonnet. Wooden spheres of variable mass and with an accelerometer mounted

inside represented the head. Unfortunately, due to the limited height of the pendulum device used to project the spheres into the vehicle components, the impact speeds available were limited to below 16 mph, and consequently the accelerations recorded would probably not have been injurious. Nevertheless, the study did establish the practicality of this method, and with a slightly more realistic head it could provide a relatively cheap and easy way of evaluating the effect of vehicle design on the initial head impact.

(ii) Ross et al (1974) "Drop tests of dummies on a mock vehicle exterior". A series of controlled tests were conducted by dropping instrumental anthropomorphic dummies on an idealised vehicle exterior. High speed cameras were used to record the dummy's kinematics and nine accelerometers measured accelerations at various body locations. The primary objective was to obtain a data base from which the TTICVS (Texas Transportation Institute's Collision Victim Simulator) could be validated in the pedestrian mode. A secondary objective was to obtain a measure of the effectiveness of a relatively soft vehicle exterior in minimising pedestrian-vehicle impact severity. It was found that a 6 inch layer of polyurethane foam did not reduce the severity of impact appreciably (as compared with impacts with conventional cars).

(iii) Severy (1970) "Vehicle exterior safety". Among the more important conclusions from this carefully-planned series of experiments in which vehicles were driven into instrumented anthropometric dummies were the following: compared with the pedestrian weight, the weight of the striking car is not a significant factor, but the pedestrian height to car-profile relationship determines, in large measure, the struck pedestrian's movements and injury exposure. Thus wedge-shaped front-ends increase upward projection of the pedestrian and the subsequent

injury potential from striking the ground, whereas blunt front-ends decrease upward projection of the pedestrian but have the disadvantage of providing higher initial head and chest impact forces. Collision accelerations for a pedestrian struck by a car travelling at 20 mph are more severe than those sustained by unbelted motorists within cars colliding head-on, each travelling at 30 mph. In one experiment the conventional sheet-metal fender was replaced by one of crushable fibre-glass foam plastic; there was no reduction in the accelerations to head or chest.

(iv) Japan Automobile Manufacturers' Association (1968)

"Experiments on the behaviour of a pedestrian in collision with a motor vehicle". Three models of car were used in this series of experiments (conducted at speeds of up to 40 km/h), each in both a conventional and a modified form - Nissan Cedric (2000 cc class), Isuzu Bellet (1000 cc class), and Honda N360 (360 cc). The modified versions were - Nissan: (a) shock-absorbing bumper, (b) specially-designed (tapered) front end; Isuzu: shock-absorbing bumper; Honda: shock-absorbing bonnet. Considering peak accelerations at the initial impact (for dummies of adult size), the modifications to the Nissan and the Isuzu succeeded in reducing accelerations of the legs, but had no, or even an adverse, effect on the head accelerations; there was some indication that the shock-absorbing bonnet of the Honda did reduce head accelerations by about 15%. It is doubtful whether accelerations at the ground impact were sufficiently reproducible to draw conclusions, but for the Isuzu there was a considerable increase in peak head acceleration when the modified vehicle was tested. For the child dummy, the results were fairly similar, except that both with the Nissan with the tapered front, and with the Isuzu with the shock-absorbing bumper, there was some indication of a reduction in peak head acceleration.

Some experiments on the behaviour of a dummy when struck by a heavy truck with or without a guard-rail were also conducted. They confirm the reduced probability of runover.

(v) Taneda, Kondo and Higuchi (1973) "Experiment on passenger car and pedestrian dummy collision". This paper reports the behaviour of a dummy when struck by three models of differing frontal design - Volkswagen Beetle (low, sloping nose), Publica 790 cc (fairly low front, bonnet composed of two planes sloping towards the windscreen, and Bluebird 1300 cc (comparatively high front, horizontal bonnet). For the Volkswagen, impact to the pelvis is slight; severity of head impact depends strongly on speed: at 20 km/h and less, it strikes the bonnet which has good energy-absorbing properties and can deform greatly (this was also clear from films of the Cornell component experiments), but at 30 km/h and over the head strikes the bonnet near where it meets the windscreen or the windscreen itself and these structures give rise to a much more severe impact. Impact to the pelvis is slight also with the Publica, but the short bonnet of this car means that the disadvantageous impact of the head with the upper bonnet or windscreen occurs at lower speeds than with the Volkswagen. The higher front of the Bluebird leads to a much larger impact with the pelvis than for the other two models, but the head is more likely to strike the relatively advantageous bonnet.

The attitude of the pedestrian when he strikes the ground is of great importance, and is also affected by front-end design. The models with low fronts appeared to be advantageous here.

2.5.3 Accident studies

The desirability of eliminating pointed projections and smoothing sharp edges is well-known: Wakeland (1961) reports a case in which a boy died from puncture of the heart after running into the tail fin of an American car. But only a small minority of pedestrian injuries are of this local type (McCarroll et al, 1962; Fisher and Hall, 1972) and the association of injury with larger-scale variations in vehicle design is more difficult.

(i) Fisher and Hall (1972) "The influence of car frontal design on pedestrian accident trauma". Included in this Australian study was a comparison of the severity and distribution of injuries to pedestrians struck by three models of car of differing frontal profile: Ford Falcon and Fairmont (combined; high, square front), Morris Mini and 1100 (combined; low, square front), and the Volkswagen Beetle (low, sloping front). The results were: significant differences in fatality rates between the models (deaths over-represented for Beetles and under-represented for Fords) but no significant differences in injuries per accident; some tendency for a higher proportion of head injuries with Beetles and a low proportion with the Fords. Estimated speed of collision was highly correlated with both injury per accident and fatality per accident, and the distributions of speeds for the different models were similar, the average being just over 20 mph.

(ii) McLean (1972) "Car shape and pedestrian injury". This study used data from New York State, and compared pedestrian injuries after being struck by a Volkswagen or by a Cadillac. (The Cadillac was chosen as the representative of conventional frontal design because all models of Cadillac have similar fronts.) It is especially valuable because

both pedestrian age and estimated impact speed were taken into account in the analysis. Although the differences between the two models were not statistically significant, McLean interprets them as true differences, with the Cadillac being the more dangerous. Such differences as there were arose largely from differences in the proportion of fatalities, and the estimated chance of being killed (as opposed to injured) was twice as great for the Cadillac as for the Volkswagen.

(iii) Other work. Other studies which have commented on pedestrian injuries after impact with cars of different frontal design are Mackay (1965, 1972), Ryan and McLean (1966), Robertson et al (1966), Jamieson et al (1971), and Culkowski et al (1971).

2.5.4 Discussion

To summarise the literature reviewed above: both simulation and crash testing indicate that a car with a low, sloping front may be less injurious to pedestrians than one of conventional box shape, at low speeds of impact; this is not necessarily the case at high speeds, since with low fronts there may be an increased tendency for the pedestrian to strike his head on the windscreen or its supports, or even to go cart-wheeling over the top of the car. Accident studies have not yet confirmed or disproved these conclusions.

McLean's analysis explicitly took account of the estimated impact speed of each accident; and though Fisher and Hall apparently did not do this, they were able to say that the average impact speed was virtually the same for the models they compared. In the present study, no information on speed was available, and consequently our conclusions must be cautious. But it does seem unlikely that in built-up areas the

common models of car should differ in average speed, and consequently we can attribute the statistically significant differences shown earlier in this Chapter to differences in design. On the other hand, by using this routinely-collected information, the sample sizes in the present study are an order of magnitude larger than those of McLean or Fisher and Hall.

What are the reasons for the differences evident in our data? This we cannot answer as yet, but a possible approach for further research would be to correlate the percentage severely injured with various parameters of car design: parameters of the geometric design such as bonnet height, bonnet width (because it may affect the proportion of pedestrians who receive a glancing blow rather than a full one), and bonnet length (because it may affect the proportion of pedestrians striking the windscreen); measures of speed or power such as maximum speed, maximum acceleration, and power to weight ratio; the stiffness of the bodywork; braking efficiency; and driver parameters (age of driver may be associated both with model of car driven and speed). A further extension of this work will be to examine the proportion of pedestrians killed, rather than the proportion severely injured. Although the results would be expected to be similar, they would not necessarily be identical; and indeed the proportion of fatalities might be more sensitive to changes in average injury level, since deaths are further out in the tail of the injury distribution.

Two papers at the 4th Conference on Experimental Safety Vehicles (Kyoto, Japan, 1973) presented devices for reducing pedestrian injury. Marumo and Maeda, discussing the Nissan ESV, made the following points: leg fracture from bumper contact will be reduced by using a hollow bumper covered with thick urethane. The bonnet of this ESV has a

built-in honeycomb for better impact energy absorption, and, in addition, a special device retains the pedestrian in contact with the vehicle, preventing him falling to the ground. The device described by Lister (1973) also is designed to retain the pedestrian on the vehicle. It consists of a metal tube shaped to conform to the periphery of the vehicle, forming the leading edge of the bonnet when in the rest position. On impact the tube is raised to a height sufficient to prevent the pedestrian from sliding off the vehicle.

The use of small low-powered cars in city streets has often been urged on the grounds of more efficient use of road space, low power consumption, and reduced pollution (see, for instance, the report "Cars for Cities", Ministry of Transport, 1967), Tables 2.22 and 2.29 suggest they may have the added benefit of reducing pedestrian injury, since they would probably be similar in relevant ways to existing three-wheeled cars. And although in mixed traffic the occupants of light vehicles sustain more injuries in non-pedestrian accidents than those of more massive vehicles - because of the greater velocity change of a smaller body in collision with a larger body, see Chapter 8 - in traffic consisting very largely of small vehicles occupant injury would not necessarily be any worse than at present.

Most attention has been given here to the design of private cars, since they are the type of vehicle most commonly involved in pedestrian accidents. But another area where countermeasures should be concentrated is the installation of side guards on heavy commercial vehicles, to prevent pedestrians, cyclists and motor-cyclists falling underneath the rear wheels (Gissane, 1962; Hogstrom et al, 1973).

So far in this Chapter no discussion has been made of the nature

of the pedestrian's injury, although in our discussion of recent trends in traffic injury in section 1.3 we found an increasing importance of internal injury for all age groups and a decreasing proportion of child pedestrian deaths assigned to skull fracture. Figure 2.1 shows the causes of death in 1972 of pedestrians killed in traffic accidents as a function of their age. The domination of head injury at all ages is clear, accounting in total for some 53% of pedestrian deaths. A considerable amount has appeared in the medical press on traffic injuries, and head injury has usually been emphasised as being the most common life-threatening injury, though leg fracture is very common amongst those seriously but not critically injured. Internal injuries are also a problem, particularly since they may be missed on hurried examination in the emergency room (McCarroll et al, 1962, 1965; Waddell and Drucker, 1971; Bolton et al, 1973). Multiple injuries are very often observed, particularly in those killed. Gogler (1965, p. 135) and Alexander et al (1961) mention that the remarkable resilience of the child's body can sometimes lead to a much less severe injury after runover than an adult would have sustained. For further details on medical aspects on traffic accidents, see, for example, Gissane (1962), Grattan and Clegg (1973), Huelke and Davis (1969), Jamieson et al (1971), Mackay (1969), Sevitt (1973), Slatis (1962), and Solheim (1964). Although they are comparatively rare, because of their severity and medico-legal interest runover injuries have sometimes received special attention from the medical and medico-legal point of view: see Alexander et al (1961), Kamiyama (1961-1964), and Katsura et al (1972).

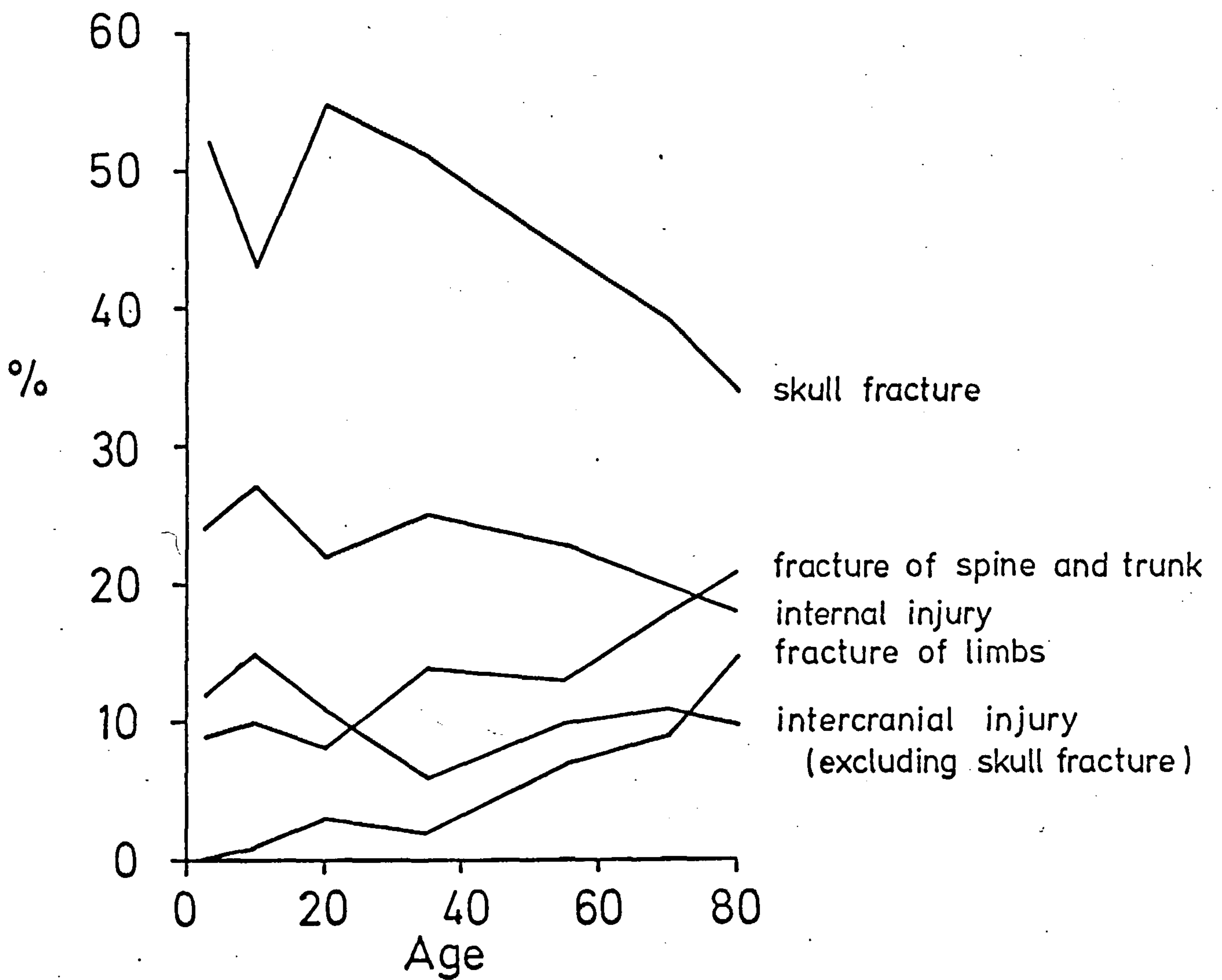


Figure 2.1: influence of age of pedestrian on the medical cause of his death
(source: Registrar-General of England and Wales, Report for 1972)

CHAPTER 3: A STUDY OF PEDESTRIAN ACCIDENTS IN LONDON

The great drawback with the data discussed in the previous Chapter is that there is no indication of the speed of impact of the vehicle with the pedestrian. Thus differences between models of car could be due to differences in design or to differences in impact speed. An attempt was made to use police reports of pedestrian accidents to estimate the speed. However, although police reports of fatal accidents could be used to estimate speeds with a high probable degree of reliability, they were much less useful for injury accidents. Accordingly, the chief purpose of this study was not fulfilled: the results are summarised in figure 3.1, which gives the estimated distribution of impact speeds in slight and severe (fatal plus serious) accidents separately, for accidents to child (5-9 years) and adult (15-49 years) pedestrians separately, and for two models of car (Ford Cortina and BMC Mini). The results are clearly sensible in that speeds are higher in the more serious accidents, but the numbers involved are too small to permit accurate graphs of severity versus speed to be prepared.

There were two aspects of the data on fatal accidents that merited special study: the estimates of speeds by witnesses to the accidents, and the distribution of survival times among the fatally-injured pedestrians. These are discussed in sections 3.1 and 3.2 below.

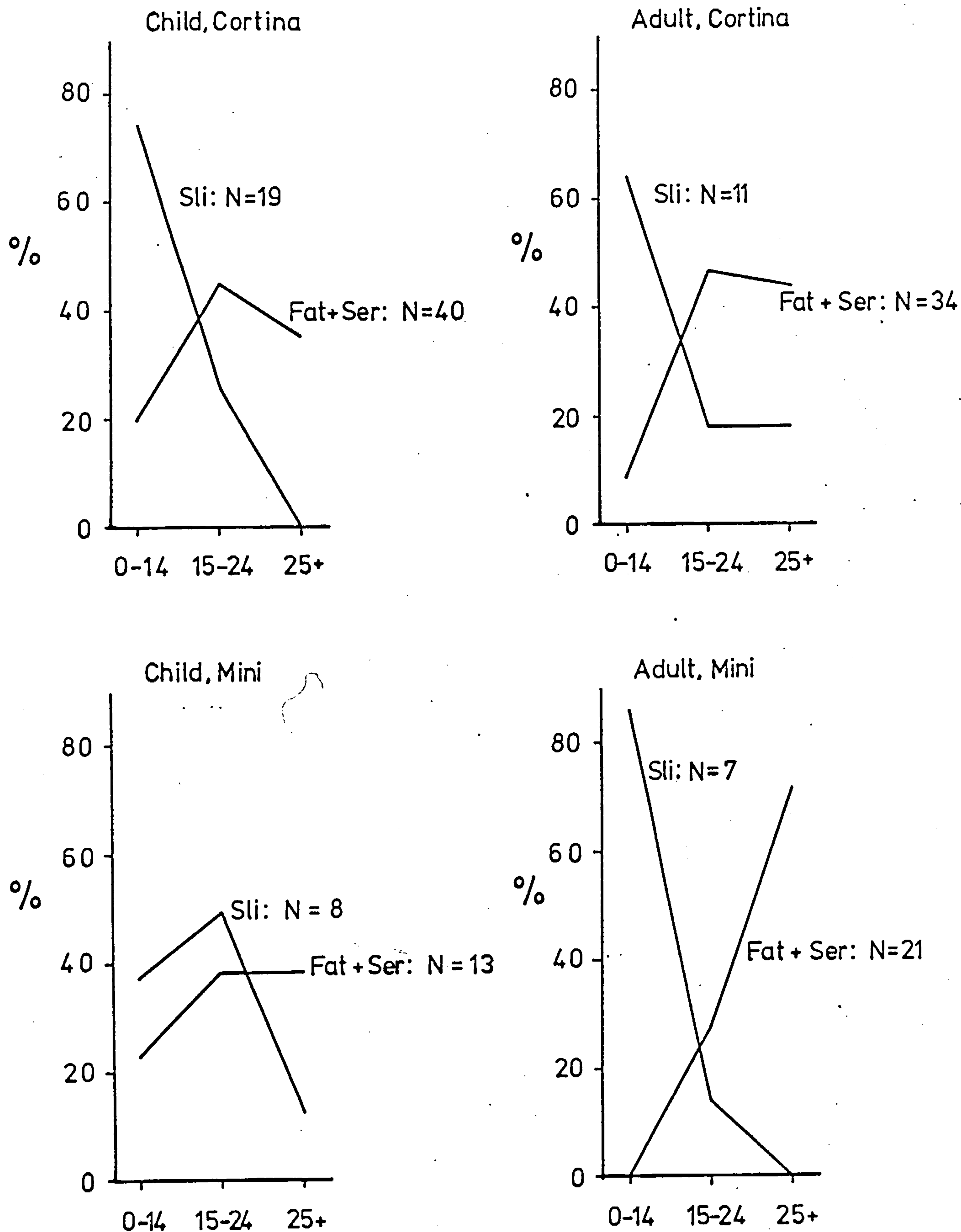


Figure 3.1: Distributions of estimated impact speeds from the London pedestrian accident study. Effect of speed on severity is clearly seen.
(N = total number in sample.)

3.1 Witnesses' estimates of the speeds of pedestrian accidents

3.1.1 Introduction

The data base used consists of all fatal pedestrian accidents occurring in the years 1970-71 in the Metropolitan Police District (Greater London) and reported to the police. There were about 850 of these, and reports of about 90% were available. The amount of information contained in them varied considerably, from only the report book filled in by the policeman at the time and scene of the accident, to a sheaf of papers several inches thick.

Only quantitative estimates of speed are considered here, descriptions such as "fast", "slow", "normal", being excluded. These estimates have been classified into four types: estimates made by independent witnesses to the accident, by the driver of the vehicle involved, and by passengers in the vehicle involved, and the length of skid made by the vehicle in stopping. Accidents of an atypical nature (such as the collapse of a vehicle on a man working underneath it) are excluded. Two approaches are used in establishing the reliability of the speed estimates: first, the quantitative estimates of speeds are intercorrelated; second, my ultimate judgement of the impact speed between the vehicle and the pedestrian is correlated with the damage to the vehicle. Firstly, however, a review is made of related work.

3.1.2 Review

I know of no directly comparable study. There have been some investigations of the perception of vehicle speed in experimental situations where the observer knows that his task is to judge speed;

less accuracy would be expected of (unprepared) witnesses to an accident. Semb (1969) investigated the perception of vehicle speed as seen in three different ways: observers experienced linear motion across the field of view, the approach motion of an oncoming car, and "en route" motion as a passenger. The aim of the experiment was to discover the form of the function relating subjective experienced speed to true speed, and so the accuracy as such of the subjects' estimates was not discussed, though very high correlations are apparent from the data presented. Instead, estimated and actual speeds were plotted on logarithmic scales, from which it was found that estimated speed was proportional to some power of the actual speed, with the exponents being 1.0, 1.35 and 1.4 for linear, approach and en route speed respectively. In view of the finding that pedestrians give themselves less time to cross the road in front of faster vehicles than in front of slow ones (Moore, 1956; Transport and Road Research Laboratory, 1963) - though the hypothesis of a constant time gap is more closely true than that of a constant distance gap - it is perhaps surprising that the exponent for approach speed should be greater than one.

Denton (1966, 1967) has also considered the relation of subjective to objective speed when driving, and found the exponent to be greater than one, though he also discovered that it was influenced by methodological factors and by the speed itself; he went on to fit an alternative model. The significance of his results for the present study is that he found that drivers under-estimate their speed when decelerating. But it is unlikely that such effects will be important compared to other sources of inaccuracy of speed estimation after seeing an accident.

There are other studies of velocity perception in the

psychological literature (for instance: Rachlin, 1966; Ellingstad, 1967) but these are of less relevance as they use artificial stimuli such as a dot moving across a screen.

I know of four experiments in which observers standing at the side of the road were asked to estimate the speed of a vehicle as it passed them. The figure given in Appendix II of Green (1954) shows a very high correlation between actual and estimated speed, though there was some tendency to under-estimate (by about 5 mph).

Hurford (1968) asked each of eleven subjects to judge the speed of each of three vehicles (a saloon car, a sports car, and a van) which were driven past at a known speed. Each vehicle made ten runs at different speeds of up to 100 mph. There were very high (usually over 0.95) correlations between true and estimated speed within each subject-vehicle subgroup. But all subjects consistently under-estimated the speeds - the average overall error was 11 mph of which 10 mph was the surplus of under-estimates over over-estimates. The total of 330 estimates comprised 26 over-estimates, 26 correct estimates and 278 under-estimates. Hurford also carried out an analysis of variance on the errors (differences between true and estimated speeds) made by the subjects and found significant effects between subjects, between vehicles, and between speeds: the van's speed was the most accurately estimated and the saloon car's the least, average error increased with increasing speed but percentage error decreased and subjects differed in their accuracy ("accuracy" being taken to include both bias and scatter). In addition the vehicle x speed and subject x speed interactions were also statistically significant. However, the restricted nature of Hurford's sample of observers (who were members of a university department) makes the generality of his conclusions

questionable.

A note of discord is sounded by Hinch (1967) who stopped pedestrians on the street and compared their estimates of the speeds of vehicles with radar speedometer readings. His sample was also biased, but towards women and retired people; the working population was either unavailable or uncooperative. He concluded that pedestrians are unable to estimate vehicle speeds with any accuracy; no bias towards under- or over-estimation was found in this study.

Goodwin, Hutchinson and Wright (1975) carried out an experiment in which seven untrained observers estimated the speeds of vehicles passing them, the true speeds being measured with a radar meter. The average speed of the vehicles observed was 32 mph with standard deviation 9 mph. The correlation coefficient between the estimated and true speeds was close to 0.8 for all subjects (see table 3.1, in which the errors are also given). Figure 3.2 shows estimated against true speed. Although this experiment suggested that observers could estimate speeds to within 5 mph most of the time, it should be said that the subjects were volunteers from the Traffic Studies Group whose numeracy, age, experience and occupation were not necessarily representative of the general population.

Although the experiments discussed in this literature review are not directly comparable with the present study, they do throw some light on the potential abilities for estimating speed and, with the exception of Hinch's investigation, show that observers who know they are taking part in an experiment to estimate speed can do so with great consistency and probably little bias.

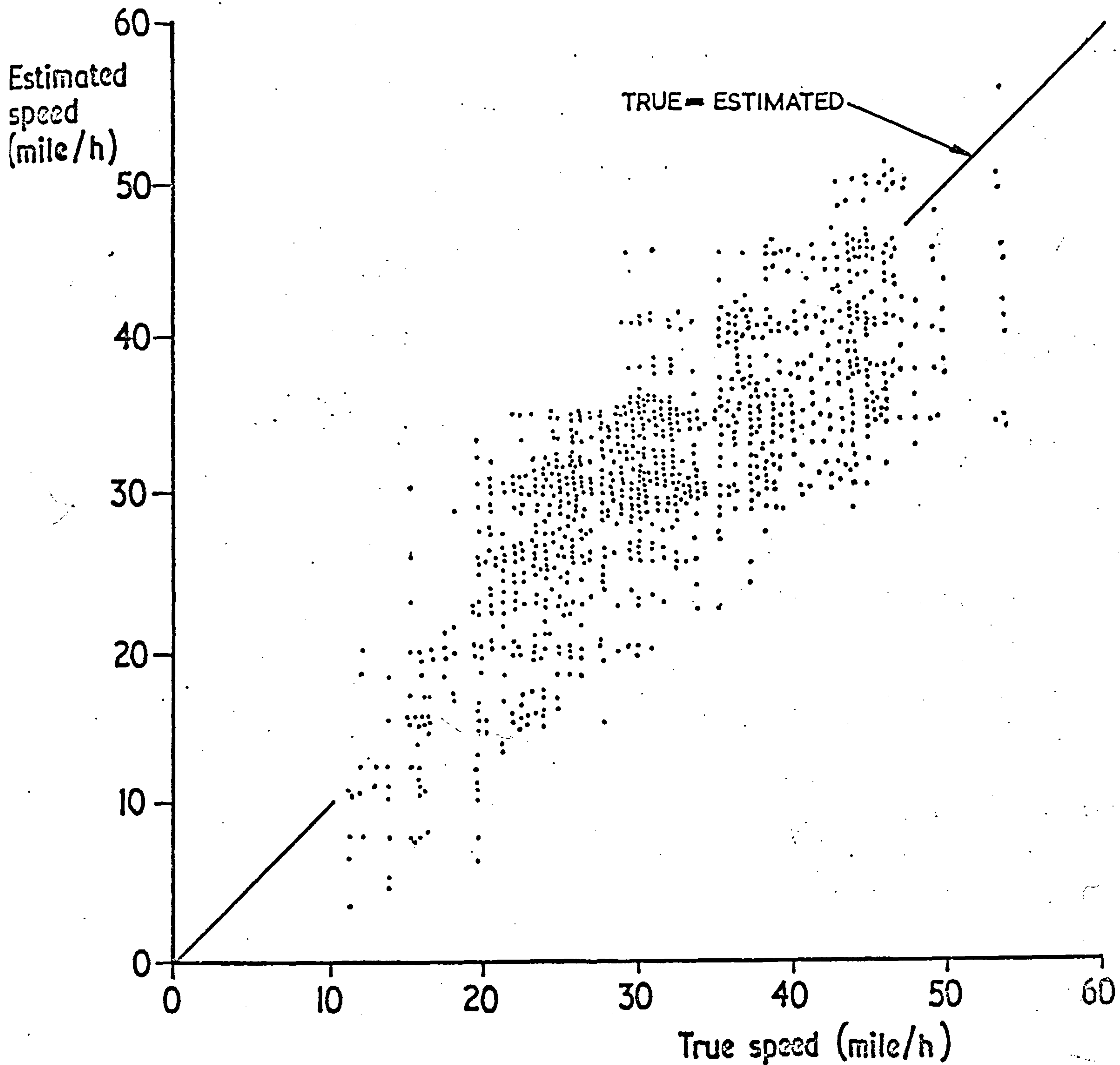


Figure 3.2: True vehicle speeds and the estimates made by all seven observers for both sites. Many of the data points are slightly displaced from their true positions so as to avoid superposition.

<u>Subject:</u>		<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
Error (estimated minus true speeds) (mph)	Mean	-0.7	+2.4	-5.6	+2.0	-0.9	+2.3	-1.0
	Standard deviation	5.2	4.9	5.2	4.7	4.6	4.5	5.1
Correlation coefficient between estimated and true speeds		0.81	0.83	0.81	0.78	0.87	0.78	0.74

Table 3.1: Summary of results of a speed estimation experiment by Goodwin, Hutchinson, and Wright (1975).

3.1.3 Intercorrelations of speed estimates

Table 3.2 shows in how many cases estimates of speed were given in the police files: the number of estimates is about three-quarters the number of accidents. For those accidents in which more than one speed estimate was given, linear regression was performed between each of the four types of speed estimate, taken two at a time, and the resulting correlation coefficients are given in table 3.3. All are statistically significant at least at the 1% level, except that between (skid length)^{1/2} and passenger's speed estimate, for which $P = 10\%$ (there were only 13 cases in this regression). Two examples are given in figures 3.3 and 3.4.

The square root of the skid length was used in the regression because the length taken to stop from speed v under constant deceleration is proportional to v^2 . When correlating the estimates of speed made by two or more independent witnesses, a slightly unusual procedure has to be adopted because there is nothing to identify one of the estimates as the x-variable and the other as the y-variable, or the other way round. The procedure is known as intra-class correlation and is closely related to the one-way analysis of variance, see section 3.1.7.

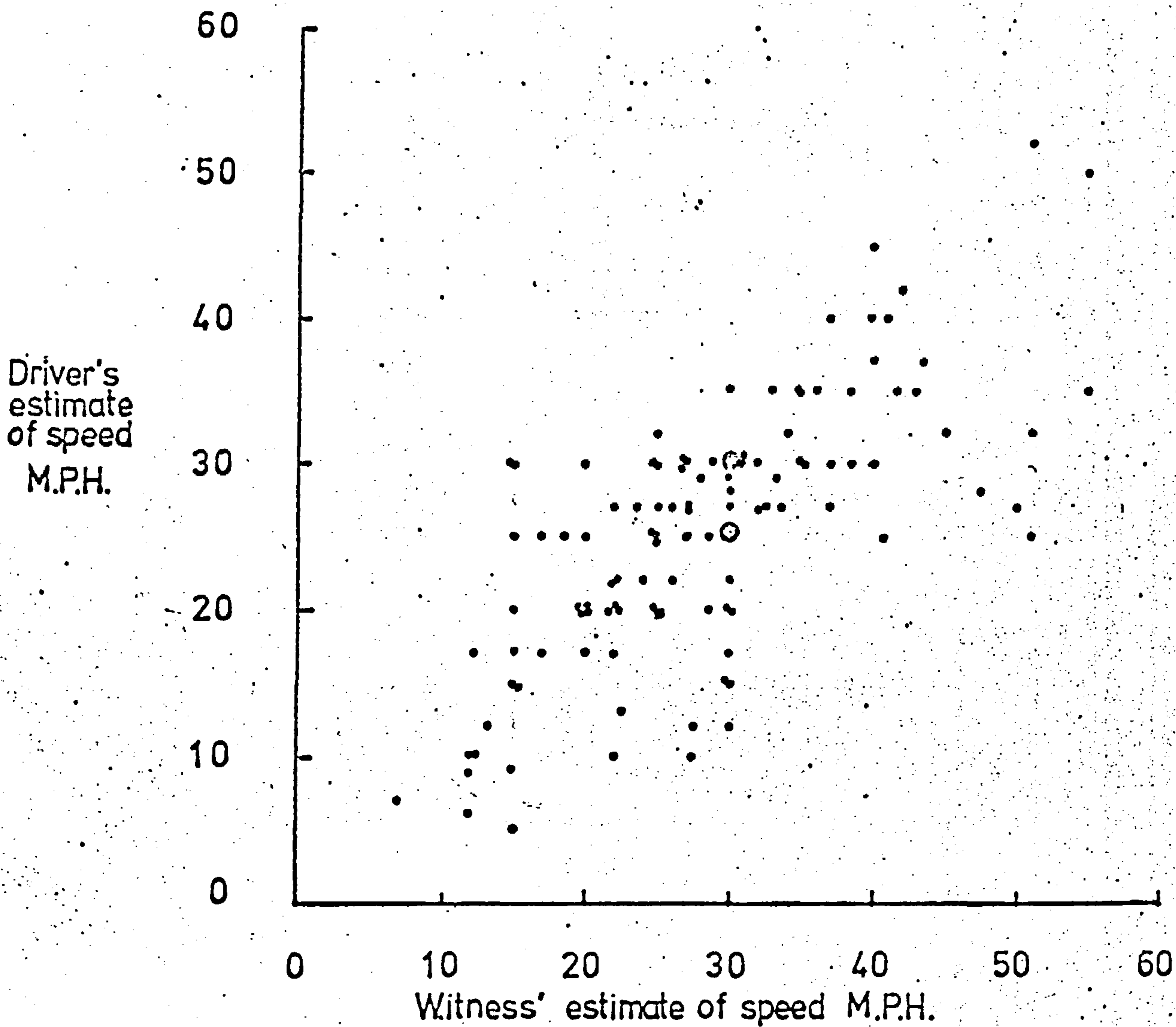


Figure 3.3: Relation between witness' and driver's estimates of speeds
 Several points are clustered together at (30,25) and (30,30).

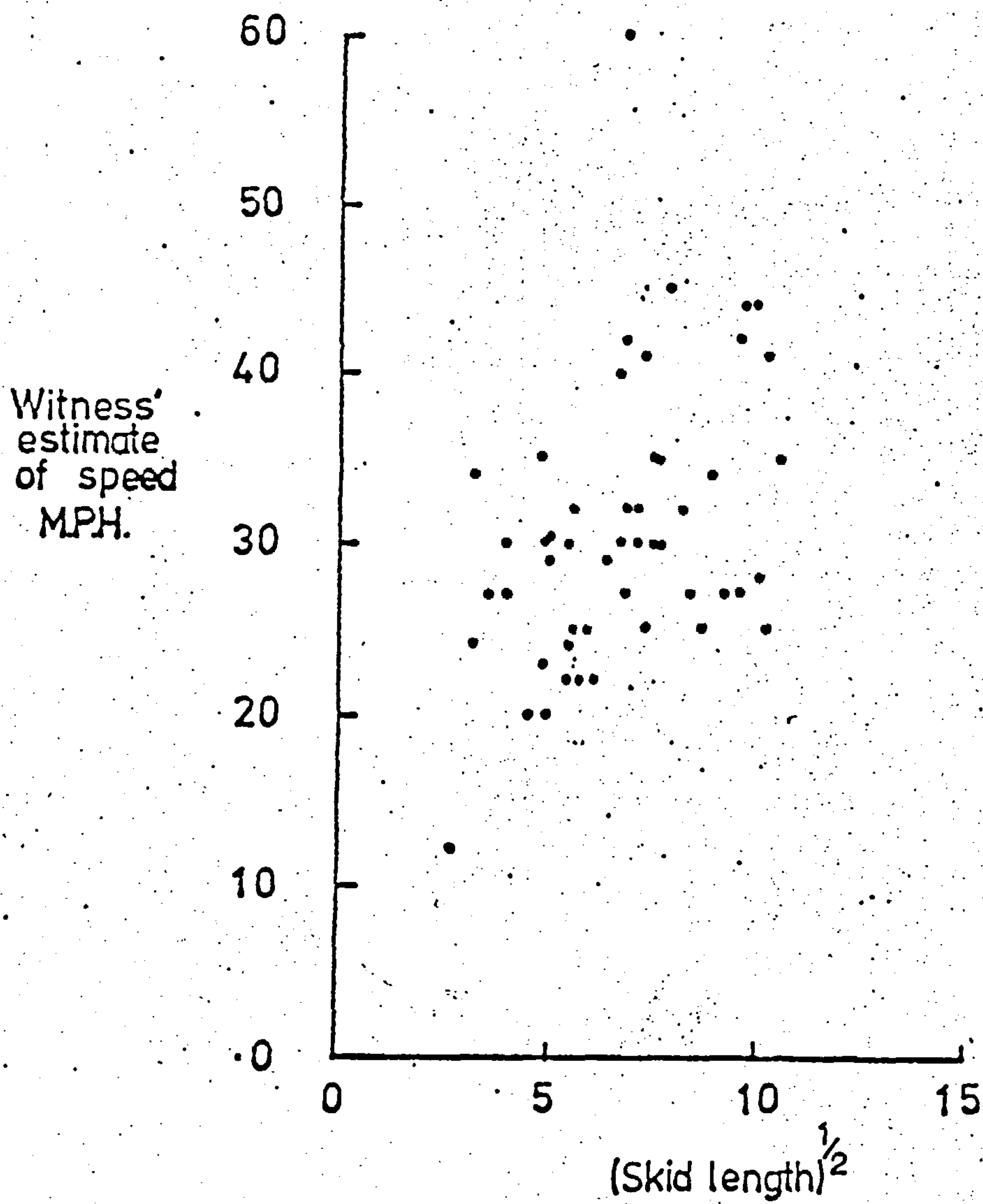


Figure 3.4: Relation between skid length and witness' estimate of speed.

<u>Source of speed estimate</u>	<u>Number of cases</u>
Police file not available	85
No quantitative estimate	276
Witness	273
Vehicle driver	182
Passenger	53
Length of skid	89
At least two quantitative estimates, total	232
Total accidents in sample	853

Table 3.2: The information on speeds in the accidents studied.

	<u>Witness</u>	<u>Driver</u>	<u>Passenger</u>
(skid length) ^{1/2}	0.42	0.64	0.48
Witness	0.67*	0.73	0.67
Driver			0.67

* intra-class correlation coefficient

Table 3.3: Intercorrelations between the four sources of quantitative estimates of speeds.

<u>Source of estimates</u>		<u>Av(x)</u>	<u>Av(x-y)</u>	<u>SD(x-y)</u>
<u>x</u>	<u>y</u>	(mph)	(mph)	(mph)
witness	driver	28.5	2.7*	6.6
witness	passenger	32.5	4.6*	7.4
driver	passenger	26.7	0.1	4.0

* significantly different from zero

Table 3.4: Showing that independent witnesses tend to give a higher estimate of speed than do drivers or their passengers.

When other estimates were being correlated with those of independent witnesses, if more than one of the latter was available the mean of them was entered in the regression.

The statistically significant and quite high correlations shown in table 3.3 provide good evidence that observers of accidents and the people involved do make reasonably consistent estimates of speed. Further evidence comes from the relation between estimates from people who saw or were involved in the accident and the length of skid of the vehicle involved. It was found that

$$L = \frac{v_w^2}{21} \quad L = \frac{v_d^2}{16} \quad L = \frac{v_p^2}{17}$$

where L = length of skid (ft), and v_w , v_d , v_p are respectively the estimates made by independent witnesses, the driver, and passengers (mph). Since the stopping distances given in the Highway Code correspond to $L = \frac{v^2}{20}$, it seems likely that estimates of speed made by people who saw or were involved in the accident are not merely related to the actual speed, but are close to it in magnitude.

3.1.4 Differences between estimates

It is also of interest to examine the differences between estimates of speed from different sources - for instance, it is plausible that a driver might tend to underestimate his speed. Table 3.4 shows that drivers do indeed tend to give a lower estimate of their speed than do independent witnesses, and passengers agree with their drivers.

It might be supposed there would be a tendency for drivers to tell the truth about their speeds if obeying the speed limit, but to

deliberately under-estimate them when breaking the law. Evidence for this is provided by the slope of the line relating driver's speed estimate to the witnesses' being significantly less than 1 - the equation found was $v_d = .65v_w + 8$. The explanation suggested above is by no means the only one, however - for instance it might be that the drivers' estimates are the true speeds, and independent witnesses tend to exaggerate the speed away from the norm.

It should be remarked, however, that these differences are small in magnitude. Bearing in mind the accuracy of estimation implied by the correlations of around 0.6 found in the previous section, they are probably of negligible importance.

3.1.5 Vehicle damage

A vehicle which strikes a pedestrian usually receives little damage, and there is probably an element of chance in whether the reporting policeman observes or notes any minor dents. But there is no reason to think that the probability with which he does so is influenced by the speed of the impact, and thus the proportion of cases in which damage is noted will reflect (though possible under-estimate) the proportion which are in fact damaged.

It is assumed that the harder the impact, the greater the chance of damage to the vehicle. That being so, if a relation is found between damage and estimated impact speed, that will help to validate the estimates of the latter (assuming that the extent of damage did not influence the estimation).

After a careful reading of the police file, an estimate of the

speed of the vehicle at the moment of impact with the pedestrian was made. In many cases there were one or more quantitative estimates of the speed to help, in others the people involved merely used adjectives like "fast" or "normal speed", and in others a judgement was made on the basis of the description of the circumstances of the accident and a knowledge of local conditions at the site. In about 25% of cases no estimate of impact speed could be made, either because of lack of detail in the accident report, or because it was far from clear how much the vehicle had braked before the impact. Cases in which the vehicle struck another object such as a lamp post or another vehicle have been excluded from this section of the analysis, and it is restricted to pedestrian impacts with cars (because goods vehicles may be of stronger construction and two-wheeled vehicles often fall to the road surface).

Speed has been grouped into six categories (the distribution of estimated impact speeds is given in table 3.5) and two measures of vehicle damage have been considered: the proportion of cars dented, and the proportion with damaged (usually shattered) windscreens. Figure 3.5 shows that both these measures increase with increasing estimated impact speed, and that the proportion of cars said to be undamaged decreases correspondingly. This strongly suggests that the estimated impact speed is related to the actual impact speed, though it does not prove that it is correct in absolute terms.

Considering only one measure of vehicle damage now (for instance, the proportion dented), one has a 2×6 contingency table giving the numbers of cars dented and not dented at each of six impact speeds. An appropriate way of describing the correlation between speed, a continuous variable (neglecting the fact that this is classified into

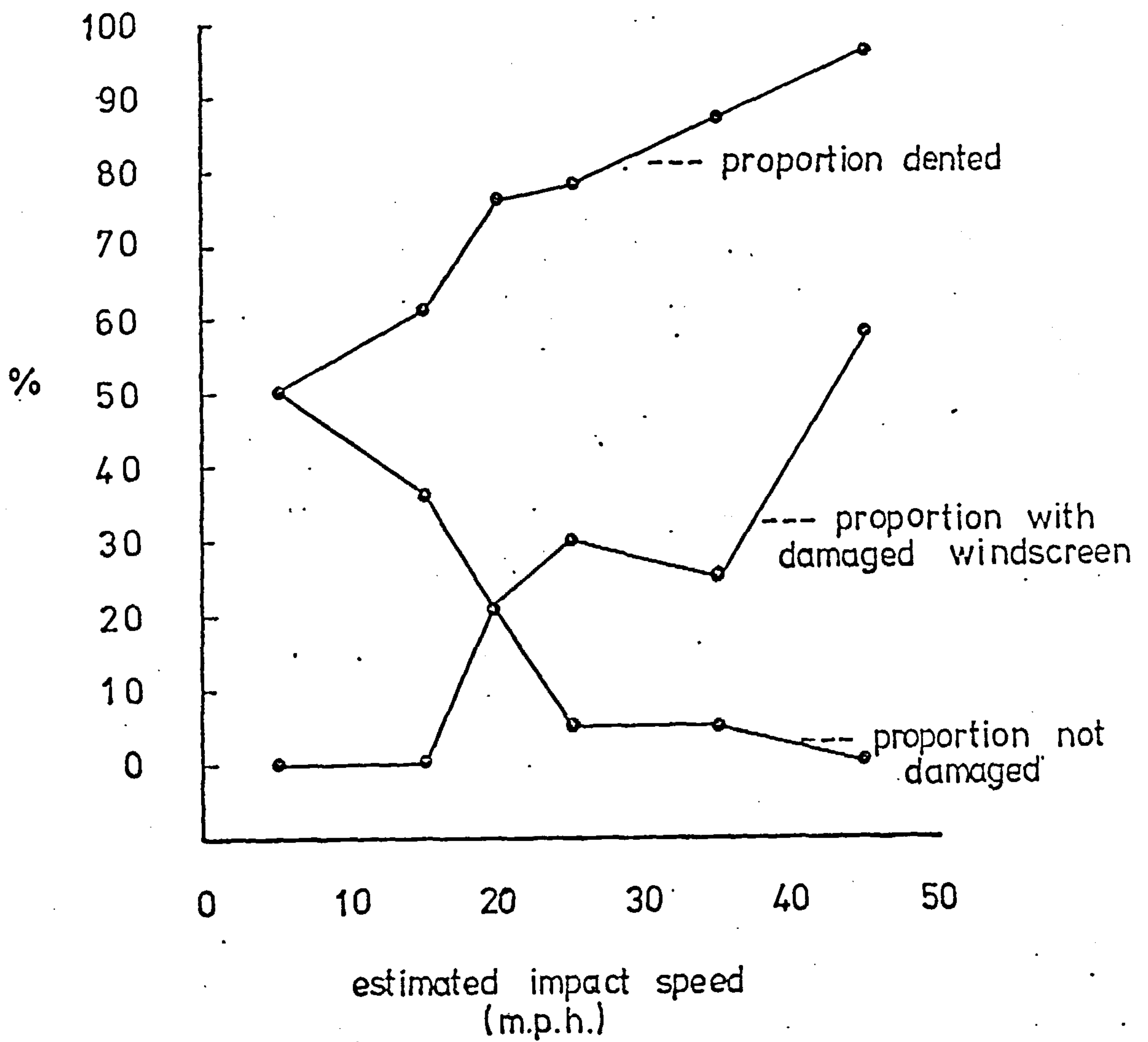


Figure 3.5: proportion of cars damaged as a function of estimated speed of impact.

a finite number of categories), and damage, a variable having only the two categories damaged or not, is point-biserial correlation (see section 3.1.7). The resulting point-biserial correlation coefficients are .26 between speed and denting, and .29 between speed and windscreen damage. With 324 cases, both these values are very highly significant. That they are not very high should not be surprising; no allowance has been made for the make and model of the car, or whether it struck the pedestrian full on or only a glancing blow, how large the pedestrian was, or any of the other details of impact and resulting trajectory of the pedestrian which might affect whether the vehicle is damaged.

<u>Estimated speed of impact</u>	<u>Proportion of cases</u>
(mph)	(%)
0 - 9	2
10 - 19	11
20	15
21 - 29	23
30 - 39	34
40+	15

Table 3.5: Distribution of estimated speeds of impact of cars with pedestrians (fatal accidents in London, 1970-71).

3.1.6 Discussion

The aim of this section has been to determine how much agreement about speeds there is between different observers of the same road accident, and to show that in this sample of fatal pedestrian accidents there is already in police reports a considerable amount of information about vehicle speeds which is likely to be sufficiently accurate at least for comparative purposes within the sample of accidents studied. Since we do not know the true speed of vehicles involved in accidents (i.e. we do not have a criterion with which to compare our estimates), evidence for this has to be indirect and rely upon the assumption that when two independent measures of a variable X are correlated, this is because they are both correlated with the true value of X. Section 3.1.3 demonstrated that in cases where two or more quantitative estimates of speed were available, these correlated quite highly. And there was some evidence that they were quite closely correct in absolute magnitude, because of the value of the average ratio of witness' estimate to the square root of skid length. (But this must be taken tentatively, because of indications from experimental studies - see section 3.1.2 - that pedestrians tend to slightly underestimate the speeds of cars passing them.) However, this tells us little about how accurately the speed at impact is likely to be estimated, and in any case there are many instances in which only one or even no quantitative estimate of speed is available. Section 3.1.5 attempted to rectify this by relating car damage to impact speed, and the correlations found there are highly significant. Nevertheless, it should be pointed out that we have given no very strong evidence for the correctness, in absolute terms, of the estimates made - the same correlations could be obtained if the estimates were, on average, double the true speeds. It is partly for this reason (and partly because of the subjective

element involved in interpreting witnesses' statements) that the caveat "at least for comparative purposes within the sample of accidents studied" was included in the first sentence of this paragraph. In other words, estimated speed will be suitable for use as a control variable where we are primarily interested in the effect of something else when the effect of speed is removed; but we have not shown that it is accurate enough for analysis in its own right.

This investigation was made on a sample of accidents of a particular type, and reservations should be made about the generality of the findings - what is true for the records of the Metropolitan Police does not necessarily apply in other countries, or even to other English police forces; there is not as much speed information in reports of injury accidents as in reports of fatal ones; and what is true for pedestrian accidents (which occur very largely in urban areas) does not necessarily apply to other types (for which a greater proportion occur in rural areas with a presumably lower density of witnesses than urban areas). Injured drivers might well not be able to give as reliable account of their speeds prior to their accident, and those involved in single-vehicle accidents may wish to conceal their true pre-accident speed. On the other hand, for accidents involving two or more vehicles, there will be at least two people involved who have experience in judging speeds (something which is not true of pedestrian accidents) - the vehicle drivers.

In addition, it has not been shown that accidents for which an estimate of speed can be made have the same distribution of speeds as those whose speeds cannot be estimated - it is possible, for instance, that the latter might contain a higher proportion of ordinary, unremarkable speeds which no one noticed. And it is possible that some

of the correlations in table 3.3 might be inflated somewhat as a result of discussions between the people involved - the police try to get independent accounts of what happened, but in many cases it is impracticable to prevent contact between the people involved. This objection is most likely to apply to the witness-witness and driver-passenger correlations, and it is less plausible for the others.

Nevertheless, there is no doubt that the chief message to emerge here - that there is a substantial degree of agreement about speeds between those seeing a road accident - is an encouraging one.

3.1.7 Appendix: explanation of (a) intra-class correlation, and (b) point-biserial correlation

(a) Intra-class correlation is useful where we wish to compare the amount of variation within a class of observations (speeds estimated by different independent witnesses to the same accident) with that between classes (speeds of different accidents), and express the result as a correlation coefficient describing the association between one independent witness's estimate and that of another independent witness to the same accident.

Say we have twenty accidents, each of which has two independent witnesses estimating the speed of the vehicle involved. Our correlation table has two variables, both speed, but in order to complete it we must decide which estimate is to be related to which variable. We might decide to take the lower estimate first, or make a random choice. But the former would give us the correlation between the lower estimate and the higher estimate - not that between estimates in general, which is what we want; and the latter would give a slightly different result

if we repeated the calculations on the same data but with a different random assignment of observations to variables.

The problem is solved by entering in the correlation table both possible pairs, i.e. those obtained by taking each estimate first. So if we had twenty accidents with two witnesses each there would be forty entries in the table. The calculation of the coefficient is then identical with that of the ordinary product-moment correlation coefficient. The test of its statistical significance is different, though, since the observations in the correlation table are not all independent.

In terms of the analysis of variance, the intra-class correlation coefficient is given by

$$\frac{s_b^2 - s_w^2}{s_b^2 + (m-1)s_w^2}$$

where s_b^2 is the average between-class sum of squares, s_w^2 is the average within-class sum of squares (so that $F = s_b^2/s_w^2$) and m is the average number of observations per class. Thus

$$r = \frac{F - 1}{F + (m-1)} \quad (\text{see McNemar, 1969}).$$

(b) Point-biserial correlation is appropriate when correlating a continuous variable (in our case, speed) with one that is merely divided into two classes (for instance, damaged and not damaged). If v_1 and v_2 are the average speeds of the damaged and undamaged groups, and σ is the standard deviation of speed in the entire sample, and p is the proportion of the entire sample which is damaged, the point-biserial correlation coefficient is given by

$$\frac{(v_1 - v_2) \{p(1 - p)\}^{\frac{1}{2}}}{\sigma}$$

There is some doubt about whether it is quite proper to regard damaged/not damaged as a true dichotomy (it could be argued that it is a continuous scale which we have arbitrarily dichotomised), but the level of statistical significance attained is so high that this need not worry us - with $n = 324$ and $r = .26$, $t = 4.8$ and $P < .000002$.

3.2 Times till death

The purpose of this section is to elucidate some of the factors which affect the survival time distribution of pedestrians killed in road traffic accidents. Such data has been previously published by several workers, but apparently only Robertson and Tonge (1968) and Sevitt (1973) have sought to relate the time till death to other features of the accident.

The police records very often (in about 75% of cases) included the time of death of the pedestrian, in the great majority fairly exactly (to the nearest hour). Where death occurred within half an hour of the accident, time till death has been taken as $1/4$ hour. Thus although not as satisfactory as hospital data, we can have a good deal of confidence in the figures given. Moreover, it does give us data on type of vehicle striking the pedestrian, and an indication of its speed, which are not usually available from hospital data. It is possible that the time of death is more likely to appear in police records if it is immediately after the accident than if it is delayed some time, but the proportion of cases in which it is unknown is probably not large enough for this effect to be of much significance. Furthermore, the primary interest of this section is in comparing different groups with respect to the time till death, and it is unlikely there is any bias differentially affecting these groups.

A further indication that any bias is small is given by figure 3.6, in which the distribution of survival time in the present sample is compared with previous findings (all relating to pedestrians) and is seen to be very similar. As suggested by Smeed (1968a), the percentage dead by time t is related approximately linearly to $\log t$.

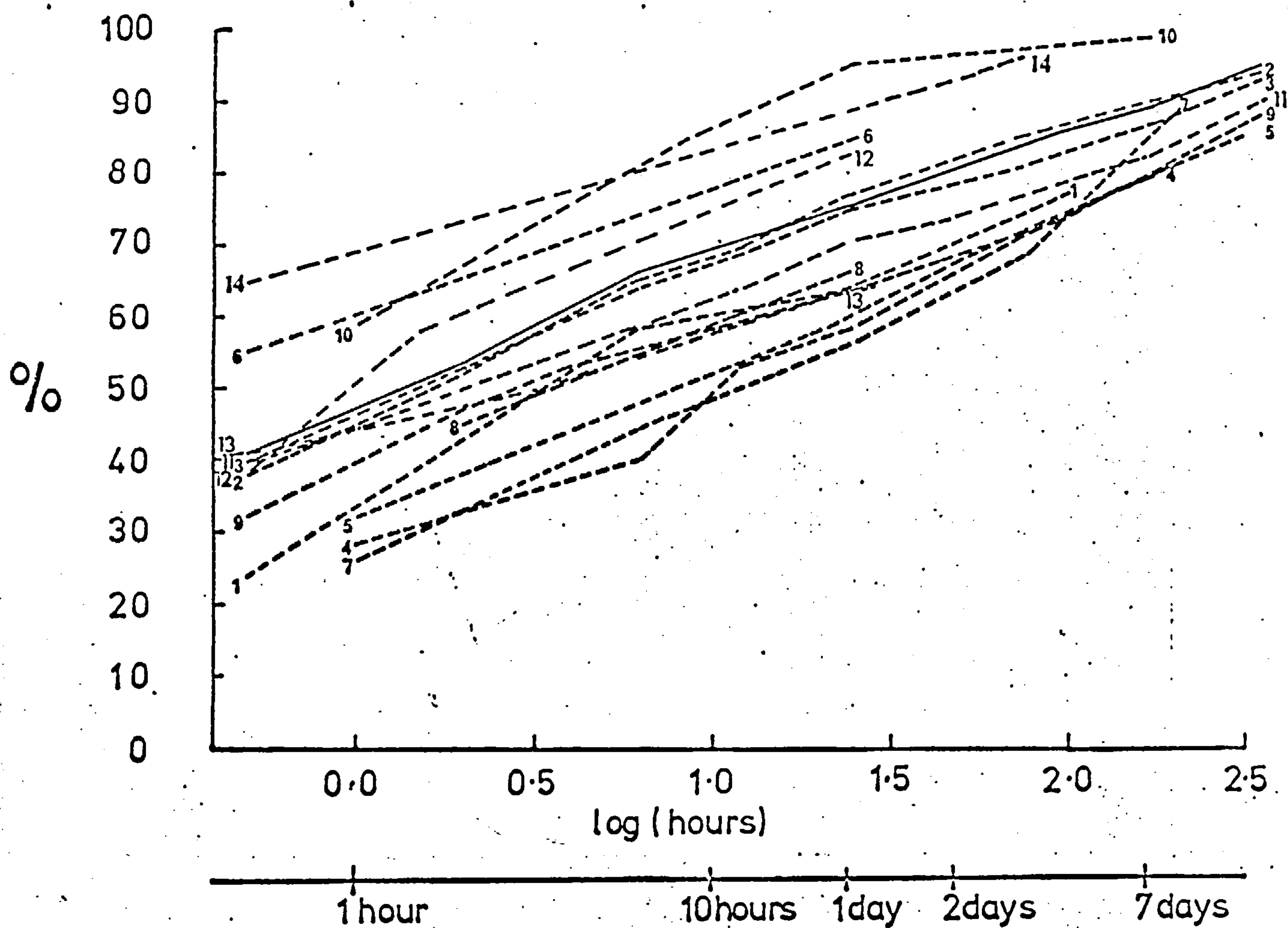


Figure 3.6: cumulative distribution of times till death. Solid line is data from this study, dotted lines are from previous studies - for key to code numbers, see Table 3.6.

<u>Code no. in fig.3.6</u>	<u>Source</u>	<u>Area</u>	<u>Sample size</u>
10	Wright (1958)	Los Angeles	81 (all children)
7	Haddon et al (1961)	Manhattan	50 (all adults)
8	McCarroll et al (1962)	Manhattan	200
1	Slatis (1962)	Helsinki	192
13	Solheim (1964)	Oslo	168
2	Robertson and Tonge (1968)	(Brisbane	864
3		(Adelaide	282
4	J.P. Bull, quoted in Smeed (1968a)	Birmingham	about 200
5	Smeed (1968a)	S.E. England	about 250
11	Gross (1969)	Manhattan	126
6	Huelke and Davis (1969)	Detroit	232
14	Morocco (1970)	Morocco	419
9	Sevitt (1973)	Birmingham	160
12	Giraldo (1973)	Medelin	135

Table 3.6: Giving a key to the code numbers of figure 3.6, together with the areas of study and sample sizes of the investigations.

This formula was used also by Robertson and Tonge (1968) and Sevitt (1973), but it is not entirely satisfactory because the fraction dead by time t becomes greater than 1 at some finite time. An alternative formula for which this objection does not hold, but which lacks the simplicity of Smeed's suggestion, is

$$f = \exp(-at^n) \quad (3.1)$$

where f is the proportion of those who eventually die who are surviving at time t . This formula implies a linear relationship (of slope $n-1$) between $\log \phi$ and $\log t$, where ϕ is the rate of dying of those still alive at time t (that is, $-\frac{1}{f} \frac{df}{dt}$). For the present data, a and n are respectively about 0.63 and 0.25. Figure 3.7 compares the observations with this formula.

In actuarial work, where t is a matter of years rather than hours, ϕ is known as the force of mortality. In engineering, where we are dealing not with people but with components of machines, it is known as the hazard rate or failure rate. We may also remark that equation 3.1 is known as the Weibull distribution. It is in frequent use in reliability and quality control work, usually, as here, with no explicit theoretical reasoning behind it. See Johnson and Kotz (1970a, chapter 20).

3.2.1 Results: impacts with cars

In this section the effect of estimated impact speed and age of pedestrian on his time till death will be considered, with the accidents restricted to the 293 cases in which the pedestrian was struck by a car (as distinct from any other type of vehicle) and for which all of the following items of information were available: pedestrian age, an

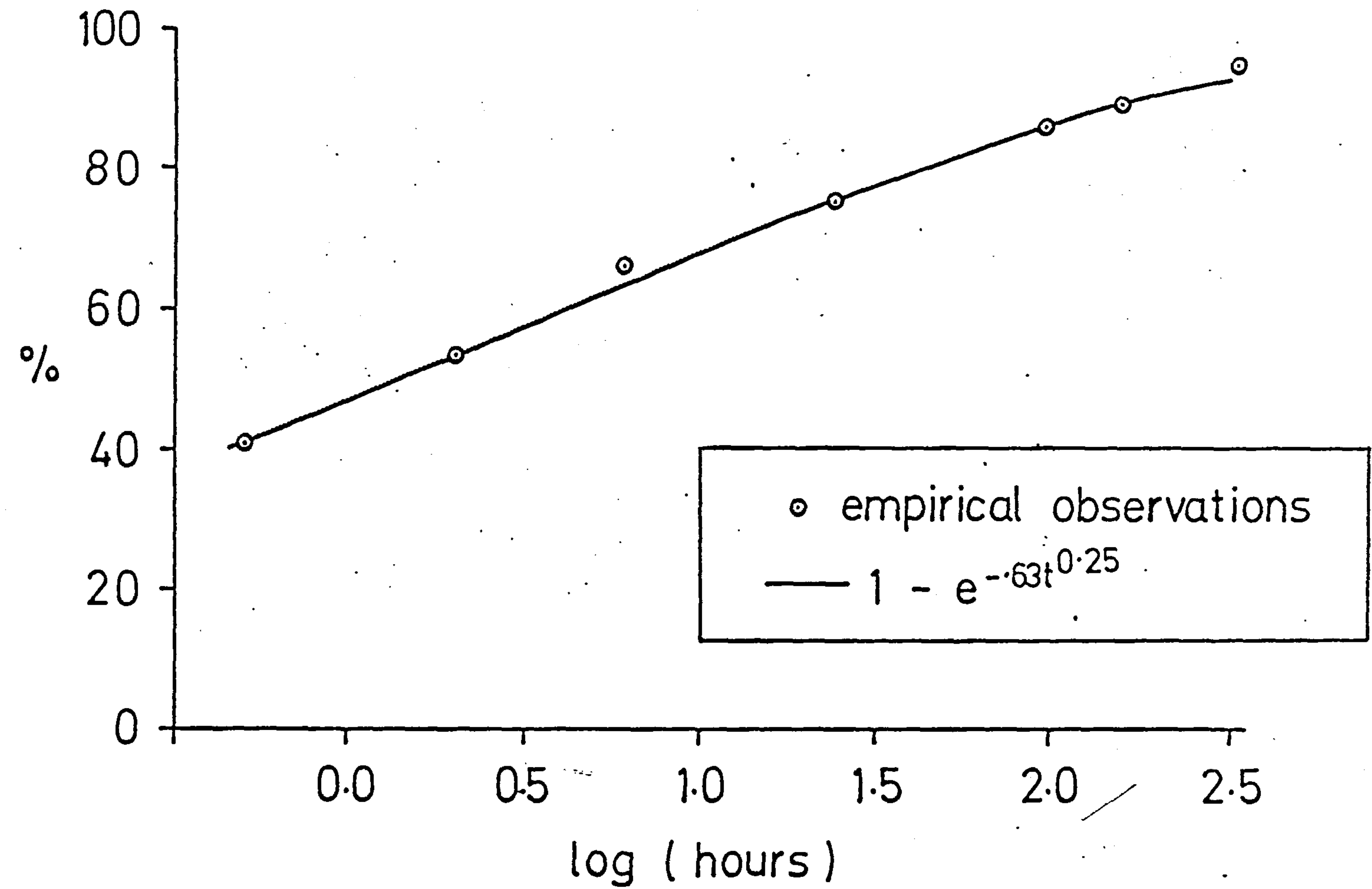


Figure 3.7: comparison of empirical and fitted cumulative distribution of times till death.

estimate of the impact speed, and time till death. As can be seen from table 3.7, 15% of these cases were children, 16% were adults under 50 years of age, 29% were between 50 and 69 years, and 41% were 70 or more years old. The succeeding section will examine the effect of type of vehicle.

Firstly, without subdividing accidents according to speed, we can examine the effect of age - grouped as follows: less than 15 years, 15-49, 50-69, 70 and over - on average time till death. Figure 3.8 shows the cumulative distribution of time till death for each age group separately, all impact speeds being combined. The median time till death, that is the time by which 50% of fatal cases have died, for each age group is given in the final column of table 3.8. From this we can see that half of the child fatalities are dead within 3 hours of the accident, half of the adults in the 15-49 and 50-69 year groups who die do so within 1 hour of the accident, whereas for the over-seventies the corresponding figure is 2 hours. We can test the statistical significance of the difference between these figures by using Kruskal and Wallis's test (which is effectively a nonparametric one-way analysis of variance - it is advisable not to use the ordinary analysis of variance because the times till death are not normally distributed. For details of the Kruskal-Wallis test, see, for instance, Langley, 1968). The result is significant at the 1% level, thus showing an association between age and time till death, children and the most elderly dying after the longest interval.

However, we will now show that this result is an artefact arising from an association between age and impact speed, and an association between impact speed and time till death. Table 3.7 shows that the different age groups have different distributions of speed of impact.

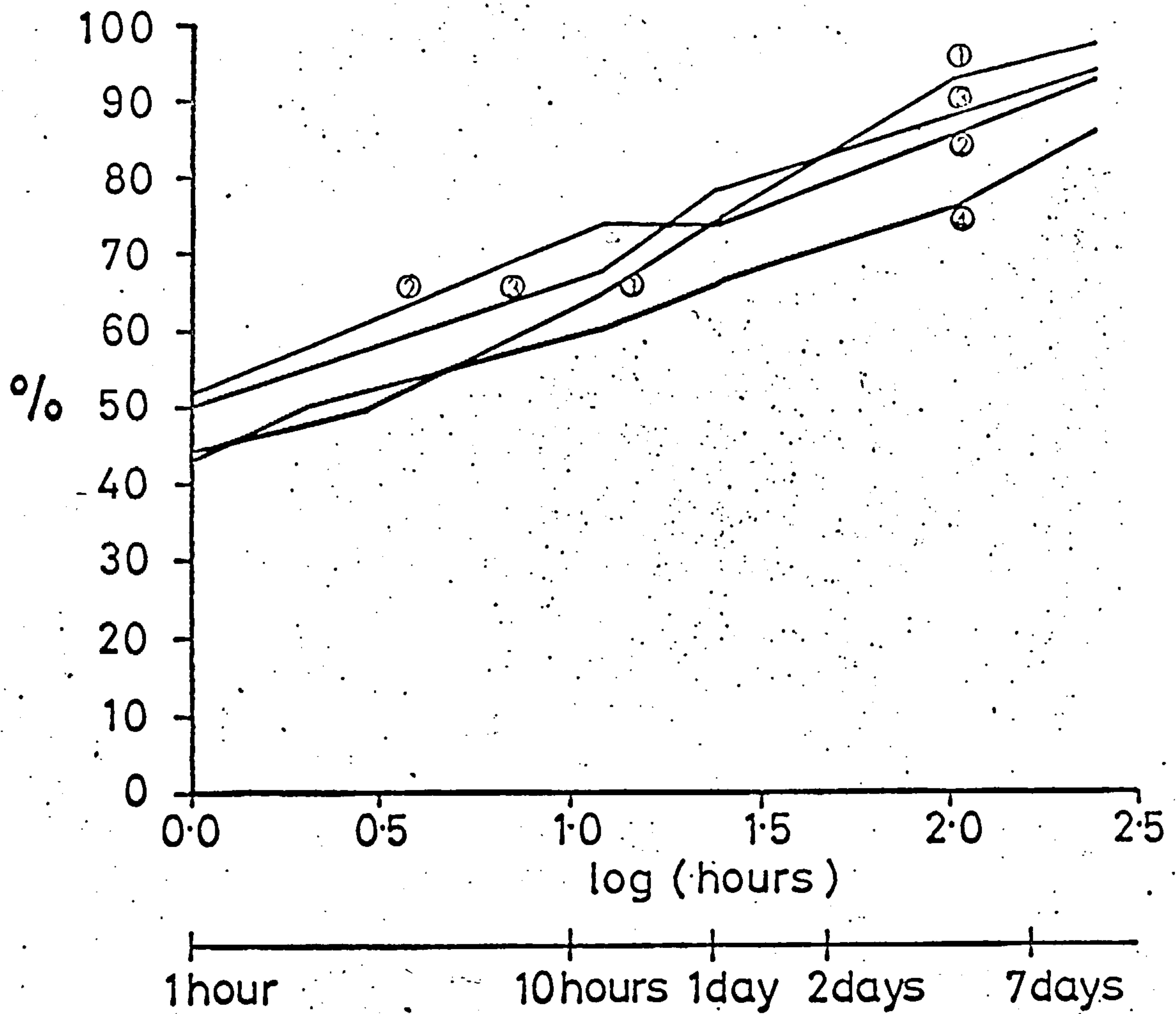


Figure 3.8: Cumulative distribution of times till death of pedestrians struck by cars, showing results for each age group separately.

- 1 0-14 years, sample size 43
- 2 15-49 years, sample size 46
- 3 50-69 years, sample size 84
- 4 70+ years, sample size 119

As the numbers of cases in the age groups vary from 43 to 119, the standard error of the mean speed at impact is between one-sixth and one-eleventh of the standard deviations quoted in table 3.7, that is between 0.9 and 1.3 mph. Thus the average speeds differ significantly at the 1% level. Remember that all the cases are fatal ones: the probable reason for the lower speed of accidents to older people is that the young survive the low speed impacts that prove fatal to the elderly.

Table 3.8 gives the median time till death of each age-speed subgroup. An association between speed and time till death is clear, at least in the two elderly age groups; the higher the speed, the greater the chance of dying quickly. This is also apparent from figure 3.9, in which all age groups are combined. Spearman's coefficient of rank correlation (calculated on the raw data) between speed and time till death is -0.32 for the 50-69 age group and -0.40 for the 70+ age group, both significant at the 1% level.

The effect of age on time till death, with the effect of speed eliminated, can be tested with a nonparametric test due to Benard and van Elteren (1953), see section 4.3; times till death are ranked within each speed group across all age groups, and then the ranks are summed across speed groups. If age has any effect, this will show up in the rank totals. The result is that there is no significant effect of age for the age categories considered here.

To sum up this section: speed of impact influences time till death, but age of injured person does not. But because the elderly are killed by low speed impacts which the young survive (or, alternatively, are not involved in - speeds of injury accidents, as opposed to fatal ones, would be needed to answer this question), when all speed groups are

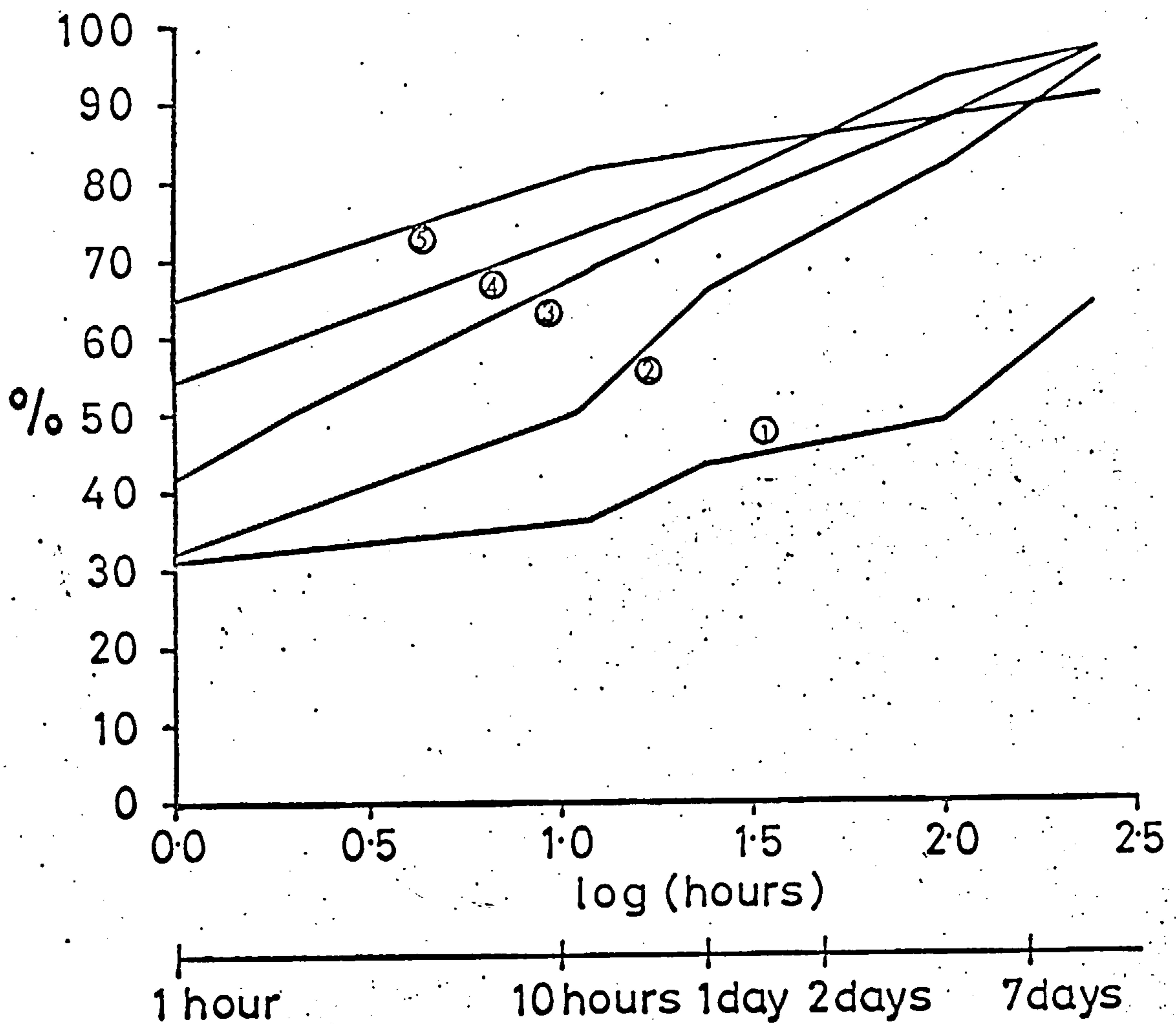


Figure 3.9: Cumulative distribution of times till death of pedestrians struck by cars, showing results for each impact-speed group separately.

- 1 0-19 mph, sample size 39
- 2 20 mph, sample size 44
- 3 21-29 mph, sample size 67
- 4 30-39 mph, sample size 99
- 5 40+ mph, sample size 43

combined, an association between age and time till death is found, with the elderly living longer.

<u>Age group of pedestrian</u>	<u>Estimated speed of impact (mph)</u>						<u>Mean and standard deviation of estimated impact speed (mph)</u>	
	<u>0-19</u>	<u>20</u>	<u>21-29</u>	<u>30-39</u>	<u>40+</u>	<u>Total</u>		
0 - 14	5	8	11	17	2	43	28	8
15 - 49	1	3	12	19	11	46	33	9
50 - 69	10	13	18	26	17	84	30	10
70+	23	20	26	37	13	119	27	10

Table 3.7: Numbers of cases, classified according to age and speed group, together with the mean and standard deviation of estimated speed at impact for each age group. (Cases, involving cars, for which pedestrian age, estimated impact speed, and time till death were all available.)

<u>Age group of pedestrian</u>	<u>Estimated speed of impact (mph)</u>					
	<u>0-19</u>	<u>20</u>	<u>21-29</u>	<u>30-39</u>	<u>40+</u>	<u>All</u>
0 - 14	0.25	30	1.3	0.7	2.2	2.9
15 - 49	108*	0.25*	2.8	0.25	0.25	0.9
50 - 69	82	5	4.2	0.4	0.25	1.0
70+	250	12	1.2	1.3	0.25	2.0
All	108	11	2.0	0.7	0.25	1.6

* sample size 4 or less

Table 3.8: Median times till death (hours) of pedestrians, struck by cars, according to their age and the speed of impact

3.2.2 Results: effect of type of vehicle

Table 3.9 compares the median times till death after impact with different types of vehicle, with different speed groups being separated but all age groups combined. Applying Friedman's test (Langley, 1968) to this table, we find an effect of type of vehicle on time till death, significant at the 5% level. Survival tends to be longer when struck by a motorcycle and shorter when struck by a heavy goods vehicle.

<u>Speed (mph)</u>	<u>2-wheelers</u>	<u>cars</u>	<u>light goods</u>	<u>medium goods</u>	<u>heavy goods</u>	<u>public service vehicle</u>
0-19	150*	108	11	0.25	0.25	1.5
20	54*	11	1.5	48	0.25	1.1
21-29	0.7*	2	0.8	0.25*	0.25*	0.25*
30-39	2.4*	0.7	2.9	0.25*	0.8	26*
All	12	1.6	2.1	0.4	0.25	2.0

* sample size 4 or less

Table 3.9: Median hours till death of pedestrians killed by different types of vehicle.

3.2.3 Discussion

Robertson and Tonge (1968) examined times till death from several points of view, and some of their comparisons were found to be statistically significant in the large sample from Brisbane, though they were not confirmed in their Adelaide data. Thus they found a tendency for pedestrians to survive longer than car occupants and for older pedestrians to survive longer than younger ones. The latter finding is also true of the present study, when all speed groups are

combined. Sevitt (1973) also found that pedestrians tend to survive longer than vehicle occupants, though in his sample this did not quite reach statistical significance, and he notes a tendency for pedestrians dying more than seven days after the accident to be elderly and have relatively modest injuries. In the Brisbane study a tendency was found for accidents occurring between midnight and 4 am to be more rapidly lethal than those at other times, and for those in the suburbs or 'highway' to be more rapidly lethal than those in the city. For these comparisons, however, all classes of road user were grouped together. Nevertheless, if the present finding of association between high speed and quick death were to be true of other road users besides pedestrians, these findings would be readily interpreted, as one would expect accidents in the early hours of the morning to be faster than average, and those in the city to be slower than those in freer conditions.

National accident statistics from Morocco (1970) permit the examination of the effects of pedestrian age and locale (urban vs rural) on time till death. Table 3.10 gives the percentage of cases dead at the scene of the accident separated according to age and according to whether the police authority concerned was the "Surete National" or the "Gendarmerie Royale" (these corresponding roughly to urban and rural areas). The higher percentage of quick deaths in rural areas can be clearly seen. Possible reasons for this are more violent impact and delayed medical attention. There is some indication that, in urban areas at least, young adults die more quickly than the elderly.

The association of high speed with quick death is very plausible, the presumed mechanism being an increased incidence of critical initial injury, which will usually be to the head in pedestrians. It is

noteworthy that, when speed is allowed for, age has no effect on time till death. One might expect the elderly to be more fragile than the young, which would lead to them sustaining a worse injury from an impact of given energy; on the other hand, one might expect them to be more susceptible to complications. Evidently these two effects roughly cancel out, and the elderly are more likely to die at any time after the accident than the young are. Finally, type of vehicle has been shown to affect time till death, with pedestrians struck by motorcycles dying after a longer time, and those struck by heavy goods vehicle a shorter time, than those struck by cars.

<u>Age group of pedestrian.</u>	<u>Surete National</u> (urban)	<u>Gendarmerie Royale</u> (rural)
0 - 6	52	79
7 - 13	51	77
14 - 19	58	82
20 - 39	49	77
40 - 69	42	74
70+	40	80

Table 3.10: Percentages of fatally-injured pedestrians who died at the scene of the accident, separately for each of six age groups and two locales.

3.3 General comments on pedestrian accident research

It is often said that pedestrian safety is comparatively neglected in comparison with that of other road users. There is certainly some truth in this: even simple accident rates to pedestrians are usually expressed in terms of accidents per person in the population being considered, rather than per mile walked. Thus the well known fact that children have more accidents than average, for example, could be attributed to the possibility that they may walk more.

Recently, Goodwin and Hutchinson (1975) have gone some way to remedy this. They used National Travel Survey data collected by the Department of the Environment in 1972-3: information on virtually all journeys made during a week was obtained for a sample of households, and for one day of this week, all walking trips were included (not excluding those of under, say, one mile which is the usual procedure for such surveys). By comparing this data with national pedestrian accident data, Goodwin and Hutchinson were able to obtain accident rates for different age and sex groups, and for different hours of the day, days of the week, and months of the year. They used walking time rather than distance in the denominator of the rate since internal evidence from the survey suggested times were reported more accurately than distances. (In any case, time is arguably as good a measure of "exposure" as distance is.)

Figure 3.10 reproduces their results relating accident rate to age and sex. As to overall figures, respondents to the National Travel Survey indicated they spent about 19 minutes per day on walking trips. Grossing this figure up and assuming an average walking speed of about 3 mph enabled the pedestrian accident rate for the whole country to be

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Figure 3.10: pedestrian accident rate related to age and sex
(from Goodwin and Hutchinson, 1975)

expressed in the same terms as for other modes of transport, as shown in table 3.11. Goodwin and Hutchinson also showed that, for daylight hours, the number of pedestrian accidents is approximately proportional to the product of vehicle and pedestrian flows.

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Table 3.11: Accident risk while travelling by different modes
(from Goodwin and Hutchinson, 1975)

Returning now to the question of secondary safety (injury severity rather than accident rate), it does seem that it is more difficult to allow for speed at impact in pedestrian accidents than in others. British (and most other) accident data capture systems do not routinely include speeds, and even the original police reports are very incomplete in this respect, as discussed at the beginning of this Chapter. For vehicle crashes, damage can be compared with that in controlled impacts (reviewing the results of such tests conducted at TRRL, Grime,

Hutchinson, and Mollart found the equation $D = .048V - .65$ between the permanent deformation D of the car (feet) and speed of impact V into a massive concrete block (ft/sec).) For vehicle crashes also, it would be possible to fit some device onto a few thousand vehicles, that when it was decelerated sharply (in a crash) would indicate the change of velocity which the vehicle underwent. For collisions involving two vehicles, it should be possible to take advantage of the correlation between the injuries to the two drivers (as discussed in Chapter 8): if the other driver is more seriously hurt than usual where a particular model of car is concerned, we might infer that the speeds of the crashes that model is involved in are higher than usual, and take account of that when considering the degree of injury to its own driver. (Actually, there is an alternative, that the model concerned is more "aggressive" than usual. For further discussion, see section 8.5.)

It is not easy to see how any of these methods could be adapted to pedestrian accidents. Perhaps it will be best to rely on detailed on-the-spot studies, though the numbers are usually very small when considering individual models of car, and experimental and theoretical studies, as discussed in sections 2.5.2 and 2.5.3. There needs to be considerably more replication of experiments in this field. Reading reports such as those of Severy (1970) and Japan Automobile Manufacturers' Association (1968) one gets a strong impression of how few in number the tests were, how scattered the results, and how tenuous the conclusions.

CHAPTER 4: SOME STATISTICAL METHODS OF ANALYSING ACCIDENT DATA

In Chapter 2, where an examination was made of a large number of variables which might affect pedestrian injury severity, the statistical tests used were simple, consisting for the most part of analyses of $2 \times n$ contingency tables by means of the χ^2 test. There was an independent variable, such as road surface condition or model of car, and a dependent variable, the severity of injury. If another variable were associated both with the independent variable and with the dependent variable, then if the independent variable was found to affect injury severity, it could be due to the action of the third variable. Consequently the strategy used in that Chapter was to filter the cases included on all variables which might interfere in this way.

Thus when the effect of model of car was examined (table 2.30), the accidents included were restricted to those with a 30 mph speed limit (because it might be that models of car are driven at different speeds when the speed limit is higher or absent, and speed will certainly be related to degree of injury) and those injured were all in the 15-49 years age group, since age is so strongly related to degree of injury beyond about age 50 (table 2.1). It is not very plausible that there should be any consistent association between model of car and age of pedestrian hit, but when the effect of age on severity is as strong as in table 2.1, even quite small random variation in average age of injured pedestrian between models of car could affect the apparent effect of the latter.

This strategy was chosen because the number of independent variables to be investigated was large, and any interactions (the extent to which the effect of one independent variable on injury

severity is different according to the category assumed by another independent variable) were thought to be probably small and uninteresting. Furthermore, with the large total number of cases, it was feasible to quite markedly restrict those included in a particular table while still retaining an adequate sample size.

However, there are many instances in accident research where a more complex statistical test is needed, and two such techniques of general usefulness will be discussed in this Chapter. Others of more specialised application are mentioned at appropriate points in other Chapters (section 3.1.7, an explanation of intra-class correlation and point biserial correlation; section 6.5, a new test involving the combination of two-tailed rank correlation statistics).

Section 4.1 will outline by means of a number of examples from traffic and accident research a general method of analysing tables of frequencies, using a computer program developed at the Department of Biostatistics, University of North Carolina, called CATLIN: the examples given are chosen to lead from simple analyses which could be treated by well-known techniques to complex ones involving regression of proportions and missing data. Example 1 is a test for interaction in an ordinary, two dimensional, contingency table. Examples 2 and 3 use data on the interrelationships between type of accident, model of car, and degree of injury. Examples 4 and 5 both take data published by other workers (on the flow of traffic at a junction and the response rate to a questionnaire on vehicle usage respectively) and analyse them according to linear and quadratic regression models respectively. CATLIN is put to use in Chapters 5 and 6, in the former using the data which, in collapsed form, provides Examples 2 and 3. Thus sections 5.2.2, 5.2.3, and 6.2 may be used as additional examples for understanding CATLIN.

Section 4.2 will turn to a nonparametric test which subsumes a number of simpler ones used elsewhere in this thesis: Friedman's, Kruskal and Wallis's, and Spearman's. Since the test is rather involved computationally (as is clear even from the simple example given using contrived data), a FORTRAN program is presented which carries out the test. This test has already been used in section 3.2.1 and will be used again in section 5.2.4 to confirm the results of the analysis using CATLIN.

4.1 The program CATLIN

4.1.1 Introduction

There are a number of computer programs and packages available that will perform general linear model analysis on measurement-data, the best known of which is probably BMD (Dixon, 1971). However, in accident analysis we do not often have quantitative data: categorical (qualitative) and ordinal data are more common.

Very often our data consists of a number of observations simultaneously classified according to several factors. For instance, we might have single-vehicle accidents classified according to type of vehicle involved (motorcycle, car, or commercial vehicle), site of the accident (urban or rural), age of the driver (less than 30, 30-50 years, over 50) and severity (fatal, serious, or slight). Recently, the analysis of such tables has been made much easier by a program developed at the Department of Biostatistics, University of North Carolina*, which provides for categorical data the same sort of facilities that general linear model programs such as BMD06V do for measurement-data. This program, called CATLIN, is described by Grizzle, Starmer, and Koch (1969) and by Forthofer, Starmer, and Grizzle (1971). The purpose of this section is to show how it can be applied to a number of situations of interest in traffic and accident research.

There are two crucial points to remember about the tables which CATLIN can analyse: (i) the entries in them are numbers of observations (not actual measurements), and (ii) the observations are independent -

* The present writer was in no way involved in the writing of the computer programs to be described herein. He is also not qualified to comment on the statistical theory involved.

an example of lack of independence would be classifying all occupants of vehicles involved in accidents according to type of accident they were involved in and type of vehicle they were occupying: for vehicles with more than one occupant they would all necessarily be classified identically on both criteria. The correct way to express such data would be to classify each vehicle rather than each occupant.

It is easiest to explain what CATLIN does by using matrix notation. We first remind the reader of the basics of matrix arithmetic.

A matrix is a rectangular array of numbers. Thus $\begin{pmatrix} 3 & 2 \\ 0 & 5 \\ 1 & 2 \end{pmatrix}$ is a three-by-two matrix. Matrices can be multiplied only if the number of columns in the first is the same as the number of rows in the second. So $\begin{pmatrix} 3 & 2 \\ 0 & 5 \\ 1 & 2 \end{pmatrix}$ multiplied by $\begin{pmatrix} 6 & 5 \\ 2 & 1 \end{pmatrix}$ is allowed, but $\begin{pmatrix} 6 & 5 \\ 2 & 1 \end{pmatrix}$ multiplied by $\begin{pmatrix} 3 & 2 \\ 0 & 5 \\ 1 & 2 \end{pmatrix}$ is not. If an $\ell \times m$ matrix is multiplied by an $m \times n$, the result is an $\ell \times n$ matrix. The elements of it are defined as in this example

$$\begin{pmatrix} 3 & 2 \\ 0 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 6 & 5 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 22 & 17 \\ 10 & 5 \\ 10 & 7 \end{pmatrix}$$

since

$$22 = 3 \times 6 + 2 \times 2$$

$$17 = 3 \times 5 + 2 \times 1$$

$$10 = 0 \times 6 + 5 \times 2$$

etc.

The ij th element of the resulting matrix, c_{ij} , is given by $c_{ij} = \sum_k a_{ik} b_{kj}$ where a_{ij} and b_{ij} are the ij th elements of the multiplying matrices.

Simultaneous equations can be expressed in matrix notation.

Thus

$$3x + 2y + z = 0$$

$$x + y - 2z = 5$$

$$2y + 3z = -4$$

can also be written

$$\begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & -2 \\ 0 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -4 \end{pmatrix}$$

A matrix with only a single column is known as a column vector and one with only one row is known as a row vector.

Further details of matrix algebra can be found in most textbooks of mathematics, for example Durrell and Robson (1964). In what follows a matrix will be denoted by underlining, so a matrix equation might be expressed $\underline{AB} = \underline{C}\underline{\beta}$.

4.1.2 Type of problem which can be analysed

The use of CATLIN will most easily be explained by a series of examples, but first it is necessary to explain further the type of problems which it can attack. It is a program for testing hypotheses, and it is necessary to tell it what the hypotheses are. (It is allowed to include unknown parameters in the hypotheses: the program will fit them to the data.) If the data consists of m populations, and each observation is classified into one of n alternative responses, then let the probability in the i th population of the j th response occurring be π_{ij} . Arrange these π_{ij} into a vector $\underline{\pi}$ thus: $(\pi_{11}, \pi_{12}, \pi_{13}, \dots, \pi_{1n}, \pi_{21}, \pi_{22}, \dots, \pi_{2n}, \dots, \pi_{m1}, \pi_{m2}, \dots, \pi_{mn})$. Thus the first n elements of this vector refer to the first population, the second n to the second population, ... and the m th n elements to the m th population. There are four slightly different forms of hypothesis that CATLIN can test: the simplest may be expressed as follows -

$$\underline{A} \underline{\pi} = \underline{0}$$

\underline{A} is a matrix supplied by the user. If \underline{A} has l rows, this equation

* $\underline{\pi}$ is actually a column vector, but it will at times be more convenient to give it in row vector form, in order to save space.

is equivalent to ℓ simultaneous linear equations in the π_{ij} 's, the right hand side of each equation being zero, $\underline{0}$ meaning a matrix all of whose elements are zero.

The second type of hypothesis is:

$$\underline{K} \log_e (\underline{A} \underline{\pi}) = \underline{0}$$

(By $\log (\underline{M})$ we mean the matrix whose elements consist of the logarithms of the corresponding elements of matrix \underline{M} .) Both \underline{A} and \underline{K} are at the user's discretion. If \underline{A} has ℓ rows, and \underline{K} has ℓ columns and k rows, the left hand side of this equation is equivalent to k expressions, each consisting of a linear combination of the natural logarithms of a linear combination of the probabilities π_{ij} .

The third type of hypothesis involves the program in fitting a number of constants to the data. It may be expressed:

$$\underline{A} \underline{\pi} = \underline{X} \underline{\beta}$$

where \underline{A} and \underline{X} are user-supplied, and $\underline{\beta}$ is a vector of constants the values of which are calculated by the program so as to provide the best fit to the data.

The final type of hypothesis is:

$$\underline{K} \log_e (\underline{A} \underline{\pi}) = \underline{X} \underline{\beta}$$

The program calculates a χ^2 (chi-squared) statistic which tells us whether the hypothesis we tested provides a good fit to the data.

(The statistic most often referred to as "chi-squared" is given by Pearson's formula $\sum (O-E)^2 / E$. (Where O is the observed number of cases falling into a particular cell and E is the corresponding number predicted by the hypothesis we are testing.) There are, however, many other formulae whose results are also distributed as χ^2 under the appropriate hypothesis. Two of the best-known alternatives are Neyman's modified χ^2 , $\sum (O-E)^2 / O$, and the log likelihood statistic, $\sum 2(O) \log_e (O/E)$, which will be briefly mentioned again in Chapter 8 (table 8.9). The conditions under which a statistic is distributed as χ^2 are discussed by Lancaster (1969) and Johnson and Kotz (1970b, Chapter 29). CATLIN uses the minimum logit χ^2 , as advocated by Berkson (1968). The formulae are given, along with the associated statistical theory, by Grizzle, Starmer and Koch (1969). This is chosen because the calculations are then directly analogous to the case where continuous (as opposed to grouped or classified) data are analysed by linear models such as multiple regression; and because calculations using maximum likelihood estimates frequently require an iterative procedure in cases where the minimum logit χ^2 gives the results directly. The procedure is asymptotically equivalent to the maximum likelihood method.)

In addition the program calculates values of the parameters, $\underline{\beta}$, which provide the best fit (in a weighted least-squares sense) to the data.

The program also allows hypotheses about these parameters to be tested. By supplying a matrix \underline{C} the hypothesis $\underline{C} \underline{\beta} = \underline{0}$ can be tested.

I hope this will become clear by means of the following examples.

4.1.3 Example 1: interaction in an ordinary contingency table

Consider table 4.1. One of the hypotheses we most often wish to test in such tables is whether there is any interaction between dimensions A and B. That is, whether the proportions classified as x, y, or z on factor A are the same for each classification of factor B; equivalently, whether the proportions which are p, q, or r on factor B are the same for each classification of factor A.

This is usually done by saying that the expected number E_{ij} of observations in the ij th cell on this hypothesis is given by

$$E_{ij} = \frac{N_{i.} N_{.j}}{N_{..}}$$

where $N_{i.}$ is the total number of cases in the i th row, $N_{.j}$ is the total number of cases in the j th column, and $N_{..}$ is the total number of cases in the whole table. Then Pearson's χ^2 statistic is calculated by the formula

$$\chi^2 = \sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where O_{ij} is the observed number of cases in the ij th cell of the table.

It can be done using CATLIN as follows: the null hypothesis, that there is no association between classification on dimension A and on dimension B can be expressed by the following equations:

$$\frac{\pi_1}{\pi_3} = \frac{\pi_4}{\pi_6} = \frac{\pi_7}{\pi_9}$$

$$\frac{\pi_2}{\pi_3} = \frac{\pi_5}{\pi_6} = \frac{\pi_8}{\pi_9}$$

where the π 's are the probabilities of falling in each of the cells of the table, the cells being numbered from left to right and top to bottom, thus:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Thus the expected number classified as being both 'q' and 'y' would be π_5 . (Notice that these equations also imply $\frac{\pi_1}{\pi_4} = \frac{\pi_2}{\pi_6}$, $\frac{\pi_1}{\pi_7} = \frac{\pi_3}{\pi_9}$, etc.) These equations can be expressed in the form $\underline{K} \log_e (\underline{A} \underline{\pi}) = \underline{0}$ if

$$\underline{\pi} = (\pi_1, \pi_2, \pi_3, \dots, \pi_9),$$

$$\underline{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{and } \underline{K} = \begin{pmatrix} 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & -1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix},$$

since $\log_e (\underline{A} \underline{\pi})$ then equals

$$\begin{pmatrix} \log_e \pi_1 \\ \log_e \pi_2 \\ \cdot \\ \cdot \\ \log_e \pi_9 \end{pmatrix} \quad \text{and so}$$

$$\underline{K} \log_e (\underline{A} \underline{\pi}) \text{ is } \begin{pmatrix} \log_e \pi_1 - \log_e \pi_3 - \log_e \pi_4 + \log_e \pi_6 \\ \log_e \pi_1 - \log_e \pi_3 - \log_e \pi_7 + \log_e \pi_9 \\ \log_e \pi_2 - \log_e \pi_3 - \log_e \pi_5 + \log_e \pi_6 \\ \log_e \pi_2 - \log_e \pi_3 - \log_e \pi_8 + \log_e \pi_9 \end{pmatrix}$$

Putting this equal to zero means that

$$\log_e \frac{\pi_1 \pi_6}{\pi_3 \pi_4} = \log_e \frac{\pi_1 \pi_9}{\pi_3 \pi_7} = \log_e \frac{\pi_2 \pi_6}{\pi_3 \pi_5} = \log_e \frac{\pi_2 \pi_9}{\pi_3 \pi_8} = \text{zero},$$

$$\text{which in turn means that } \frac{\pi_1 \pi_6}{\pi_3 \pi_4} = \frac{\pi_1 \pi_9}{\pi_3 \pi_7} = \frac{\pi_2 \pi_6}{\pi_3 \pi_5} = \frac{\pi_2 \pi_9}{\pi_3 \pi_8} = 1,$$

as required.

Thus by supplying the data of table 4.1, and the matrices A and K, this hypothesis may be tested. The resulting value of χ^2 is 26.1, and this has 4 degrees of freedom, and is much larger than could reasonably have occurred under the null hypothesis: it is statistically significant at the $P < .001$ level.

		Classification on factor A			
		x	y	z	Total
Classification on factor B	p	5	10	15	30
	q	25	10	5	40
	r	5	20	15	40
	Total	35	40	35	110

Table 4.1: An example of a two-dimensional frequency table

4.1.4 Example 2: a contingency table with an empty cell

Table 4.2 gives the numbers of accidents of four different types in which several different models of car were involved, taken from the study reported in full in Chapter 5. The condition for an accident to be included in this sample was that one of the five models of car A - E was involved. The "others" category refers to other models of

car which collided with one or other of the five selected models. Thus in this table, the sampling procedure used means that the number of cases in one cell of the table (other models, single-vehicle accidents) is necessarily zero. How can we test whether different models of car are involved in the different types of accident in different proportions?

The null hypothesis is that the relative proportions of the four different types of accident are the same for the five models A - E, and that the relative proportions of the collision accidents are the same for "other" models as for each of the five selected models. Numbering the cells from left to right and top to bottom thus:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ & \cdot & \cdot & \cdot \\ 21 & 22 & 23 & 24 \end{bmatrix}$$

we express the null hypothesis in the form

$$\frac{\pi_1}{\pi_2} = \frac{\pi_5}{\pi_6} = \frac{\pi_9}{\pi_{10}} = \frac{\pi_{13}}{\pi_{14}} = \frac{\pi_{17}}{\pi_{18}} = \frac{\pi_{21}}{\pi_{22}}$$

$$\frac{\pi_1}{\pi_3} = \frac{\pi_5}{\pi_7} = \frac{\pi_9}{\pi_{11}} = \frac{\pi_{13}}{\pi_{15}} = \frac{\pi_{17}}{\pi_{19}} = \frac{\pi_{21}}{\pi_{23}}$$

$$\frac{\pi_1}{\pi_4} = \frac{\pi_5}{\pi_8} = \frac{\pi_9}{\pi_{12}} = \frac{\pi_{13}}{\pi_{16}} = \frac{\pi_{17}}{\pi_{20}}$$

This fits into the format $\underline{K} \log_e (\underline{A} \underline{\pi}) = \underline{0}$

$$\text{if } \underline{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 & 0 \\ & \\ & \\ 0 & 1 \end{pmatrix}$$

(identity matrix of size 24 x 24)

The result is $\chi^2 = 12.4$, with 10 degrees of freedom, which is not statistically significant.

These results suggest that there are differences between models in the types of accident they are involved in, but that these largely arise from differences in the relative proportions of single-vehicle and collision accidents.

	Collision accidents			Single-vehicle accidents
	Head-on	Rear-end	Intersection	
Model A	23	34	43	35
B ₂	8	3	13	23
B ₁	13	10	12	26
H	6	14	18	25
F	9	6	19	17
Others	67	63	92	0

Table 4.2: Types of accident in which certain models of car were involved in, the sample being drawn from serious accidents in London in 1971. The sampling procedure used ensured that there were no models other than A - E involved in single-vehicle accidents.

4.1.5 Example 3: a contingency table in which one dimension is ordered

When one dimension of a contingency table is ordered (for instance, into no injury, minor, serious, and fatal injury) it is often desired to test whether the alternative categories of the second variable differ in the level of the first variable, rather than the more general test of any difference in distribution of observations. The procedure (Cochran, 1954) involves calculating the mean and variance of the ordered variable for each category of the second variable, and comparing

the means. It is very similar to one-way analysis of variance.

A variant version of CATLIN, called LINCAT, is very suitable for analysing such problems. If we consider each category of the second dimension to be a separate population, then LINCAT is applicable if the functions of π formed by multiplying it by \underline{A} each consist of combinations of the π_{ij} formed only within populations, and furthermore, the function formed within each population is the same. That is, \underline{A} has the following form:

$$\begin{pmatrix} \underline{A}^* & \underline{0} & \underline{0} & \dots & \dots & \dots & \underline{0} \\ \underline{0} & \underline{A}^* & \underline{0} & \dots & \dots & \dots & \underline{0} \\ \underline{0} & \underline{0} & \underline{A}^* & \dots & \dots & \dots & \underline{0} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \underline{0} & \underline{0} & \underline{0} & \dots & \dots & \dots & \underline{A}^* \end{pmatrix}$$

where \underline{A}^* is a matrix with the same number of columns as the number of responses within each population, and $\underline{0}$ is a matrix of the same shape as \underline{A}^* but consisting entirely of zeros.

LINCAT is advantageous over CATLIN in such cases because it saves computer storage space and because the small matrix \underline{A}^* is input instead of the much larger \underline{A} .

What is meant by an average score when a variable is ordered, but there exists no objective measurement of it, as is the case for injury severity? To compute an average score we need to assign to each of the categories a score. One rule by which we can do this is simply to number them 1, 2, 3, An alternative is based on the midpoint within a category of the cumulative distribution of cases. Thus if there were five categories, ordered A, B, C, D, E, accounting for respectively 10%, 20%, 20%, 30%, and 20% of cases in the total

population, scores assigned would be: $\frac{10}{2} = 5$ for category A, $10 + \frac{20}{2} = 20$ for category B, $10 + 20 + \frac{20}{2} = 40$ for category C, $10 + 20 + 20 + \frac{30}{2} = 65$ for D, and $10 + 20 + 20 + 30 + \frac{20}{2} = 90$ for E. We use this latter approach in the example which follows.

In table 4.3 are given the injuries to the legs of drivers of several models of car, again referring to the study reported in Chapter 5. We wish to test whether the average leg injury severity is different for different models of car. We first define average injury severity to be $.369p_1 + .852p_2 + .983p_3$ where p_1 , p_2 , and p_3 are the probabilities of no leg injury, of minor leg injury, and of severe leg injury, respectively. Denoting the average leg injury severity to drivers of the i th model of car by S_i , the null hypothesis that we wish to test is:

$$S_1 = S_2 = S_3 = S_4 = S_5 = S_6$$

This can be done within the format

$$\underline{K} \log_e \left[\begin{pmatrix} \underline{A^*} & \underline{0} & \cdot & \cdot & \cdot & \underline{0} \\ \underline{0} & \underline{A^*} & \cdot & \cdot & \cdot & \underline{0} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \underline{0} & \underline{0} & \cdot & \cdot & \cdot & \underline{A^*} \end{pmatrix} \underline{\pi} \right] = \underline{0}$$

by setting

$$\underline{A^*} = (.369 \quad .852 \quad .983)$$

$$\underline{K} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

The result is $\chi^2 = 15.2$, which with 5 degrees of freedom is significant at the $P < .01$ level, showing that different models of car injure their drivers' legs to different degrees.

In view of the finding of an association between model of car and type of accident, it would be unwise to draw conclusions from this, particularly since type of accident is associated with degree of leg injury. It is shown in Chapter 5 how both model of car and type of accident can simultaneously be related to degree of injury.

	<u>No injury</u>	<u>Slight injury</u>	<u>Severe injury</u>
Model A	85	43	7
B	36	9	2
C	52	7	2
D	43	19	1
E	39	11	1
Others	172	43	7

Table 4.3: Degree of leg injury to drivers of certain models of car involved in frontal impacts.

4.1.6 Example 4: an example involving regression

So far, we have considered testing hypotheses of the forms $\underline{A} \underline{\pi} = \underline{0}$ and $\underline{K} \log_e(\underline{A} \underline{\pi}) = \underline{0}$. We now turn to the more general type of equation which CATLIN and LINCAT can consider, in which the right hand side of the equation is some function of some unknown parameters, expressed in matrix notation as $\underline{X} \underline{\beta}$ where $\underline{\beta}$ is a column vector of parameters which will be calculated by the program and \underline{X} is a design matrix supplied by the user.

$$\underline{X} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 1 & 6 & 0 & 0 \\ 0 & 0 & 1 & 6 \\ 1 & 7 & 0 & 0 \\ 0 & 0 & 1 & 7 \\ 1 & 8 & 0 & 0 \\ 0 & 0 & 1 & 8 \end{pmatrix}$$

(16 rows, 4 columns)

The left hand side of the equation then becomes

$$\begin{pmatrix} \log_e(\pi_{-1}) \\ \log_e(\pi_{+1}) \\ \log_e(\pi_{-2}) \\ \log_e(\pi_{+2}) \\ \cdot \\ \cdot \\ \cdot \\ \log_e(\pi_{+8}) \end{pmatrix}$$

and the right hand side is

$$\begin{pmatrix} \beta_1 + \beta_2 \\ \beta_3 + \beta_4 \\ \beta_1 + 2\beta_2 \\ \beta_3 + 2\beta_4 \\ \cdot \\ \cdot \\ \beta_3 + 8\beta_4 \end{pmatrix}$$

The parameters in the vector $\underline{\beta}$ are then interpreted as follows:

$$\begin{aligned} \beta_1 &= a \\ \beta_2 &= b \\ \beta_3 &= c \\ \beta_4 &= d \end{aligned}$$

(Remember that flow category 1 corresponds to 0-1 vehicles per half-minute, flow category 2 to 2-3 vehicles per half-minute, etc., i.e. the numbers used (i) correspond to vehicles per half-minute (v) according to the equation $v = 2i - 1\frac{1}{2}$. Thus $a + bi = a + \frac{3}{4}b + \frac{1}{2}b v$ if we want an equation directly in terms of the vehicle flow per half-minute.)

LINCAT can analyse this problem by taking $\underline{A}^* = (1 \ 0)$, and the result is $\chi^2 = 4.8$, which with 12 degrees of freedom is not statistically significant, thus telling us that this model provides a good fit to the data. The values of the parameters a, b, c, d found by the program are as follows: $a = -.534$, $b = -.185$, $c = -.041$, $d = -.155$.

We therefore have two lines, one applying to queue state + and the other to queue state - . They are plotted in figure 4.2. Are they significantly different? We can test whether their intercepts are different and whether their slopes are different. The null hypothesis that there is no difference in their intercepts can be put in the form $\underline{C} \underline{\beta} = \underline{0}$ if

$$\underline{C} = (1 \ 0 \ -1 \ 0) \ .$$

The result is $\chi^2 = 4.6$, 1 degree of freedom, $P < .05$. Thus this value of χ^2 is too large to have reasonably occurred by chance and we can conclude that the intercepts of the lines are significantly different.

The null hypothesis that there is no difference in the slopes of the lines can be put in the form $\underline{C} \underline{\beta} = \underline{0}$ if

Image removed due to third party copyright

Figure 4.1: Illustration of the traffic flows observed by Hawket (1975).

Flow (veh. per half minute)

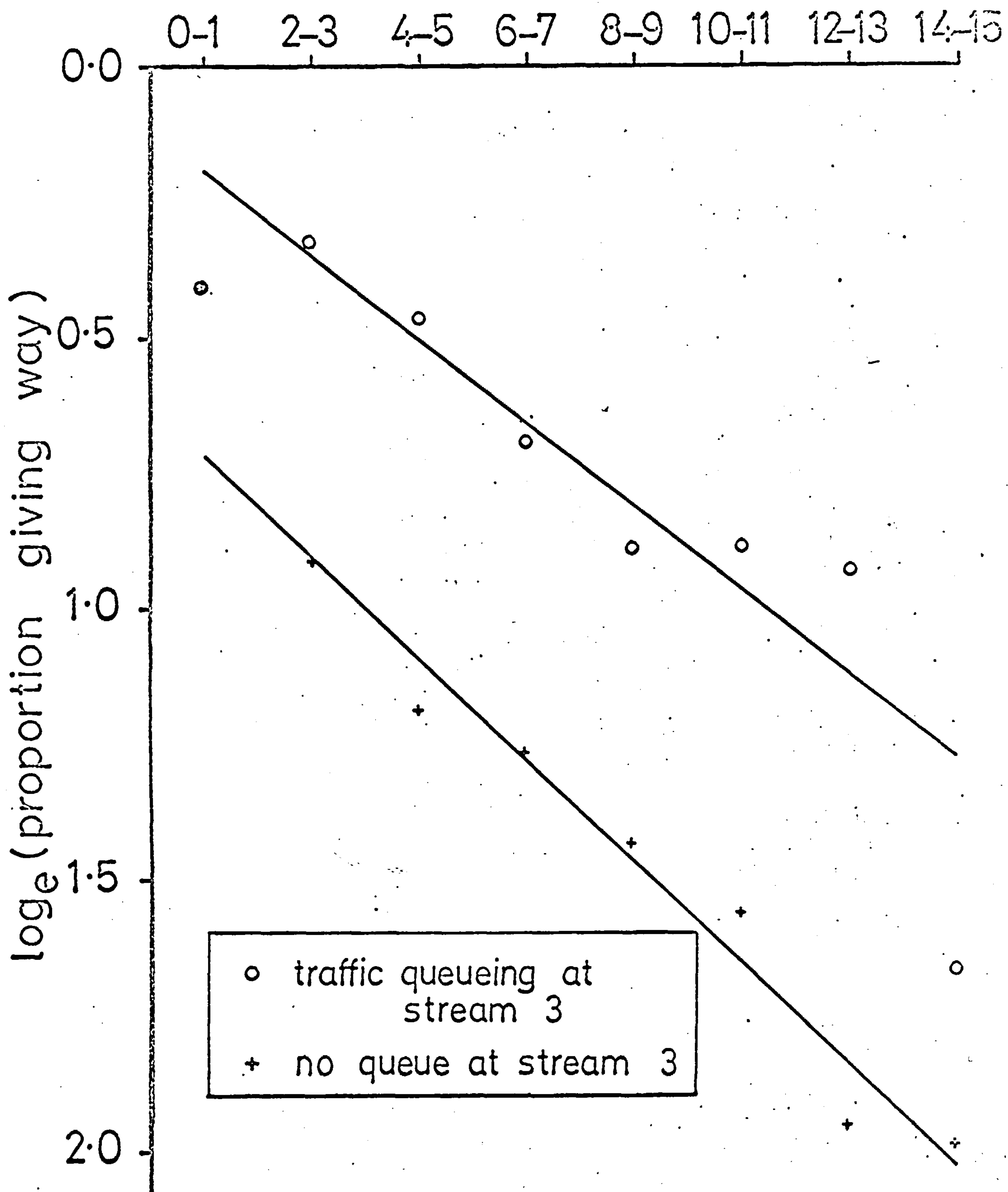


Figure 4.2: Comparison of observed proportions with the model fitted using LINCAT (Hawlett's data).

$$\underline{C} = (0 \ 1 \ 0 \ -1)$$

The result is $\chi^2 = 0.4$, 1 degree of freedom, not significant, thus showing that the two lines could have the same slope.

<u>Flow downstream of junction</u> (vehicles per half-minute)	<u>Queue state of stream 3*</u>	<u>Number giving way</u>	<u>Number not giving way</u>
0 - 1	-	2	1
0 - 1	+	2	1
2 - 3	-	12	18
2 - 3	+	18	7
4 - 5	-	28	64
4 - 5	+	42	25
6 - 7	-	45	115
6 - 7	+	66	66
8 - 9	-	56	180
8 - 9	+	43	62
10 - 11	-	29	110
10 - 11	+	39	56
12 - 13	-	15	92
12 - 13	+	11	17
14 - 15	-	10	63
14 - 15	+	6	26

*
+ = traffic queueing at ③
- = no traffic queueing at ③

Table 4.4: Numbers of drivers in stream 1 who do or do not give way to traffic from stream 2, at different levels of flow in stream 1 and 2 combined, and different queue states in stream 3.

4.1.7 Example 5: an example in which some data is mixed-up or missing

This is taken from a survey by Foldvary (1969) of vehicle mileage in Queensland, Australia. He reports on the effectiveness of three variants of a questionnaire in obtaining information about the mileage driven by the people to whom it was sent. The question at issue is whether the questionnaires (referred to as A, B, and C) differ in their effectiveness, as measured by the response rate to them. Table 4.5 has been reconstructed from part of Foldvary's Table 1. It gives the numbers of people replying and not replying to the three questionnaires, matched by the fortnightly period in which they were sent out (since season of the year might affect response rate).

There are two additional points to note about this table. Firstly, in fortnights 8 and 9 for questionnaires A and B only the combined average return rate is available. Secondly, questionnaire C was not mailed in the final two fortnights of the survey.

We first set up a model of the return rate. If p_{ij} is the proportion of questionnaires of type j returned in the i th fortnight, one model we might choose is

$$p_{ij} = \mu + \phi_i + \gamma_j \quad (4.1)$$

together with the constraints $\sum \phi_i = \sum \gamma_j = 0$, where μ is the base level of response rate, the ϕ_i are the differential effects of the fortnight periods, and the γ_j are the differential effects of the alternative questionnaires.

Note: we choose equation (4.1) rather than $p_{ij} = F_i + G_j$ because the latter has too many parameters: given one set of F 's and

Table 4.5: Data from Foldvary (1969) giving numbers of people replying to three slightly different forms of a questionnaire (A, B, C) in each fortnightly period in a total of six months.

G's, we can construct another giving exactly the same fit to the data by adding any number Δ to all the F's and subtracting the same number from all the G's. Expressing the equation instead as a base level plus differential effects gives the right number of parameters, one μ , plus (in this case) twelve ϕ 's and two γ 's (since the ϕ for the 13th fortnight is given by $-\sum_{i=1}^{12} \phi_i$ and the γ for questionnaire C is $(-\gamma_1 - \gamma_2)$).

Most of the simultaneous equations that this implies are of the form $p_{11} = \mu + \phi_1 + \gamma_1$, $p_{12} = \mu + \phi_1 + \gamma_2$, $p_{13} = \mu + \phi_1 - \gamma_1 - \gamma_2$, etc, but for fortnights 8 and 9 they are of the form $p_{8*} = \mu + \phi_8 + (\gamma_1 + \gamma_2)/2$, where p_{8*} is the response rate to the combined population of questionnaires A and B in the 8th fortnight, and for fortnights 12 and 13 we have no equations relating to questionnaire C.

If the populations are ordered by fortnight, and within each fortnight by questionnaire, as in table 4.5, we can express the equations in the form $\underline{A} \underline{\pi} = \underline{X} \underline{\beta}$ thus:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix} \begin{pmatrix} p_{11} \\ 1 - p_{11} \\ p_{12} \\ 1 - p_{12} \\ \vdots \\ p_{8*} \\ 1 - p_{8*} \\ p_{83} \\ 1 - p_{83} \\ p_{9*} \\ 1 - p_{9*} \\ p_{93} \\ 1 - p_{93} \\ \vdots \\ p_{12,1} \\ 1 - p_{12,1} \\ p_{12,2} \\ 1 - p_{12,2} \\ p_{13,1} \\ 1 - p_{13,1} \\ p_{13,2} \\ 1 - p_{13,2} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & \dots & 0 & 0 & \dots & 0 & 1 & 0 \\ 1 & 1 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 1 \\ 1 & 1 & 0 & \dots & 0 & 0 & \dots & 0 & -1 & -1 \\ 1 & 0 & 1 & \dots & 0 & 0 & \dots & 0 & 1 & 0 \\ 1 & 0 & 1 & \dots & 0 & 0 & \dots & 0 & 0 & 1 \\ \vdots & & & & & & & & & \\ 1 & 0 & 0 & \dots & 1 & 0 & \dots & 0 & .5 & .5 \\ 1 & 0 & 0 & \dots & 1 & 0 & \dots & 0 & -1 & -1 \\ 1 & 0 & 0 & \dots & 0 & 1 & \dots & 0 & .5 & .5 \\ 1 & 0 & 0 & \dots & 0 & 1 & \dots & 0 & -1 & -1 \\ \vdots & & & & & & & & & \\ 1 & 0 & 0 & \dots & 0 & 0 & \dots & 1 & 1 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 & \dots & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mu \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_{12} \\ \gamma_1 \\ \gamma_2 \end{pmatrix}$$

A π X β

(35 rows, 70 columns)

(35 rows, 15 columns)

The output from the program includes the parameters μ , ϕ_1 , ϕ_2 , ..., ϕ_{12} , γ_1 , γ_2 , together with a value of χ^2 referring to the fit of the model. $\chi^2 = 26.8$, with 20 degrees of freedom, and is not statistically significant. Thus we can accept our additive model as providing quite a good fit to the data.

We can test whether there is any effect of fortnight on response rate by testing the null hypothesis $\phi_1 = \phi_2 = \phi_3 = \dots = \phi_{12} = 0$, in the format C β = 0, by putting

$$\underline{C} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

The result is $\chi^2 = 25.6$, 12 degrees of freedom, $P < .05$. Thus we can reject the null hypothesis that fortnightly period had no effect on response rate.

The test of whether the questionnaires A, B, C differed in their response rate involves the null hypothesis $\gamma_1 = \gamma_2 = 0$:

$$\underline{C} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\chi^2 = 62.8$, 2 degrees of freedom, highly statistically significant. Thus questionnaire design did affect response rate.

The differences between the questionnaires were as follows: A and B were similar except that the registration number of the respondent's car was asked for in questionnaire A but not in B. Questionnaire C was similar to A except that certain additional questions were asked. Thus if there is a difference in response rate to questionnaires A and B we can attribute it to the question about registration number. If there is a difference between A and C we can attribute it to the presence or absence of the additional questions.

The null hypothesis that there is no difference in response rates

to questionnaires A and B can be expressed $\gamma_1 = \gamma_2$, and can be put in the form $\underline{C} \underline{\beta} = \underline{0}$ if

$$\underline{C} = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ -1)$$

$\chi^2 = 29.6$, 1 degree of freedom, highly statistically significant.

Thus presence or absence of a request for the registration number does affect response rate. (It was higher for questionnaire A, that is asking for this information increased response rate, contrary to expectation.)

The null hypothesis that there is no difference in response rate to questionnaires A and C can be expressed $\gamma_1 = -\gamma_1 - \gamma_2$, and can be put in the form $\underline{C} \underline{\beta} = \underline{0}$ if

$$\underline{C} = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 1)$$

$\chi^2 = 2.3$, 1 degree of freedom, not statistically significant.

Thus the data does not disprove the hypothesis that response rates to questionnaires A and C are equal. That is, we have no evidence that the presence or absence of the additional questions in questionnaire C affected the response rate.

In the above analysis, no account has been taken of the fact that the fortnightly periods are ordered - they are successive fortnights from July to December. If there is a true effect of fortnight, we would expect it to change gradually through the year. Indeed, it would be possible for the model discussed above to show no significant effect of fortnight, while a model which took into account the ordering of the fortnights would do so. As it is, the above model showed a fortnight effect significant at the 5% level, and the model to be discussed below

will confirm the significance of the fortnight effect at a much higher statistical level.

How should we represent a seasonal effect over a period of six months? It is unreasonable to expect a linear relationship to represent a seasonal effect over a period as long as six months, since it is likely that the six months might include a maximum or a minimum of the seasonal effect. The simplest way to include this in our model is to represent a fortnight effect as a parabola. That is, assume the ϕ_i of the previous model can be expressed as $a + bi + ci^2$. We then have $p_{ij} = \mu + bi + ci^2 + \gamma_j$, the constant a having been absorbed in μ . We put this model in the form $\underline{A} \underline{\pi} = \underline{X} \underline{\beta}$ by keeping \underline{A} the same as in the previous model, but setting

$$\underline{X} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 2 & 4 & 1 & 0 \\ 1 & 2 & 4 & 0 & 1 \\ 1 & 2 & 4 & -1 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 8 & 64 & .5 & .5 \\ 1 & 8 & 64 & -1 & -1 \\ 1 & 9 & 81 & .5 & .5 \\ 1 & 9 & 81 & -1 & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 13 & 169 & 1 & 0 \\ 1 & 13 & 169 & 0 & 1 \end{pmatrix}$$

The vector $\underline{\beta}$ will then consist of $(\mu \ b \ c \ \gamma_1 \ \gamma_2)$. The values of these found by the program are given in table 4.6. In figure 4.3 the response rate to each questionnaire is plotted against fortnight period. In addition, the predictions made by this model are shown, that is

$$P_A = .363 - .0105i + .00063i^2 + .011 = .374 - .0105i + .00063i^2$$

$$P_B = .363 - .0105i + .00063i^2 - .030 = .333 - .0105i + .00063i^2$$

$$P_C = .363 - .0105i + .00063i^2 - .011 + .030 = .382 - .0105i + .00063i^2$$

The fit of this model is tested by χ^2 , which turns out to be 36.6, which with 30 degrees of freedom is not statistically significant, thus showing the model is a good fit to the data. When the significance of the effect of questionnaire design is tested, the results are almost identical to the previous ones. The null hypothesis that there is no effect of fortnight is now $b = c = 0$, tested by $\underline{C} \underline{\beta} = \underline{0}$, where

$$\underline{C} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

The result is $\chi^2 = 15.9$, 2 degrees of freedom, significant at the $P < .001$ level.

It is clear from figure 4.3 that there is much scatter of the points about their respective curves, but the non-significant value of χ^2 tells us that this is not enough for us to be able to reject the model. And the fact that the coefficients of the linear and quadratic terms in the equation describing the seasonal effect are together significantly different from zero tells us that the curves in figure 4.3 account for a significant amount of variation in the scatter of the points.

Foldvary (1969) analysed this table by carrying out two analyses of variance - firstly comparing questionnaires A and B, matching for fortnight: and secondly comparing A with C, also matching for fortnight. In both cases the effect of questionnaire was significant but the effect of fortnight was not significant. However, Foldvary also had data

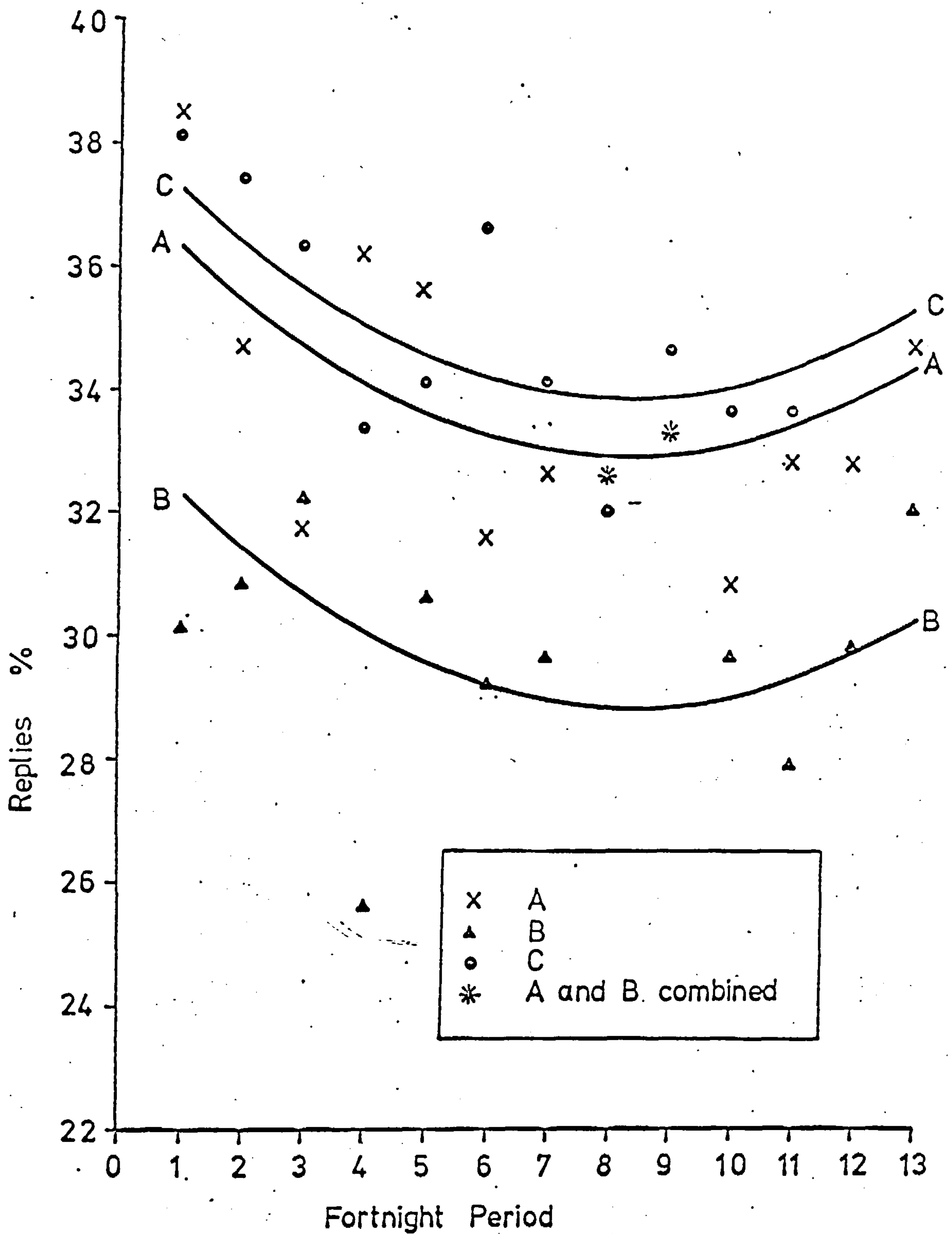


Figure 4.3: Comparison of observed proportion with the model fitted using CATLIN (Foldvary's data).

(not considered in the analyses by CATLIN) for the response to questionnaire C in the remaining 13 fortnights of the year. This group as a whole showed a significant difference from the response to questionnaire C in the other half-year, thus showing an effect of season. It is likely that the reasons why Foldvary detected no effect of fortnight were (i) he made two paired comparisons of the three response rates, rather than comparing all three questionnaires simultaneously; (ii) his analysis of variance was based on response rates rather than on the actual numbers responding and not responding - clearly a difference between rates of 30% and 35% is not significant if each are based on a total of 100, but are significant if based on a total of 1000; (iii) in his model, he did not include the information that the fortnights were ordered: when he, in effect, did this by comparing one half-year with another, the seasonal effect showed up.

Clearly it is not possible to estimate whether there is any interaction between the effect of the additional questions included in questionnaire C and the effect of asking for the registration number, since there were not any questionnaires which did have the additional questions but which did not ask for the registration number. Foldvary's statement to the contrary is based on a misinterpretation of his F statistics.

Base level of response rate	μ	.363
Coefficients of the quadratic	b	-.0105
effect of fortnight period	c	.00063
Effect of questionnaire design	γ_1	.011
	γ_2	-.030
	$\gamma_3 = -\gamma_1 - \gamma_2$.019

Table 4.6: Parameters of a model in which questionnaire design and fortnight period have an additive effect on response rate, and the effect of the fortnight period is represented by a quadratic function of time of year.

4.1.8 Discussion

The programs CATLIN and LINCAT appear to be extremely useful in extending the range of hypotheses which can be tested about tables of frequencies. They can take account of cells which are known, a priori, to be empty, as in example 2; ordering of dimensions can be taken account of, as in example 3, provided some rule can be found to assign scores to categories; linear or log-linear models can be fitted to the data, and hypotheses about the parameters can be tested. When one of the factors is ordered, as in examples 4 and 5, the analysis becomes very similar to analysis of covariance.

To summarise: the program considers hypotheses like these:

$$\underline{A} \underline{\pi} = \underline{0} \quad (4.2)$$

$$\underline{K} \log_e (\underline{A} \underline{\pi}) = \underline{0} \quad (4.3)$$

$$\underline{A} \underline{\pi} = \underline{X} \underline{\beta} \quad (4.4)$$

$$\underline{K} \log_e (\underline{A} \underline{\pi}) = \underline{X} \underline{\beta} \quad (4.5)$$

The observed vector corresponding to π is implied by the number of observations in each cell of the table which is supplied as data by the user. Matrices \underline{A} , \underline{K} , and \underline{X} are also supplied by the user. In each case the program calculates a value of χ^2 corresponding to the fit of the model. If it is large, the model is a bad fit and we can reject it. If it is small, we can accept it (until we find some other data which it does not fit). In addition, the vector $\underline{\beta}$ is calculated by the program in cases where a matrix \underline{X} is supplied. Hypotheses about the parameters in the vector $\underline{\beta}$ can be tested by supplying a vector \underline{C} . The program then tests the null hypothesis $\underline{C} \underline{\beta} = \underline{0}$ and calculates a corresponding value of χ^2 .

4.2 Nonparametric tests

By "nonparametric" tests are meant, broadly speaking, those procedures which analyse data that is ranked rather than metric. They are thus especially suitable for problems involving such factors as severity of injury. In comparison with conventional tests, they do not usually require the assumptions of Normality and homoscedasticity of errors, and they are often arithmetically easier to carry out. Furthermore most of them are never much less powerful than the corresponding Normal-theory test, and can be much more powerful in non-Normal circumstances, see section 4.2.6. Two of the best-known are the Kruskal-Wallis test for the one-way Analysis of Variance design, and Friedman's test for the two-way design. These are used several times in this thesis. (They are described in, for instance, Langley (1968) and Hollander and Wolfe (1973).) Both are special cases of a very general test developed by Benard and van Elteren (1953) which, despite its apparent usefulness, apparently remains little-known. This section will describe Benard and van Elteren's test, and present a FORTRAN program that (a) calculates the test statistic, χ^2_R (because of its generality, this test is considerably more trouble to carry out manually than most nonparametric tests), and (b) checks directly on the probability of getting so large a value under the null hypothesis of random rankings by actually assigning random ranks within the rows, calculating the test statistic, and comparing it with the observed value: if this is repeated, say, 1000 times, a fairly accurate idea of the significance level attained can be obtained.

This is important because if χ^2_R is interpreted as χ^2 with $N-1$ degrees of freedom, the significance level deduced is decidedly conservative for small k . See Friedman (1940) and table 4.7 which

presents the numbers of times in 1000 random rankings that the test statistic exceeded the 1% and the 5% significance levels of χ^2 for the case of one observation per cell, i.e. Friedman's original test. More accurate is to use Fisher's Z distribution with (non-integer) degrees of freedom determined by k and N.

Benard and van Elteren's test will be introduced by a description of Friedman's test.

	Number of columns						
	2	3	4	8	12	16	20
Number of rows	2		$\frac{0}{0}$	$\frac{0}{0}$	$\frac{0}{2}$	$\frac{0}{3}$	$\frac{0}{4}$
	4	$\frac{0}{129}$	$\frac{0}{60}$	$\frac{0}{34}$	$\frac{2}{26}$	$\frac{5}{31}$	$\frac{4}{27}$
	6	$\frac{0}{24}$	$\frac{5}{40}$	$\frac{9}{43}$	$\frac{4}{41}$	$\frac{2}{31}$	$\frac{5}{41}$
	8	$\frac{12}{70}$	$\frac{6}{43}$	$\frac{7}{46}$	$\frac{3}{39}$	$\frac{11}{42}$	$\frac{6}{39}$
	10	$\frac{4}{17}$	$\frac{11}{55}$	$\frac{5}{47}$	$\frac{7}{40}$	$\frac{4}{39}$	$\frac{8}{45}$
	20	$\frac{15}{42}$	$\frac{12}{56}$	$\frac{8}{62}$	$\frac{5}{42}$	$\frac{8}{43}$	$\frac{9}{42}$

Table 4.7: The number of times in 1000 random rankings that the test statistic exceeded the 1% (top) and 5% (bottom) significance levels of χ^2 . Thus we expect each entry above to be 10/50. (One observation per cell.)

4.2.1 The basic paradigm

When we have a two-way table of observed measurements, one per cell, and we wish to determine whether the column variable affects the level of our dependent measurement, we can use the Analysis of Variance. Alternatively, we can use a nonparametric test due to Friedman (1937).

This is performed by ranking the observations within each row, and then adding up the ranks columnwise. The squares of these column totals are then summed. If the total is sufficiently large we can conclude that our observations are dependent on the column variable. Friedman's test statistic, usually denoted by χ^2_r , is given by the formula

$$\chi^2_r = \frac{12R}{kN(N+1)} - 3k(N+1)$$

where k = number of rows

N = number of columns.

R = sum of squared column totals of the ranks.

For small k , N the exact distribution of χ^2_r is known (Owen, 1962). For other k , N it is usually recommended that χ^2_r be interpreted as χ^2 with $N-1$ degrees of freedom.

The test can also be interpreted in a slightly different way, as testing the mutual correlation or concordance of the measurements in the k rows. In order to describe this correlation, Kendall and Babington Smith (1939) and Wallis (1939) independently proposed what has become known as Kendall's Coefficient of Concordance, W . This is given by

$$W = \frac{\chi^2_r}{k(N-1)}$$

W is simply related to the average of the $\frac{k(k-1)}{2}$ Spearman rank correlation coefficients between pairs of the k rankings (\bar{r}_s):

$$\bar{r}_s = \frac{kW - 1}{k - 1}$$

Kendall and Babington Smith also propose continuity corrections for small values of k and N .

Durbin (1951) extended Friedman's test to the case of balanced incomplete blocks, and Benard and van Elteren (1953) further extended the test to cases where the numbers of observations in each cell were arbitrary. These tests are often said to use the "method of m rankings" (actually, k rankings in our notation).

4.2.2 Benard and van Elteren's test

Very often the numbers of observations in each cell are not all exactly one: some observations may be missing, leaving empty cells, and others may have been replicated. Benard and van Elteren (1953) showed how this situation could be dealt with by a generalisation of Friedman's test. In this section their test is outlined, with mathematical proofs omitted.

1. The data are arranged into a table in which there are N columns and k rows, and it is desired to test for the effect of the column classification on the dependent variable. The rankings are then carried out within each row. Let the number of replications of the j th column in the i th row be n_{ij} (all $n_{ij} \geq 0$). The total number of observations in the i th ranking is then $n_{i.}$. Tied observations are given their average rank.

2. After the i th ranking has been carried out, subtract from each rank $\frac{1}{2}(n_{i.} + 1)$, the arithmetical mean of these ranks. Benard and van Elteren call the resulting numbers the "reduced" ranks. The reduced ranks in cell (i,j) are then added together, and the result denoted by u_{ij} . (If $n_{ij} = 0$, $u_{ij} = 0$.) Evidently $\sum_{j=1}^N u_{ij} = 0$.

3. Calculate the column totals of the matrix of u_{ij} 's, and denote them by $u_{.j}$.

4. Calculate weighting factors K_i for the rows:

$$K_i = \frac{n_{i.}^3 - \sum p^3 t_{ip}}{12n_{i.}(n_{i.} - 1)}$$

where t_{ip} is the number of ties of size p in the i th ranking.

(Note that if there are no ties, the second term in the numerator is not zero: it is $\sum t_{i1} = n_{i.}$).

5. Now calculate the N by N symmetric matrix V , whose elements v_{gh} are given by

$$\text{Off-diagonal elements: } v_{gh} = - \sum_i n_{ig} n_{ih} K_i \quad (g \neq h)$$

$$\text{Diagonal elements: } v_{gg} = \sum_i n_{ig} (n_{i.} - n_{ig}) K_i$$

V is the matrix of variances and covariances of the column totals.

Consider also the matrix

$$V_u = \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1N} & u_{.1} \\ v_{21} & v_{22} & \cdots & v_{2N} & u_{.2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ v_{N1} & v_{N2} & \cdots & v_{NN} & u_{.N} \\ u_{.1} & u_{.2} & \cdots & u_{.N} & 0 \end{pmatrix}$$

In both of the matrices V and V_u , each row or column except for the last one in V_u is a linear combination of the other rows or columns.

6. Delete an arbitrary row and an arbitrary column from V , and calculate the determinant D of the resulting matrix. Delete an arbitrary row and an arbitrary column (but not the last row or column) from V_u , and calculate the determinant D_u of the resulting matrix.

7. Then $\chi^2_r = \frac{|D_u|}{|D|}$ is approximately distributed as χ^2 with $N-1$ degrees of freedom.

4.2.3 _ Example

Four keen golfers, Flatfoot, Greybeard, Hogsbody, and Ironbreast, were comparing their scores on a number of courses. They had not all played on the same ones, and on some courses had played several times. Their scores are given in table 4.8. Is there evidence that they differ in skill?

To compare golfers, we rank their scores within courses (table 4.9), and the u_{ij} 's are given in table 4.10. Table 4.11 gives the K_i together with the n_{ij} , and illustrates how the v_{gh} are calculated as a sort of weighted scalar product of two columns of this matrix. The matrix V is given in table 4.12.

It is found that $D = 909$, $D_u = -8835$, so $\frac{|D_u|}{|D|} = 9.7$, which interpreted as χ^2 with 3 degrees of freedom indicates $P < .05$.

<u>Course</u>	<u>Flatfoot</u>	<u>Greybeard</u>	<u>Hogsbody</u>	<u>Ironbreast</u>
I	79,80	83	82,89	-
II	83	-	84	84,85
III	73,79,80	-	75,80	81,83
IV	-	83,86	85,87,89	88
V	75,80	82	-	-
VI	73	76	75	-

Table 4.8: Data for the Example, scores by four golfers on six courses.

<u>Course</u>	<u>Flatfoot</u>	<u>Greybeard</u>	<u>Hogsbody</u>	<u>Ironbreast</u>
I	1,2	4	3,5	-
II	1	-	2½	2½,4
III	1,3,4½	-	2,4½	6,7
IV	-	1,3	2,4,6	5
V	1,2	3	-	-
VI	1	3	2	-

Table 4.9: The data of table 4.8 after ranking.

<u>Course</u>	<u>Flatfoot</u>	<u>Greybeard</u>	<u>Hogsbody</u>	<u>Ironbreast</u>
I	-3	1	2	0
II	-1½	0	0	1½
III	-3½	0	-1½	5
IV	0	-3	1½	1½
V	-1	1	0	0
VI	-1	1	0	0
	<u>-10</u>	<u>0</u>	<u>2</u>	<u>8</u>

Table 4.10: The matrix of u_{ij} 's and the column totals $u_{.j}$.

K_i			n_{ij}	
0.5	$\overset{\frown}{2}$	$\overset{\frown}{1}$	2	0
0.375	$\overset{\frown}{1}$	$\overset{\frown}{0}$	1	2
0.655	$\overset{\frown}{3}$	$\overset{\frown}{0}$	2	2
0.583	$\overset{\frown}{0}$	$\overset{\frown}{2}$	3	1
0.333	$\overset{\frown}{2}$	$\overset{\frown}{1}$	0	0
0.333	$\overset{\frown}{1}$	$\overset{\frown}{1}$	1	0

$$\begin{aligned}
 v_{12} &= -0.5 \times 2 \times 1 \\
 &\quad -0.375 \times 1 \times 0 \\
 &\quad -0.655 \times 3 \times 0 \\
 &\quad -0.583 \times 0 \times 2 \\
 &\quad -0.333 \times 2 \times 1 \\
 &\quad -0.333 \times 1 \times 1 \\
 &= -2.0
 \end{aligned}$$

Table 4.11: Illustrating how the v_{ij} are calculated from the K_i 's and the n_{ij} 's.

13.315	-2.0	-6.637	-4.679
-2.0	8.0	-4.833	-1.167
-6.637	-4.833	16.589	-5.119
-4.679	-1.167	-5.119	10.964

Table 4.12: The matrix V.

4.2.4 _ FORTRAN program

As was said in the introduction, this program both carries out the test described above and also calculates a significance level for it by randomisation of ranks within the rows.

Necessary input to the program includes the numbers of observations in each cell and their ranks; output includes the test statistic, the number of degrees of freedom, and (if desired) the number of times the test statistic based on randomisation exceeded the observed value of the test statistic. A listing of the program is given in the Appendix, section 4.2.7. Two subroutines have been omitted because they are standard and possibly machine-dependent: SUBROUTINE F03AAF (A,IA,M,DET, DUMMY,IFAIL) evaluates the determinant DET of the M by M matrix A; the dimensionality of A is IA, DUMMY is workspace (and is not used in the main program at all), and IFAIL is a failure indicator. A, DET, and DUMMY should be declared DOUBLE PRECISION. Also SUBROUTINE RAN01 (RAND) generates a number RAND randomly and uniformly distributed between 0 and 1 each time it is called.

No special care has to be taken to make the programs particularly efficient, but no problems with excessive CPU time have been encountered - if the problem is so large that 1000 randomisation of ranks within each row takes a lot of time, then it is likely that the χ^2 approximation will be satisfactory (though this is not necessarily so if the number of rows is small).

4.2.5 Special cases of the Benard/van Elteren test

Because the following tests are similar or equivalent to special cases of Benard and van Elteren's test, the program listed in the Appendix is also suitable when they might be used: Friedman's test, Kruskal-Wallis test, Mann-Whitney U (Wilcoxon) test, sign test for matched pairs, Spearman's rank correlation, Wilcoxon's stratified test, Meddis's test, and Jonckheere's generalisation of the Kruskal-Wallis test to the case of ordered alternatives.

1. When all n_{ij} are equal to 1, Friedman's test is produced.
2. Since this in turn is an N-sample analogue of the matched pair sign test (for which $N = 2$), this also may be carried out by our program.
3. It is also a k-block analogue of Spearman's correlation test (for which $k = 2$).
4. A special application of Spearman's test occurs when we have N ordered categories, in each of which there are several observations which are not matched across categories in any way. (That is, the paradigm of the Kruskal-Wallis test except that the categories are ordered.) Since there are two rankings of each object (albeit with many ties in one of them) this can be treated in the same way as Spearman's test by our program. Terpstra (1952) and Jonckheere (1954b) independently proposed a test of this type but which was based on Kendall's tau rather than Spearman's rho.
5. The interpretation of the case $k = 2$, all $n_{ij} = 1$, in terms

of correlation needs care over the significance level. If we are looking for a trend in a particular direction, then a one-tailed test is appropriate, i.e. we look up the area under the χ^2 curve to the right of our observed value (if we expect a positive correlation) or to the left (if we expect a negative correlation). If we want a two-tailed test we should find the smaller of the two areas into which our observed value divides the χ^2 distribution, and double it. In the case $k > 2$ we are always only interested in agreement between the several sets of observations, so a one-tailed test is appropriate. Note also that when some n_{ij} are greater than unity, the test cannot be interpreted in terms of correlation. If, for $N = 4$, $k = 2$, $n_{1j} = 1$, $n_{2j} = 4$, we found the ranks:

1	2	3	4
13,14,15,16	9,10,11,12	5,6,7,8	1,2,3,4

the value of χ^2 would probably be large and significant but this would be due to a Kruskal-Wallis type test on the second row, the opposite ordering in the first row not being strong enough to outweigh the greater number of observations in the second. The test would say nothing at all about the apparent negative correlation between the two sets of ranks. It might, however, be appropriate to assign to each measurement in the second row the corresponding rank in the first, in which case a repeat of the test with $N = 16$, $k = 2$, $n_{ij} = 1$, and the ranks

$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{1}{2}$	$6\frac{1}{2}$	$10\frac{1}{2}$	$10\frac{1}{2}$	$10\frac{1}{2}$	$10\frac{1}{2}$	$14\frac{1}{2}$	$14\frac{1}{2}$	$14\frac{1}{2}$	$14\frac{1}{2}$
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4

could be carried out, and a two-tailed significance level found.

6. The Kruskal-Wallis test itself is a special case of Benard and van Elteren's, for which the number of rows k is 1.

7. The Mann-Whitney U-test (otherwise known as Wilcoxon's sum of ranks test) is a special case of the Kruskal-Wallis test for which $N = 2$. Thus the Mann-Whitney U-test can also be looked upon as a version of Spearman's (or Kendall's) rank correlation test for which $N = 2$ and there are repeated observations of one of the variables.

8. An extension of the Mann-Whitney U-test known as Wilcoxon's stratified test (see Langley, 1968, p.190) is applicable if $N = 2$, $n_{i1} = n_{i2}$ for all i , but the n_{i1} can be different.

9. Meddis (1975) has extended Wilcoxon's stratified test to cases where $n_{i1} \neq n_{i2}$. Thus his test fulfills the same purpose as Benard and van Elteren's except that it is restricted to $N = 2$.

10. Figure 4.4 illustrates the relationships between these tests.

4.2.6 Concluding remarks

It may be useful to give some references to related topics in nonparametric ANOVA.

(i) General form of the method of m rankings. It is possible to conceive of a host of alternatives to Friedman's test, with the ranks of the observations replaced by some other scores. This has been considered by Sen (1968) and Puri and Sen (1971, section 7.2). For the case of Friedman's test (one observation per cell), let R_j , $j = 1, 2, \dots, N$, be the N scores, and let

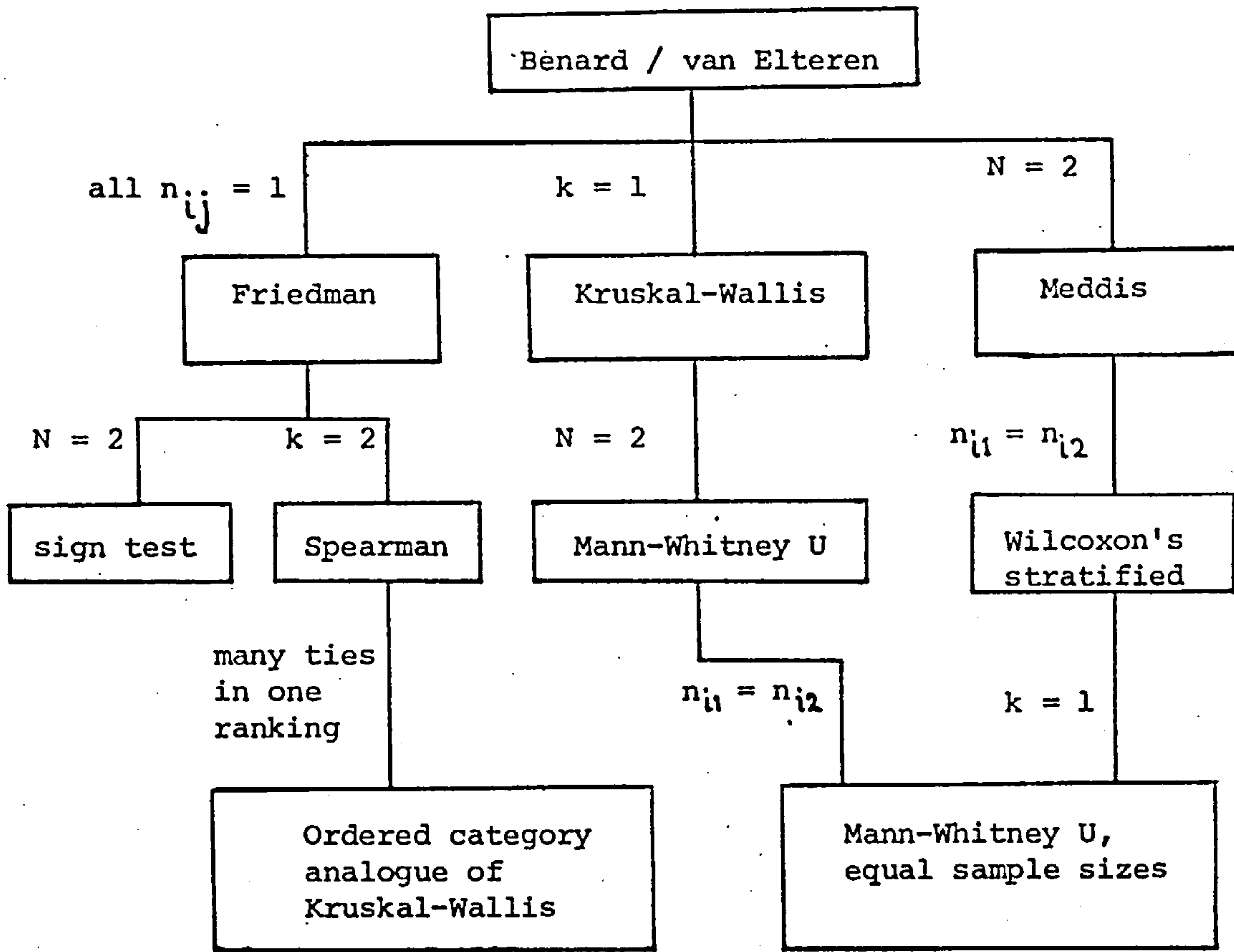


Figure 4.4: Illustration of the relationships between certain nonparametric tests.

$$\bar{R} = \frac{\sum_{j=1}^N R_j}{N} \quad \text{and} \quad A^2 = \frac{\sum_{j=1}^N (R_j - \bar{R})^2}{N - 1}$$

Define T_j 's as the column-averages of the R 's:

$$T_j = \frac{\sum_{i=1}^k R_{ij}}{k}$$

Then the statistic

$$S = \frac{\sum_{j=1}^N (T_j - \bar{R})^2}{k A^2}$$

is appropriate for testing for differences between the columns. It may be seen that Friedman's test is based on a statistic of this type for which the scores R_j are the ranks within that row.

(ii) Median tests. Brown and Mood (1951) proposed a test based on counting the number of observations in a column that were greater than the medians of their respective rows. It turns out that this also is a special case of the S statistic defined above, with $R_j = 1$ or 0 according to whether the observation was \leq the median of that row, or not.

(iii) Interactions. Brown and Mood (1951) and Bhapkar (1961) also consider generalisations of the median test to different numbers of observations per cell, testing for interaction, and regression problems. Mehra and Sen (1969) have also developed rank order tests for interactions in factorial experiments.

(iv) Asymptotic efficiency. If the assumptions of the classical ANOVA - block effects additive, errors Normal and homoscedastic - hold, the Asymptotic Relative Efficiency of Friedman's test compared with

parametric ANOVA (as the number of rows becomes large) is given by $\frac{N}{N+1} \cdot \frac{3}{\pi}$ which is an increasing function of N , from 0.64 for 2 columns (the sign test) to 0.95 for many columns. For two rows, i.e. Spearman's rank correlation compared with ordinary regression, the A.R.E. (as the number of columns becomes large) is unity. If the assumptions of classical ANOVA do not hold, then the A.R.E. of Friedman's test can be greater than unity (Sen, 1967). For Durbin's test, the formula $\frac{N}{N+1} \cdot \frac{3}{\pi}$ is replaced by $\frac{N'}{N'+1} \cdot \frac{3}{\pi}$ where N' is the number of treatments which each observer ranks, i.e. the number of observations per row (van Elteren and Noether, 1959). Sen (1968) has considered the A.R.E. of the S statistic defined above, for various R_j .

(v) The method of ranking after alignment. Because Friedman's test is based on intrablock rankings, it does not utilise the possible information contained in the interblock comparisons. It is primarily for this reason that its A.R.E. is low compared with parametric ANOVA when the number of columns is small. The method of ranking after alignment (Puri and Sen, 1971, section 7.3) consists essentially of removing the block (row) effects from each observation by subtracting the row average (or median) from each measurement. The kN transformed observations are then ranked as one block, and the column totals of these ranks are found. The test statistic is based on how different these column totals are from each other. This test reduces to Wilcoxon's signed rank test for $N = 2$. In contrast with Friedman's method, the asymptotic efficiency relative to conventional ANOVA of this test is between $\frac{3}{\pi}$ and 1.

(vi) Ordered alternative in Friedman's test. If there is a priori reason to expect the measurements in each row to be in a particular order (i.e. the column variable is ordinal), instead of calculating

$\sum u_{.j}^2$ (the $u_{.j}$'s being the column totals of the reduced ranks, as in an earlier part of this section), calculate $\sum ju_{.j}$ (the columns j being numbered in the order we expect the column totals of the ranks to be in). This test is directly related to the average Spearman rank correlation coefficient between the column ordering and the ranking within each of the rows. It was proposed by Page (1963) developing the ideas of Lysterly (1952). A similar test but based on the average of Kendall's rank correlation coefficients was proposed by Jonckheere (1954a).

4.2.7 Appendix: listing of a FORTRAN program for Benard and van Elveren's test.

Before listing the program, the data preparation will be explained.

Data preparation

I. Problem-description card. This gives the number of rows, the number of columns, and the number of randomisations of the ranks that will be performed, in (2I2,I4) format. Should the last of these be less than or equal to zero, the randomisation option will not be carried out.

Cards II - V apply to one row of the table. They are repeated for each row.

II. The number of observations in each cell of the present row, punched in I2 format.

III. The ranks of the observations in this row, the first n_{i1} applying to the first cell, the second n_{i2} to the second, etc. (If there are any ties, the average rank must be given to each of the tied observations.) F4.1 format.

IV. Ties card. The number of sets of ties in this row are punched in the first two columns (I2 format). (A 'set of ties' is a run of equal ranks: thus if ranks 5,6,7, and 8 were tied and thus assigned the average rank of 6.5, this would be one set of ties. If ranks 5 and 6 were tied, and ranks 7 and 8 also tied (but not with 5 and 6), there would be two sets of ties.)

V. If there were no ties in this row, go to VI. Otherwise the sets of ties are described. The first four columns give the lowest rank contributing to the first set of ties (F4.1), and the second four columns give the highest rank contributing to this set of ties (F4.1). The next eight columns similarly give the lowest and highest rank

contributing to the second set of ties, and so on. Thus if ranks 5 to 8 were tied this card would read

b5.bb8.b

(b meaning blank) whereas if ranks 5 and 6 were tied and so were 7 and 8, this card would read

b5.bb6.bb7.bb8.b

VI. If there are more rows to read, go to II. Otherwise go to VII.

VII. If there are more problems, go to I. Otherwise punch '0' in column 4.

Example

For the example of section 4-2.3 the cards input are as shown in figure 4.5.* As mentioned earlier, $\chi^2_T = 9.7$, $P < .05$. Using the randomisation option in the program, it was found that the test statistic exceeded the observed value 5 times in 1000 randomisations, indicating $P = .005$. This accords with the generally conservative nature of Friedman's test, already discussed.

* See p. 180.


```

      DIMENSION IA(20,20),IC(20),RIC(20),R(20,100),NSETTY(20),ST(20,10),
      1FT(20,10),RD(100),B(20,20),BCT(20),RBCT(20),DUMMY(20),DV(19,19),
      2DVU(20,20)

```

```

C   LIMITATIONS ON THIS VERSION OF THE PROGRAM: 20 ROWS, 20 COLUMNS,
C   100 OBSERVATIONS PER ROW, 10 SETS OF TIES PER ROW
      DOUBLE PRECISION DV,DVU,DENOM,DUMMY
      COMMON IB

```

```

13  IB=153

```

```

C   IB HERELY STARTS THE RANDOM NUMBER GENERATOR OFF
      READ(5,1) NROWS,NCOLS,ITERS

```

```

1  FORMAT(I2,I2,I4)

```

```

      IF(NROWS.EQ.0) STOP

```

```

C   ITERS IS THE NUMBER OF TIMES THE RANKS IN EACH ROW ARE RANDOMISED

```

```

C   NROWS IS NUMBER OF ROWS, NCOLS IS NUMBER OF COLUMNS.

```

```

C   NOW READ IN SETS OF CARDS FOR EACH ROW

```

```

      DO 4 I=1,NROWS

```

```

      READ(5,2) (IA(I,J),J=1,NCOLS)

```

```

2  FORMAT(20I2)

```

```

C   IA(IJ) IS THE NUMBER OF OBSERVATIONS IN THE IJ TH CELL

```

```

      IC(J)=0

```

```

      DO 3 ID=1,NCOLS

```

```

3  IC(I)=IC(I)+IA(I,ID)

```

```

C   IC(I) IS THE NUMBER OF OBSERVATIONS IN THE I TH ROW

```

```

      RIC(I)=(IC(I)+1.)/2.

```

```

C   RIC(I) IS THE AVERAGE RANK IN THE I TH ROW

```

```

      IDL=IC(I)

```

```

      READ(5,6) (R(I,K),K=1,IDL)

```

```

6  FORMAT(20F4.1)

```

```

C   THE R'S ARE THE RANKS OBSERVED

```

```

C   THE NEXT CARDS SUPPLY INFORMATION ABOUT THE TIES IN THE I TH ROW

```

```

      READ(5,5) NSETTY(I)

```

```

5  FORMAT(I2)

```

```

C   NSETTY(I) IS THE NUMBER OF SETS OF TIES OCCURRING IN THE I TH ROW

```

```

      IF (NSETTY(I).EQ.0) GO TO 4

```

```

      NSTIDL=NSETTY(I)

```

```

      READ(5,6) (ST(I,ID),FT(I,ID),ID=1,NSTIDL)

```

```

C   ST AND FT ARE THE FIRST AND LAST RANKS WHICH ARE TIED. THERE ARE

```

```

C   NSETTY(I) PAIRS OF THEM.

```

```

C   THIS ENDS THE INPUT FOR THE I TH ROW.

```

```

4  CONTINUE

```

```

C   INPUT IS NOW FINISHED

```

```

C   OUTPUT THE DATA TO MAKE SURE WE HAVE IT RIGHT

```

```

      WRITE(6,14)

```

```

14  FORMAT(1H1)

```

```

      DO 26 I=1,NROWS

```

```

      J=1

```

```

      IST=1

```

```

22  IF(IA(I,J).NE.0) GO TO 23

```

```

      WRITE(6,24) I,J,IA(I,J)

```

```

24  FORMAT(1X,'ROW',I3,'COLUMN',I3,'THIS HAS',I3,'ENTRIES')

```

```

      GO TO 25

```

```

23  WRITE(6,20) I,J,IA(I,J)

```

```

20  FORMAT(1X,'ROW',I3,'COLUMN',I3,'THIS HAS',I3,'ENTRIES WHICH ARE:')

```

```

      ISF=IST+IA(I,J)-1

```



```

6      WRITE(6,21) (R(I,K),K=IST,ISF)
7      21 FORMAT(1X,20F6.1)
8      IST=ISF+1
9      25 J=J+1
0      IF (J.LE.NCOLS) GO TO 22
1      WRITE(6,29) NSETTY(I)
2      29 FORMAT(1X,'TIES IN THIS ROW (NSETTY=',I3,'):')
3      IF (NSETTY(I).EQ.0) GO TO 26
4      NSTIDL=NSETTY(I)
5      DO 28 ID=1,NSTIDL
6      28 WRITE(6,27) ST(I,ID), FT(I,ID)
7      27 FORMAT(20X,2F6.1)
8      26 CONTINUE
C      END OF DATA OUTPUT
9      DO 9 I=1,NROWS
0      IDL=IC(I)
1      DO 15 ID=1,IDL
2      15 RD(ID)=R(I,ID)
3      9 CALL EVROW(RD,NCOLS,IA,I,RIC(I),B)
C      SUBROUTINE EVROW CALCULATES THE VECTOR OF REDUCED RANKS FROM THE
C      SUPPLIED RANKS (RD) AND THEIR DISTRIBUTION AMONG THE COLUMNS (IA)
C      AND RETURNS THE ANSWER IN B.
4      DO 10 J=1,NCOLS
5      10 BCT(J)=0
6      DO 11 J=1,NCOLS
7      DO 11 I=1,NROWS
8      11 BCT(J)=BCT(J)+B(I,J)
C      BCT(J) IS THE SUM OF REDUCED RANKS IN THE J TH COLUMN
C      PRELIMINARY CALCULATIONS NOW COMPLETE.
9      CALL EVMATS(NCOLS,NROWS,NSETTY,FT,ST,IC,IA,DV,DVU)
C      SUBROUTINE EVMATS CALCULATES THE MATRICES V AND VU, WITH ONE ROW
C      AND COLUMN DELETED FROM EACH, AND PUTS THE RESULTS IN DV AND DVU
0      WRITE(6,105)
1      105 FORMAT(///1X,'THE COLUMN TOTALS OF REDUCED RANKS'/)
2      WRITE(6,104) (BCT(J),J=1,NCOLS)
3      104 FORMAT(1X,10F12.3)
4      IFAIL=1
5      CALL F03AAF(DV,19,NCOLS-1,DENOM,DUMMY,IFAIL)
C      SUBROUTINE F03AAF CALCULATES THE DETERMINANT OF MATRIX DV AND
C      PUTS THE ANSWER IN DENOM
6      CALL RATIO(NCOLS,NROWS,BCT,DV,DVU,ANS,DENOM)
C      SUBROUTINE RATIO CALCULATES THE TEST STATISTIC, CALLED ANS
7      DF=NCOLS-1
8      WRITE(6,30) ANS,DF
9      30 FORMAT(1H1,'ANS =',E12.4/1X,'D.F.',F6.1)
0      IF(ITERS.LE.0) GO TO 13
1      NGT=0
2      KOUNT=0
3      DO 31 ID=1,ITERS
4      CALL SIMRAN(NCOLS,NROWS,NSETTY,FT,ST,IC,IA, RIC,RBCT)
C      SIMRAN GENERATES RANDOM ARRANGEMENTS OF RANKS WITHIN ROWS, AND
C      THE CORRESPONDING COLUMN TOTALS OF REDUCED RANKS, RBCT
5      CALL RATIO(NCOLS,NROWS,RBCT,DV,DVU,DRAT,DENOM)
6      IF(DRAT.GT.(ANS+.000001)) NGT=NGT+1

```

```
31 IF (DRAT.GE.ANS ) KOUNT=KOUNT+1
    PROB=FLOAT(KOUNT)/FLOAT(ITER)
    WRITE(6,32) KOUNT,ITER,PROB,NGT
32 FORMAT(1H1,'RESULTS FROM RANDOMISATION'///1X,'KOUNT =',I4/
1 1X,'ITER =',I5/1X,'THEREFORE PROB =',F7.4//1X,' (NUMBER OF TIMES
2RANDOMISED TEST STATISTIC GREATER THAN OBSERVED TEST STATISTIC PLU
3S .000001 WAS',I4,') ')
    GO TO 13
END
```

```
      SUBROUTINE EVROW(A,NC,M,I,AVR,RES)
C      THIS SUBROUTINE CALCULATES THE VECTOR OF REDUCED RANKS FOR A GIVEN
C      ROW AND PUTS IT IN THE VECTOR RES. A IS THE VECTOR OF RANKS WHICH
C      IS SUPPLIED , WHICH ARE ARRANGED AMONG THE NC COLUMNS AS DESCRIBED
C      BY THE I TH ROW OF THE MATRIX M. AVR IS THE AVERAGE RANK FOR THE ROW
      DIMENSION RES(20,20),M(20,20),A(100)
      DO 1 ID=1,NC
1 RES(I,ID)=0.
      J=1
      ID=1
4 IK=0
2 IF(IK.GE.M(I,J)) GO TO 3
      RES(I,J)=RES(I,J) + A(ID)
      ID=ID+1
      IK=IK+1
      GO TO 2
3 J=J+1
      IF (J.LE.NC) GO TO 4
      DO 5 J=1,NC
5 RES(I,J)=RES(I,J) - M(I,J)*AVR
      RETURN
END
```

```

LEVEL 20          EVHATS          DATE = 75084          16/52/41

SUBROUTINE EVHATS(NCOLS,NROWS,NSETTY,FT,ST,IC,IA,DV,DVU)
THIS SUBROUTINE EVALUATES THE MATRIX V
DIMENSION DK(20),PSK(20),NSETTY(20),FT(20,10),ST(20,10)
DIMENSION RK(20),IC(20),V(20,20),IA(20,20),DV(19,19),DVU(20,20)
DOUBLE PRECISION DV,DVU
C CALCULATING THE WEIGHTING FACTORS (K) CALLED HERE RK.
DO 3 I=1,NROWS
DK(I)=0
3 PSK(I)=0
DO 2 I=1,NROWS
IF (NSETTY(I).EQ.0) GO TO 10
NSTIDL=NSETTY(I)
DO 1 ID=1,NSTIDL
T=FT(I,ID)+1-ST(I,ID)
PSK(I)=PSK(I)+T*T*T
1 DK(I)=DK(I)+FT(I,ID)+1-ST(I,ID)
DK(I) IS NOW THE NUMBER OF OBSERVATIONS IN THE I TH ROW WHICH ARE TIED.
10 PSK(I)=PSK(I)+IC(I)-DK(I)
PSK(I) IS THE SECOND TERM IN THE NUMERATOR OF THE WEIGHTING FACTORS
2 RK(I)=((IC(I)*IC(I)-PSK(I))/(12.*IC(I)*(IC(I)-1))
C THE K'S HAVE NOW BEEN CALCULATED. NOW CALCULATE THE MATRIX (V)
C OF THE VARIANCES AND COVARIANCES OF THE COLUMN TOTALS.
DO 5 L=1,NCOLS
DO 5 K=1,NCOLS
5 V(K,L)=0.
C ELEMENTS OF V HAVE NOW BEEN INITIALISED TO ZERO
DO 6 K=2,NCOLS
KIDL=K-1
DO 6 L=1,KIDL
DO 13 ID=1,NROWS
13 V(K,L)=V(K,L)-IA(ID,K)*IA(ID,L)*RK(ID)
6 V(L,K)=V(K,L)
C OFF-DIAGONAL ELEMENTS OF V HAVE NOW BEEN CALCULATED
DO 4 K=1,NCOLS
DO 4 ID=1,NROWS
4 V(K,K)=V(K,K)+IA(ID,K)*(IC(ID)-IA(ID,K))*RK(ID)
C DIAGONAL ELEMENTS OF V HAVE NOW BEEN CALCULATED
C OMIT FIRST ROW AND FIRST COLUMN FROM V TO GET DV, WHOSE DETERMINANT
C WILL BE THE DENOMINATOR WHEN CALCULATING CHI-SQUARED.
NCIDL=NCOLS-1
DO 11 L=1,NCIDL
DO 11 K=1,NCIDL
DV(K,L)=V(K+1,L+1)
11 DVU(K,L)=DV(K,L)
WRITE(6,105)
105 FORMAT(1H1)
DO 104 IZ=1,NROWS
104 WRITE(6,101) IZ,RK(IZ)
101 FORMAT(1X,'K(',I2,') =',F7.3)
WRITE(6,106)
106 FORMAT('//1X,' THE MATRIX V' /)
DO 103 K=1,NCOLS
103 WRITE(6,102) (V(K,L),L=1,NCOLS)
102 FORMAT(1X,10E12.4)

```


V G LEVEL 20

EVMATS

DATE = 75084

16/52/41

RETURN
END

V G LEVEL 20

RATIO

DATE = 75084

16/52/41

```

SUBROUTINE RATIO(NCOLS,NROWS,      BCT,DV,DVU,ANS,DENOM)
DIMENSION DVU(20,20),DV(19,19),BCT(20),DUMMY(20),DVUDUM(20,20)
DOUBLE PRECISION DV,DVU,DENOM,RNUM,DUMMY,DVUDUM
COMMON IB
C THE DETERMINANT OF MATRIX DVU WILL BE THE NUMERATOR WHEN
C CALCULATING CHI-SQUARED.
NCIDL=NCOLS-1
DO 8 K=1,NCIDL
DVU(K,NCOLS)=BCT(K+1)
8 DVU(NCOLS,K)=BCT(K+1)
DVU(NCOLS,NCOLS)=0.
DO 1 J=1,NCOLS
DO 1 I=1,NCOLS
1 DVUDUM(I,J)=DVU(I,J)
C EVALUATE THE DETERMINANT OF DVU.
IFAIL=1
CALL F03AAF(DVUDUM,20,NCOLS,RNUM,DUMMY,IFAIL)
IF(IFAIL.EQ. 0) GO TO 14
RNUM=0.
14 ANS=RNUM/DENOM
ANS=ABS(ANS)
RETURN
END

```


G LEVEL 20

SIMRAN

DATE = 75084

16/52/41

```

SUBROUTINE SIMRAN(NCOLS,NROWS,NSETTY,FT,ST,IC,IA, RIC,RBCT)
C THIS SUBROUTINE GENERATES RANDOM ARRANGEMENTS OF THE RANKS AMONG
C THE COLUMNS, AND THE REDUCED RANK TOTALS.
  DIMENSION IC(20),RRAN(100),NSETTY(20),ST(20,10),FT(20,10),
1 RIC(20), RB(20,20),RBCT(20),IA(20,20)
  DO 3 I=1,NROWS
    CALL RANRAN(IC(I),RRAN)
C RANRAN PUTS FIRST IC(I) INTEGERS INTO ARRAY RRAN IN A RANDOM ORDER
C IF NSETTY IS NOT ZERO CHANGE THE RANKS WHICH ARE TIED IN THE REAL DATA
  IF (NSETTY(I).EQ.0) GO TO 7
  ICIDL=IC(I)
  DO 9 ID=1,ICIDL
    NSTIDL=NSETTY(I)
    DO 9 IE=1,NSTIDL
9 IF ((RRAN(ID).GE.ST(I,IE)).AND.(RRAN(ID).LE.FT(I,IE))) RRAN(ID)=
1 (ST(I,IE)+FT(I,IE))/2.
  7 CALL EVROW(RRAN,NCOLS,IA,I, RIC(I),RB)
  3 CONTINUE
  DO 4 J=1,NCOLS
4 RBCT(J)=0.
  DO 5 J=1,NCOLS
  DO 5 I=1,NROWS
5 RBCT(J)=RBCT(J)+RB(I,J)
C RBCT(J) IS THE SUM OF THE REDUCED RANKS IN THE J TH COLUMN
  RETURN
  END

```

G LEVEL 20

RANRAN

DATE = 75084

16/52/41

```

SUBROUTINE RANRAN(NUM,RES)
C THIS SUBROUTINE PUTS THE FIRST NUM INTEGERS INTO THE VECTOR RES
C IN A RANDOM ORDER.
  DIMENSION IT(100),L(100),RES(100)
  DO 3 K=1,NUM
3 IT(K)=0
  DO 2 K=1,NUM
1 CALL RAN01(RAND)
  L(K)=RAND*NUM
  L(K)=L(K)+1
C L(K) IS A RANDOM INTEGER BETWEEN 1 AND NUM
  IF (IT(L(K)).EQ.1) GO TO 1
  IT(L(K))=1
2 RES(K)=L(K)
  RETURN
  END

```

10										20										30										40									
6 41000																																							
2 1 2 0																																							
1. 2. 4.					3. 5.																																		
0																																							
1 0 1 2																																							
1. 2.5 2.5 4.																																							
1																																							
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1. 3. 4.5 2. 4.5					6. 7.																																		
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0 2 3 1																																							
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Figure 4.5: Data cards for the example of section 4.2.3.

CHAPTER 5:

EFFECTS OF VEHICLE AND TYPE OF ACCIDENT ON CAR DRIVERS' LEG AND HEAD INJURIES

5.1 Introduction

In this Chapter, an examination is made of whether different models of car differ in (a) the leg injury, and (b) the head injury received by their drivers in accidents. If these can be demonstrated and related to design features of the interior of the vehicle then useful design guides will have been provided.

Because national accident statistics give no information about which part of the body of a casualty is injured, the source of the data is the notebook the policeman fills in at the scene of the accident. Even so, the injury description is poor in comparison with data from medical sources, but details of the accident are not usually available from the latter.

The aim was to include in the accident sample all the serious non-pedestrian single-vehicle accidents involving five selected models of cars, plus all their collisions with other cars, which occurred in the Metropolitan Police District (Greater London) in 1971. In practice, reports of 64% of these were available and were analysed. The sample was restricted to serious accidents to try to obtain an appreciable number of major leg injuries (those involving broken bones).

The models selected for study are among the most common ones in Britain. They are all of similar small size (in the weight range 1400 - 1800 lb). Models B₁ and B₂ are virtually identical, but the design of the fascia and parcel shelf is different; the effect of this

on leg injury problems will be discussed later.

The circumstances of the accident might be expected to affect the severity of injury. Therefore the analysis took into account the type of accident as well as the model of car, since a low degree of injury could result from a particular model being involved in a higher than usual proportion of rear-end accidents (which are typically less severe than most other types) as well as from having an interior which is less injurious. The four types into which accidents were classified were head-on, rear-end, and intersection collision between two cars, and single-vehicle accidents. Impacts in which the main line of force is from front to back along the car ("frontal impacts") are most single-vehicle accidents, head-on accidents, and to the striking vehicle in rear-end and intersection accidents, and it is these to which most attention will be devoted.

5.2 Results: leg injuries

5.2.1 Data

Despite the restriction of the sample to serious accidents, broken legs constituted only 3.4%, and in 74% of cases no leg injury at all was mentioned. (These figures refer to frontal impacts, with which the main analysis will be concerned.) The leg injury was classified into three groups: no leg injury, leg injury apparently not involving broken bones, and cases involving broken bones. When more than one injury of the legs was mentioned, the injury was classified as that of the most severe of them. In this first analysis, injury to any part of the leg is included; later a distinction will be made between injuries to the knee and upper leg as one category, and to the lower leg, ankle, and foot as the other.

The importance of taking into account the type of accident is emphasised by the results of sections 4.1.4 and 4.1.5, in which it was shown that type of accident affects the driver's leg injury, and that there was some indication that models of car differ in the relative proportions of the four types of accident. But it may be said that the average injury severity (as defined in section 4.1.5) does differ significantly between models (table 5.1).

Table 5.2 gives the results, classified according to both type of accident and model of car. Because there are so many cells with few observations in them, the "major" and "slight" categories of leg injury were combined in the analysis. This used the program CATLIN in the following way.

<u>Model</u>	<u>No injury</u>	<u>Slight injury</u>	<u>Major injury</u>
A	85	43	7
B ₂	36	9	2
B ₁	52	7	2
H	43	19	1
F	39	11	1
Others	172	43	7

Table 5.1: Degree of leg injury in frontal impacts to different models of car.

<u>Model</u>	<u>Type of accident*</u>	<u>Degree of leg injury</u>		
		<u>None</u>	<u>Slight</u>	<u>Major</u>
A	HO	14	9	0
	RE	27	7	0
	Int	28	13	2
	SV	16	14	5
B ₂	HO	5	3	0
	RE	2	1	0
	Int	10	3	0
	SV	19	2	2
B ₁	HO	10	3	0
	RE	10	0	0
	Int	10	1	1
	SV	22	3	1
H	HO	5	1	0
	RE	9	5	0
	Int	13	5	0
	SV	16	8	1
F	HO	3	6	0
	RE	6	0	0
	Int	18	1	0
	SV	12	4	1
Others	HO	44	19	4
	RE	55	7	1
	Int	73	17	2

* HO = head-on
 RE = rear-end
 Int = intersection
 SV = single-vehicle

Table 5.2: Frontal impacts classified by model of car, type of accident, and degree of leg injury.

5.2.2 Application of CATLIN

Consider each accident-type/model combination as being a separate population, and for each there are two responses, injured or not injured. In this example there are 23 populations - 4 types of accident (head-on, rear-end, intersection, and single-vehicle) for each of five models of car, and 3 types of accident (single-vehicle being omitted) for a sixth category, other models.

Let p_{ij} be the probability of the driver's leg being injured in the j th model when involved in the i th type of accident. It might be thought that a reasonable model to fit to the data is that p_{ij} should be a linear combination of an effect due to model of car and an effect due to type of accident, that is $p_{ij} = F_i + G_j$. This model says that being involved in one type of accident rather than another changes p_{ij} by the same amount whatever the model of car is; similarly being in one model rather than another changes p_{ij} by the same amount whatever the type of accident is. However, as has been mentioned in section 4.1.6, the additive model in this form has too many parameters since the same predicted values of p_{ij} would be obtained if some constant k was added to each of the F_i , provided that the same constant was subtracted from each of the G_j . Instead, express the model in the form

$$p_{ij} = \mu + \phi_i + \gamma_j, \quad \sum \phi_i = \sum \gamma_j = 0.$$

The interpretation of this is that μ is the base level of proportion injured (the mean of the proportions injured in each population, not weighted by the number of cases in each population), ϕ_i is the differential effect of the i th type of accident, and γ_j is the differential effect of the j th model of car. Since there are 4 types of accident and 6 categories of model of car,

$$\phi_4 = -\phi_1 - \phi_2 - \phi_3 \quad \text{and} \quad \gamma_6 = -\gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5.$$

This reduces the number of parameters from 10 to 9 ($\mu, \phi_1, \phi_2, \phi_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$) and means that the model is completely specified. This, then, is the model we wish to fit:

$$p_{ij} = \mu + \phi_i + \gamma_j, \quad i = 1, 2, 3, \quad j = 1, 2, 3, 4, 5.$$

There are 23 simultaneous equations of this form that we wish to fit to our data, one for each population:

$$p_{11} = \mu + \phi_1 + \gamma_1 \quad p_{12} = \mu + \phi_1 + \gamma_2 \quad \dots \quad p_{15} = \mu + \phi_1 + \gamma_5 \quad p_{16} = \mu + \phi_1 - \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5$$

$$p_{21} = \mu + \phi_2 + \gamma_1 \quad p_{22} = \mu + \phi_2 + \gamma_2 \quad \dots \quad p_{25} = \mu + \phi_2 + \gamma_5 \quad p_{26} = \mu + \phi_2 - \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5$$

$$p_{31} = \mu + \phi_3 + \gamma_1 \quad p_{32} = \mu + \phi_3 + \gamma_2 \quad \dots \quad p_{35} = \mu + \phi_3 + \gamma_5 \quad p_{36} = \mu + \phi_3 - \gamma_1 - \gamma_2 - \gamma_3 - \gamma_4 - \gamma_5$$

$$p_{41} = \mu - \phi_1 - \phi_2 - \phi_3 + \gamma_1 \quad p_{42} = \mu - \phi_1 - \phi_2 - \phi_3 + \gamma_2 \quad \dots \quad p_{45} = \mu - \phi_1 - \phi_2 - \phi_3 + \gamma_5$$

These equations may be expressed in the form $\underline{A} \underline{\pi} = \underline{X} \underline{\beta}$ thus:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & & & & & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{pmatrix} \begin{pmatrix} p_{11} \\ 1 - p_{11} \\ p_{12} \\ 1 - p_{12} \\ p_{13} \\ 1 - p_{13} \\ \vdots \\ p_{45} \\ 1 - p_{45} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & & & & & & & & \vdots \\ 1 & 0 & 1 & 0 & -1 & -1 & -1 & -1 & -1 \\ \vdots & & & & & & & & \vdots \\ 1 & -1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & -1 & 0 & 1 & 0 & 0 & 0 \\ \vdots & & & & & & & & \vdots \\ 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mu \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \end{pmatrix}$$

\underline{A}

$\underline{\pi}$

\underline{X}

$\underline{\beta}$

(23 rows,
46 columns)

(46 rows,
1 column)

(23 rows,
9 columns)

(9 rows,
1 column)

The model may be tested by supplying LINCAT with $\underline{A}^* = (1 \ 0)$, \underline{X} as above, and the data arranged by type of accident, within each accident-type by model of car, and within each model of car the numbers of drivers injured and the number not injured.

The output from the program consists of the vector of parameters $\underline{\beta}$ (see table 5.3), together with the value of χ^2 due to deviations from the model. This χ^2 tells us whether the model fits the data. In this case $\chi^2 = 19.5$, which with 14 degrees of freedom is not statistically significant. So we can say that this sample of data agrees fairly well with our hypothesised model of the effect of accident type and model of car on driver's leg injury.

The parameters given in table 5.3 have a direct interpretation: they are the amount of increase or decrease in the probability of leg injury that is added to the base level probability, $\mu = .238$, as a result of being in one type of accident rather than another, or in one model of car rather than another. Thus the probability of the driver receiving a leg injury in a head-on accident in model A is $.238 + .099 + .136 = .473$, whereas in a rear-end accident in model B₂ it is $.238 - .108 - .044 = .086$, for instance.

It is of interest to determine whether the effect of type of accident is statistically significant, that is whether all the ϕ 's could be zero, and in the form $\underline{C} \underline{\beta} = \underline{0}$, this can be done by setting

$$\underline{C} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Then it is found that $\chi^2 = 21.1$ which with 3 degrees of freedom is highly statistically significant. This tells us that the deviations

Base level: injury severity, μ .238

Differential effects of type of accident, the ϕ_i :	Head-on	.10
	Rear-end	-.11
	Intersection	-.05
	Single-vehicle	.06

Differential effects of make and model of car, the γ_j :	Model A	.14
	B ₂	-.04
	B ₁	-.10
	H	.07
	F	-.07
	Others	.01

Values of χ^2 :

fit of model: 19.5, 14 d.f., N.S.

differences between types of accident:

21.1, 3 d.f., $P < .001$

differences between all six models of car:

22.4, 5 d.f., $P < .001$

differences between the five selected models:

22.3, 4 d.f., $P < .001$

Table 5.3: The parameters of a model in which type of accident and model of car have an additive effect on the probability of a leg injury to the driver. Remember that

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 = 0$$

was specified as part of the model.

from the hypothesis of no effect of accident type are too large to have reasonably occurred by chance.

Similarly, to test whether there is any statistically significant effect of model of car, we set

$$\underline{C} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\chi^2 = 22.4$ with 5 degrees of freedom is the result, highly significant, telling us that model of car does significantly affect injury.

A third hypothesis of interest concerns whether the five selected models of car differ significantly among themselves in the injury they give rise to. (They are all smaller than most other cars. Thus it might be that the significant effect of model of car arises from a difference between them as a group and the other models category.)

The null hypothesis is $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5$ which may be put in the form $\underline{C} \underline{\beta} = \underline{0}$ if

$$\underline{C} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

The result is $\chi^2 = 22.3$, 4 degrees of freedom, highly significant.

Thus there are significant differences between the selected models.

To test whether there are significant differences between two models, for instance model B_1 and model B_2 we put

$$\underline{C} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix}$$

The result is $\chi^2 = .5$, 1 degree of freedom, not significant. Thus we have not shown that model B_1 and model B_2 differ in the injury they give rise to.

The relative sizes of the ϕ_i in table 5.3 roughly reflect the expected velocity changes in the different types of accident - head-on and single-vehicle accidents being more severe than rear-end or intersection accidents. But in view of the selected nature of the sample of accidents considered, a number of reservations should be made, which are discussed in section 5.4.

We can see from table 5.3 that the effect of an accident being a head-on one is to produce an increase of some 40% ($10/23.8$) in the likelihood of leg injury. The effect of model of car can be considered in a similar way - adding μ on to each of the γ_j in turn, we get .37, .19, .14, .31, .16, .25. Thus there is a range of from 14% to 37% of drivers suffering leg injuries (averaged over accident type) for the six categories of model of car. It seems fair to call this a substantial effect of model of car.

5.2.3 An alternative model

Table 5.3 gave the result of fitting a model of the form $p_{ij} = \mu + \phi_i + \gamma_j$. It might be thought equally appropriate to fit one in which $\log_e(p_{ij}) = \mu + \phi_i + \gamma_j$. (That is, $p_{ij} = kA_iB_j$.) The definition of p_{ij} was the same as for table 5.3, that is p_{ij} = probability of suffering some leg injury. The results are shown in table 5.4. Qualitatively, the results are very similar to those of table 5.3.

The implication of this model is that, since $p_{ij} = cA_iB_j$,

$$\frac{p_{ij}}{p_{ik}} = \frac{cA_iB_j}{cA_iB_k} = \frac{B_j}{B_k}.$$

Thus the ratio of the probability of being injured in model j to that of being injured in model k is the same for all types of accident.

That is, being in one model of car rather than another increases the chance of leg injury by a constant factor, irrespective of type of accident. And similarly,

$$\frac{p_{ij}}{p_{hj}} = \frac{A_i}{A_h},$$

and being in one type of accident rather than another increases the chance of injury by a constant factor that does not depend on the model of car. This is in contrast to the model of table 5.3, where the chance of injury is changed by an additive constant when accident type or make and model is changed. As a numerical example, if $p_{11} = .1$, $p_{12} = .2$, and $p_{21} = .15$, the model of table 5.3 would imply $p_{22} = .25$, whereas the model of table 5.4 would imply $p_{22} = .3$.

Mean log (probability of leg injury), μ : -1.32

Differential effects of type of accident, the ϕ_i :	Head-on	.37
	Rear-end	-.41
	Intersection	-.13
	Single-vehicle	.17

Differential effects of make and model of car, the γ_j :	Model A	.38
	B ₂	-.14
	B ₁	-.60
	H	.25
	F	.27
	Others	-.16

Values of χ^2 :

fit of model:	15.8,	14 d.f.,	N.S.
differences between types of accident:			
	15.2,	3 d.f.,	P < .01
differences between all six models of car:			
	17.1,	5 d.f.,	P < .01
differences between the five selected models:			
	11.9,	4 d.f.,	P < .05

Table 5.4: Results from fitting the model $\log_e(\text{probability of leg injury}) = \mu + \phi_i + \gamma_j$.

5.2.4 Nonparametric analysis

Benard and van Elteren's test may be used to determine whether models of car differ in the severity of leg injury. The four types of accident constitute the rows, and the six models the columns, and there are very many ties within each row. Since this test is appropriate for fully-ranked data (i.e. with no ties), there is no need to combine the "major" and the "slight" categories as there was when using CATLIN. It was found that $\chi^2 = 15.1$, 5 degrees of freedom, $P < .01$. Using the randomisation option in the program listed in section 4.2.7, it was found that in 5 cases out of 1000 the observed test statistic was exceeded, indicating $P = .005$. This confirms that the models do differ in the degree of leg injury to their drivers.

The advantage of using a nonparametric test is that it does not need to assume a particular model for the combination of model and accident type effects, as was necessary to obtain the results of tables 5.3 and 5.4. The disadvantage is that numerical measures of the effects of each model are not obtained.

5.2.5 Non-frontal impacts

So far, frontal impacts only have been considered. Data for the struck car in intersection accidents (i.e. suffering a side impact) and the struck car in rear-end accidents (i.e. suffering a rear impact) are given in table 5.5. The low level of injury in the latter case is very noticeable. (The total numbers are not the same as for frontal impacts in the same type of accident because in some accidents more than two cars were involved.)

	<u>No injury</u>	<u>Slight injury</u>	<u>Major injury</u>
<u>Side impacts in intersection accidents</u>			
Model A	24	10	0
B ₂	17	3	0
B ₁	7	2	1
H	8	0	0
F	16	1	1
Others	82	16	0
<u>Rear impacts in rear-end accidents</u>			
Model A	14	1	0
B ₂	4	0	0
B ₁	6	0	0
H	4	0	0
F	2	0	0
Others	62	4	0

Table 5.5: Leg injuries in non-frontal impacts.

5.2.6 Location of injury

Injuries to the lower and to the upper leg may be affected differently by variations in design of the fascia and parcel shelf. Indeed, there was some evidence in Lister and Wall (1970) that model B₂ (with a strengthened fascia) gave rise to an exceptionally high incidence of injuries to the hip and pelvis, but as its parcel shelf is very weak and placed well forward under the fascia the incidence of fractures to the tibia and fibula was very low.

The police data used in the present study in many cases did not allow the site of the injury to be identified. For those in which an injury could be attributed to the upper (knee, thigh, hip) or lower (foot, ankle, tibia-fibula) leg, the data are given in table 5.6, and the results of the analysis in table 5.7. Differences between types of accident and between models of car show up for lower leg injury, but not for injury to the upper leg.

It was thought that there might be some tendency for the driver's right leg to be more severely injured than his left, since it will strike the parcel shelf or fascia at a point closer to where it is supported by the side of the car. But as table 5.8 shows, no difference of this sort was in fact found. This might be due to the presence of the steering column, but the absence of any difference between injuries to the left and right legs of front seat passengers (table 5.9) reinforces the conclusion that any difference is likely to be small.

<u>Model</u>	<u>Type of accident</u>	<u>Lower leg</u>		<u>Upper leg</u>	
		<u>Not injured</u>	<u>Injured</u>	<u>Not injured</u>	<u>Injured</u>
A	HO	16	7	21	2
	RE	29	5	32	2
	Int	31	12	40	3
	SV	17	18	34	1
B ₂	HO	6	2	7	1
	RE	3	0	2	1
	Int	11	2	11	2
	SV	19	4	23	0
B ₁	HO	11	2	12	1
	RE	10	0	10	0
	Int	11	1	11	1
	SV	22	4	26	0
H	HO	5	1	6	0
	RE	11	3	12	2
	Int	17	1	14	4
	SV	21	4	19	6
F	HO	4	5	8	1
	RE	6	0	6	0
	Int	18	1	19	0
	SV	14	3	15	2
Others	HO	54	13	57	10
	RE	58	5	60	3
	Int	84	8	81	11

Table 5.6: Injuries to upper and lower leg, frontal impacts only.

		<u>Lower leg</u>	<u>Upper leg</u>
Base level injury severity, μ :		.164	.08
Differential effects of type of accident, the ϕ_i :	Head-on	.06	.04
	Rear-end	-.07	-.02
	Intersection	-.06	.01
	Single-vehicle	.06	-.03
Differential effects of make and model of car, the γ_j :	Model A	.14	-.02
	B ₂	.00	-.02
	B ₁	-.05	-.03
	H	-.03	.10
	F	-.03	-.04
	Others	-.02	.01

Values of χ^2 :

fit of model	14.1 (N.S.)	9.4 (N.S.)
accident types	16.2 (P < .001)	4.9 (N.S.)
all six models	17.5 (P < .01)	7.8 (N.S.)
the five selected models	16.3 (P < .01)	6.9 (N.S.)

Table 5.7: Results from fitting linear models to the lower leg and upper leg injury data (upper leg = knee and above, lower leg = below knee).

<u>Model</u>	<u>Leg* most severely injured (drivers)</u>		
	<u>Left</u>	<u>Equal**</u>	<u>Right</u>
A	19	105	11
B ₂	3	41	3
B ₁	2	58	1
H	3	50	10
F	1	46	4
Others	13	193	16
Total	41	493	45

* Whole leg, including knee and above

** The equal category means both legs injured or both uninjured, or not clear which leg injured

Table 5.8: Showing which of the driver's legs was most severely injured (frontal impacts only).

<u>Model</u>	<u>Leg* most severely injured (passengers)</u>		
	<u>Left</u>	<u>Equal**</u>	<u>Right</u>
A	3	40	5
B ₂	3	13	1
B ₁	4	20	1
H	3	16	3
F	2	13	1
Others	7	66	7
Total	22	168	18

* Whole leg, including knee and above

** Both legs injured, or both uninjured, or not clear which leg injured, in cases where a passenger is known to have been present

Table 5.9: Showing which of the front seat passenger's legs was most severely injured (frontal impacts only).

5.3 Results: head injury

Although this research was primarily orientated towards leg injury, since the data was already available it was decided to analyse head injury as well, in view of the outstanding importance of such trauma as evidenced in section 1.3.

It was found possible to classify head injury into three categories: none, slight, and severe, the last corresponding to concussion or worse, and "head" injury including injury to the face. Table 5.10 gives the data. This was analysed by defining an average injury score s_{ij} for the (ij) th population as $.244p_1 + .673p_2 + .929p_3$ where p_1 , p_2 , and p_3 are the proportions of cases falling in the three injury categories. (The weighting factors were chosen on the same basis as described in section 4.1.5.)

It is quite true that the numbers of observations in the "severe" column of table 5.10 are quite small, but recent studies have shown (Craddock and Flood, 1970; Roscoe and Byars, 1971) that the chi-squared approximation to the test statistic is frequently very robust to smallness of sample (although their work is not exactly applicable here, since LINCAT does not use the conventional Pearson chi-squared). Moreover, the cells with the small sample sizes are never used alone, being always combined with the other categories by means of the function $s_{ij} = .244p_1 + .673p_2 + .929p_3$.

By using LINCAT with $\underline{A}^* = (.244 \ .673 \ .929)$ the model $s_{ij} = \mu + \phi_i + \gamma_j$ was fitted to table 5.10, with results as shown in table 5.11.

<u>Model</u>	<u>Type of accident</u>	<u>No injury</u>	<u>Slight injury</u>	<u>Severe injury</u>
A	HO	7	13	3
	RE	17	12	5
	Int	22	16	5
	SV	6	18	11
B ₂	HO	4	4	0
	RE	0	3	0
	Int	5	7	1
	SV	11	9	3
B ₁	HO	6	5	2
	RE	4	2	4
	Int	6	5	1
	SV	6	11	9
H	HO	1	4	1
	RE	5	8	1
	Int	8	7	3
	SV	8	14	3
F	HO	4	3	2
	RE	3	2	1
	Int	8	8	3
	SV	3	10	4
Others	HO	37	22	8
	RE	45	11	7
	Int	66	21	5

Table 5.10: Severity of head injury according to model of car and type of accident (frontal impacts only).

Base level injury severity, μ : .535

Differential effects of type of accident, the ϕ_i :	Head-on	.01
	Rear-end	-.03
	Intersection	-.05
	Single-vehicle	.07

Differential effects of make and model of car, the γ_j :	Model A	.03
	B ₂	-.01
	B ₁	.03
	H	.01
	F	.04
	Others	-.10

Values of χ^2 :

fit of model: 16.5, 14 d.f., N.S.

differences between types of accident:

18.2, 3 d.f., $P < .001$

differences between all six models of car:

29.4, 5 d.f., $P < .001$

differences between the five selected models:

1.4, 4 d.f., N.S.

Table 5.11: Results from fitting a model in which average head injury severity is a linear combination of accident type effects and model of car effects.

As can be seen there, a significant difference in head injury severity appears between different types of accident, with head-on and single-vehicle accidents being more severe than intersection or rear-end accidents. There is also a significant effect of model of car, but this arises purely from cars in the "others" category giving rise to less head injury than the five selected models, and there is no significant difference between these five. It seems likely that this arises because the five selected models are all fairly small - as Grime (1971) has shown, the velocity change to a vehicle in an accident is highly dependent on the ratio of its mass to the mass of the other vehicle involved, and this would be expected to be reflected in higher injury severity to its occupants. Grime (1971) has confirmed empirically that this is so.

Table 5.12 gives the head injuries suffered in impacts other than frontal ones. The chief features of this table are that the level of injury for side impacts in intersection accidents is roughly the same as for the striking car, and the low degree of injury in the struck car in rear-end accidents.

Side impacts in intersection accidents	Degree of head injury		
	<u>None</u>	<u>Slight</u>	<u>Severe</u>
Model A	21	10	3
B ₂	12	8	0
B ₁	5	5	0
H	3	3	2
F	10	8	0
Others	65	25	8

Rear impacts in
rear-end
accidents

Model A	12	2	1
B ₂	4	0	0
B ₁	6	0	0
H	3	1	0
F	2	0	0
Others	60	3	3

Table 5.12: Head injuries in non-frontal impacts.

5.4 Discussion

The influence of both accident type and model of car on leg injuries sustained by drivers has been demonstrated. The sample of accidents concerned was drawn from serious accidents in London in 1971, and it is possible that different results would be found in rural accidents. It is certainly likely that the different types of accident occur in different proportions in urban and rural areas (Grime and Jones, 1973), and that rural accidents are more severe than urban ones, but it seems unlikely that the relative degrees of injury for different models of car or different types of accident would be altered.

But it is now time to return to a consideration of the relative sizes of the ϕ_i in table 5.3 which, as has already been mentioned, roughly reflect the expected velocity changes in the different types of accident - head-on and single-vehicle accidents being more severe than rear-end or intersection accidents. It is perhaps surprising that this should be so clearly seen in this sample, since it was restricted to serious accidents - omitting the slight accidents would tend to equalise the severity of those remaining. Another reason for not paying too much attention to the relative severities of the types of accident as calculated from this sample is the different numbers at risk in the different accident types - for single-vehicle accidents there is only the driver and his passengers, one of whom must be seriously injured in order for the accident to be included in this sample, whereas for head-on accidents there are two drivers and their passengers. Thus in our sample the severity of injury to the one driver in a single-vehicle accident is likely to be increased relative to that to the drivers in a head-on accident. Rear-end accidents are similar to single-vehicle accidents in this respect, since virtually all injuries are to occupants

of the striking car, and intersection accidents are similar to head-on accidents since injury is roughly the same in the striking as in the struck car. Thus we would expect that the true values of the ϕ_i in table 5.3 (that is, derived from a random sample of accidents, not a sample of serious accidents) would be modified as follows: for single-vehicle and rear-end accidents, decreased relative to those for head-on and intersection accidents. It is perhaps better, therefore, to regard the ϕ_i as correction factors for the types of accident the different models are involved in, rather than as parameters of interest in their own right.

Models B_1 and B_2 are virtually identical except that the lower edge of the fascia of model B_2 is supported by a rigid box section metal beam which is absent in model B_1 . This is presumably why model B_2 gives rise to a higher degree of leg injury, though the difference between these two models is not actually statistically significant. It is also of interest that the effects of model, the γ_j , are of the same order of magnitude as the effects of accident type, the ϕ_i , thus showing that model of car is roughly as important as type of accident in determining leg injury.

It should also be said that we cannot with absolute confidence attribute differences in apparent severity of leg injury to differences in design of the leg impact area, since it could be, for instance, that differences in the design of the steering wheel, windscreen, and roof, could affect head injury; and if head injury were reduced then a higher proportion of serious injuries would involve leg injuries, even though for a given severity of impact the chance of a leg injury would be unchanged. Nevertheless - particularly in view of the lack of evidence for differences between the selected models in severity of head injury -

the simplest explanation is that design of leg impact area affects leg injury sufficiently for differences to be apparent in police reports with their crude descriptions of injury.

The injuries to front seat passengers have not been discussed because in cases where no injury to a passenger was mentioned, it was frequently not clear whether this was because there was no passenger, or because he was present but uninjured. (This difficulty does not apply when considering differences between right and left legs.)

Nahum et al (1968) studied 290 collisions involving 464 front seat car occupants, of whom 405 were injured. 186 of these received a leg injury, 141 from contact with the instrument panel. The distribution of leg injuries in this study was: 14% hip, 12% upper leg, 52% knee, 16% lower leg, 6% ankle and foot. This contrasts sharply with the distribution in rear seat casualties (Nahum et al, 1967): 17% hip and upper leg, 20% knee, 46% lower leg, and 17% ankle and foot. Whereas in front seat occupants the instrument panel damages the knee, in rear seat passengers the lower leg becomes trapped under the back of the front seat.

A regression analysis against various vehicle, occupant, and accident factors was performed, and it was concluded that (a) later model year cars are less injury producing, (b) more injury results at higher speeds, (c) older occupants are more susceptible to injury, and (d) slightly less injury results if the occupant is in a heavier vehicle. Nahum et al commented on improvements in instrument panel design that had recently occurred by removing and flattening protruding objects and control knobs, by relocation or removal of supporting structures that added rigidity to the panel, and by the use of materials

with excellent energy-absorption characteristics such as sheet metal.

Nahum et al (1968) also discussed the chief mechanisms of injury to each of the bones from the pelvis to the foot, and gave a number of case examples to illustrate their conclusions.

In their study based on the on-the-spot studies of TRRL, Grattan and Hobbs (1968) described the mechanisms by which certain skeletal injuries to the legs occur. Their chief conclusions were that (i) it is important to optimise the energy absorption characteristics of fascia panels, parcel trays, etc, in both the horizontal and vertical planes, because the great majority of serious injuries occur from contact with these rather than from contact with small projections or sharp edges (though clearly the elimination of these too is desirable); (ii) the varieties of lower limb injury were similar in both the restrained and the unrestrained occupant, though there is a substantial overall reduction in serious injury in restrained occupants. The great majority of their cases received their injuries in frontal impacts: the only type of leg injury which appeared to be specifically associated with side impacts was central fracture dislocation of the hip (see also Grattan and Hobbs, 1967). Table 5.13 summarises their data on the origin of the leg injury.

"By taking an undamaged vehicle structure and reproducing, experimentally under controlled laboratory conditions, the damage which has been caused to a similar structure by an occupant injured in an accident it is possible to determine the load exerted between the occupant and the structure. This load is of course that which caused the injury to the occupant and the damage to the structure. Provided a sufficiently large number of accident cases are investigated it should

be possible to determine the minimum load required to produce any particular injury." This quotation is taken from Lister and Wall (1970) who, in addition to comparing leg injuries to a limited sample of occupants of some models of car, carried out dynamic tests on the fascias of two models of car and on the parcel shelf of a third model. By comparing the occupants' injury or lack of it with the damage to the fascia, they concluded that the threshold value for skeletal injury to the knee-thigh-hip complex lies between 2 and 4 kN.

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Table 5.13: Point of contact with car causing injury to different parts of the leg, taken from Table 4 of Grattan and Hobbs (1968). These occurred in a total of 426 seriously injured vehicle occupants, a few of whom were rear-seat passengers.

CHAPTER 6: DRIVER AND VEHICLE EFFECTS ON TYPE OF ACCIDENT

6.1 Introduction

The chief purpose of this aspect of the study was to determine whether model of car has a significant effect on the type of accident it is involved in. The dependent variables studied were (i) the relative numbers of single- and two-car accidents, and (ii) the proportion of overturning in single-car accidents. The independent variables were (i) age of driver, (ii) sex of driver, (iii) model of car, and (iv) locale of accident (urban or rural). The data base used consists of accidents occurring in Great Britain during 1969-72.

Since it is clear that the distribution of accidents over models is different for different age groups of drivers for both single- and two-car accidents (as can be seen by examining respectively the numerators and the denominators in table 6.1) it is necessary to use a method of analysis that separates factors related to model of car from those related to driver age group. Otherwise, a high ratio of single- to two-car accidents (for instance) for one model of car when all age groups of driver are combined could be due to that model being predominately driven by drivers in an age group that is associated with a high ratio of such accidents, as well as the alternative of that model genuinely being prone to a higher ratio than other models. A suitable method using the program CATLIN (described in section 4.1) was devised for the statistical analysis, which was made separately on accidents occurring in urban and in rural areas. (An urban area is here defined to be one where a 30 or 40 mph speed limit is in operation, and rural areas are where the speed limit is higher or absent.)

The application of CATLIN to the analysis of this data is discussed in section 6.2. Results for the relative proportions of single- and two-car accidents are presented in section 6.3, and section 6.4 gives the results for the proportion of overturning in single-car accidents. The Chapter concludes with a discussion section.

6.2 Derivation of statistical model

The application of CATLIN to analysing the relative numbers of single- and two-car accidents will now be discussed. The populations are each combination of age group of driver and model of car (36 populations). The two responses are single- and two-car accidents.

For convenience, we first slightly extend the notation associated with CATLIN and LINCAT.

When \underline{A} is of the form

$$\begin{pmatrix} \underline{A}^* & \underline{0} & \underline{0} & \dots & \underline{0} \\ \underline{0} & \underline{A}^* & \underline{0} & \dots & \underline{0} \\ \underline{0} & \underline{0} & \underline{A}^* & \dots & \underline{0} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ \underline{0} & \underline{0} & \underline{0} & \dots & \underline{A}^* \end{pmatrix}$$

in which \underline{A}^* is a constant matrix with 2 columns, so that the populations are kept separate in the linear functions of the $\pi_{\mu\nu}$ which are formed on the left-hand side of the equation, and, moreover, the function is the same for all populations so that LINCAT may be used, we shall write

$$\underline{A} = (\underline{A}^*)^{36}.$$

Similarly, when $\underline{A} = \underline{I}$ (the identity matrix) and \underline{K} is of the form

$$\begin{array}{ccccccc}
 \underline{K^*} & \underline{0} & \underline{0} & . & . & . & \underline{0} \\
 \underline{0} & \underline{K^*} & \underline{0} & . & . & . & \underline{0} \\
 \underline{0} & \underline{0} & \underline{K^*} & . & . & . & \underline{0} \\
 . & . & . & . & . & . & . \\
 . & . & . & . & . & . & . \\
 . & . & . & . & . & . & . \\
 \underline{0} & \underline{0} & \underline{0} & . & . & . & \underline{K^*}
 \end{array}$$

in which $\underline{K^*}$ is a constant matrix with 2 columns, so that the populations are kept separate in the logarithmic functions of the $\pi_{\mu\nu}$, and the function is the same for all populations, we shall write $\underline{K} = (\underline{K^*})^{36}$.

The chief hypothesis of interest is derived as follows: let ${}_1N_{ij}$ be the number of single-car accidents to model j driven by drivers in age group i , and ${}_2N_{ij}$ be the corresponding number of two-car accidents. Let M_{ij} be the mileage driven by drivers in age group i and model j . Assume that driver age and model of car have independent effects on accident rates, in the sense that

$$\begin{aligned}
 {}_1N_{ij} &= M_{ij} b_i c_j \\
 {}_2N_{ij} &= M_{ij} d_i e_j
 \end{aligned}$$

where the b_i and d_i are the age factors for single- and two-car accidents respectively, and c_j and e_j are the vehicle factors for the two types of accident. Then

$$\frac{{}_1N_{ij}}{{}_2N_{ij}} = \frac{b_i c_j}{d_i e_j} = f_i g_j$$

and

$$\log_e \frac{{}_1N_{ij}}{{}_2N_{ij}} = \mu + \phi_i + \gamma_j \quad (6.1)$$

Since

$$\frac{{}_1N_{ij}}{{}_2N_{ij}} = \frac{{}_1N_{ij} / ({}_1N_{ij} + {}_2N_{ij})}{{}_2N_{ij} / ({}_1N_{ij} + {}_2N_{ij})} = \frac{\pi_{(ij)1}}{\pi_{(ij)2}}$$

(in which the bracketed subscript denotes the population and the unbracketed subscript the response), the left hand side of equation (6.1) can be put in a suitable form for CATLIN (equation (4.5)) by putting $\underline{A} = \underline{I}$ (the identity matrix), and $\underline{K} = (\underline{K}^*)^{36}$ where $\underline{K}^* = (1 \ -1)$.

If we therefore choose \underline{X} to be the 36 x 14 matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ & & \cdot & & & & \cdot & & & & \cdot & & & \\ & & \cdot & & & & \cdot & & & & \cdot & & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 0 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix} \quad (6.2)$$

the fourteen parameters in the vector $\underline{\beta}$ are interpreted as follows:

$$\begin{aligned} \beta_1 &= \mu \\ \beta_2 &= \gamma_1 \\ \beta_3 &= \gamma_2 \\ &\vdots \\ &\vdots \\ \beta_{12} &= \gamma_{11} \\ \sum_{12} \beta &= -\gamma_{12} \\ \beta_{13} &= \phi_1 \\ \beta_{14} &= \phi_2 \\ \beta_{13} + \beta_{14} &= -\phi_3 \end{aligned}$$

To test whether there is any significant effect of model of car, we wish to know whether $\gamma_1 = \gamma_2 = \gamma_3 = \dots = \gamma_{12}$, that is whether

$\beta_2 = \beta_3 = \beta_4 = \dots = \beta_{12} = 0$, which we can do by specifying \underline{C} to be

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & & & & & & \\ & & & & & & & & & & & & & \\ & & & & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Similarly, to test whether there is any significant effect of age of driver, i.e. $\beta_{13} = \beta_{14} = 0$, we specify \underline{C} to be

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Analysis of the proportion of overturning in single-car accidents is similar. The 36 populations are the same, and the two responses are single-car accidents in which overturning occurred, and those in which overturning did not take place. (Since the observations in the different cells of the table must be independent, it would be incorrect to input to the program as two responses the number of overturning accidents and the total number of single-car accidents, since the former are included in the latter.) The hypothesis of primary interest is whether age and vehicle factors have independent effects on the proportion of overturning (in the sense of combining multiplicatively), that is $\pi_{(ij)OT} = f_i g_j$, which can be put in the form of equation (4.5) if $\underline{A} = \underline{I}$, the identity matrix, and $\underline{K} = (\underline{K}^*)^{36}$ where $\underline{K}^* = (1 \ 0)$. \underline{X} is matrix (6.2) as before, and hypotheses about the β 's can be tested using \underline{C} as before.

See also table 6.13 for a specification of these and other, alternative, models.

6.3 Results: relative numbers of single- and two-car accidents

The numbers of all injury accidents reported to the police in Great Britain in 1969-72 that were either single-car non-pedestrian or two-car non-pedestrian accidents were extracted from national accident tapes. Table 6.1 presents the results for the twelve most common models, for male drivers only, and for accidents in urban areas only. This data was input to CATLIN and the results of table 6.3 obtained when using the statistical model described in the previous section. The following conclusions may be drawn:

- (i) We have no reason to reject the model described above, since the value of χ^2 describing the deviation of the data from the model is only 29.1, which is not statistically significant.
- (ii) Age of driver significantly affects the ratio of single- to two-car accidents, with young drivers being involved in a higher proportion of the former than are older drivers.
- (iii) Model of car significantly affects the ratio of the two types of accident, and the "car factors" which quantify this have been determined for twelve models of car.
- (iv) The effect of age group is stronger than the effect of model of car.

The ratio of single- to two-car accidents for a particular age group of driver, corrected for the association between driver age and model of car, may be obtained from table 6.3 by adding the appropriate age factor to the base level μ and taking the antilogarithm. Correspondingly this ratio for a particular model of car, corrected

for the effect of driver age, is obtained by adding the appropriate car factor to μ and taking the antilogarithm.

By comparing ratios calculated thus with the crude overall ratios for each age group and model, the practical importance of correcting the age ratios for model effects and vice versa may be assessed. It turns out that the ratio for each age group averaged over models is altered only in the third decimal place; for models, the root-mean-square difference between the crude ratio and the age-corrected ratio is 0.02, the largest difference being for model A, which has a crude ratio of .286 but a corrected ratio of .228 ($= e^{-1.46 - 0.02}$).

Tables 6.2 and 6.4 give the corresponding results for accidents at rural sites. Here, the value of χ^2 describing the deviation of the data from the model is larger than in the urban case, being 61.1, which is very highly statistically significant. However, it is much smaller than the values of χ^2 obtained when testing for differences between models of car and between age groups, and since the numbers of observations in table 6.2 are large, a large and significant value of χ^2 could be the result of quite small deviations from the model. So the model is probably acceptable (at least as a first approximation) for accidents at rural sites as well as for urban accidents. Figure 6.1 shows the actual and predicted (from the best-fit vector $\underline{\beta}$) values of $\underline{X} \underline{\beta}$ for the 36 populations for rural areas. The good agreement evident there supports the contention that the large χ^2 is due to the large number of observations rather than to the model being seriously deficient.

As for the urban data, the "corrected" age ratios are only slightly different from the "crude" ratios, differing only by about 0.01. For

models, however, there are some substantial differences between the age-averaged figures in the final column of table 6.2 and the ratios with the effect of driver age removed. The largest difference is again for model A (crude ratio, 0.75; corrected ratio, 0.56) and the root-mean-square difference is 0.08.

Table 6.5 gives the natural antilogarithms of the parameters listed in tables 6.3 and 6.4: these are the values of k , A_i , and B_j in the equation ${}_1N_{ij}/{}_2N_{ij} = k A_i B_j$. (So $k = \exp(\mu)$, $A_i = \exp(\phi_i)$, and $B_j = \exp(\gamma_j)$.) Thus we estimate the ratio of single- to two-car accidents for drivers less than 25 years old in model E in urban areas to be $.23 \times 1.72 \times .92 = .36$, compared to the figure of .37 in table 6.1.

Figure 6.2 shows the high correlation which exists between the car factors for the urban data and those for the rural data. By showing that models of car which are prone to single-vehicle accidents in rural areas are also prone to them in urban areas, this provides more evidence (besides, that is, the significance of the differences between models found for the urban and rural data separately) of the existence of vehicle factors predisposing towards one or other type of accident. It is also clear that model of car has a greater effect on the proportion of single-vehicle accidents in rural areas than it does in urban areas.

Driver age group	<u>Single-car accidents</u> <u>car-car accidents</u>							
	≤ 24		25 - 34		35+		All ages	
<u>Model</u>								
E	$\frac{1373}{3677}$.373	$\frac{957}{4728}$.202	$\frac{1065}{8146}$.131	$\frac{3395}{16551}$.205
A	$\frac{2568}{6527}$.393	$\frac{722}{3458}$.209	$\frac{464}{3139}$.148	$\frac{3754}{13124}$.286
B	$\frac{1335}{2872}$.465	$\frac{634}{2800}$.226	$\frac{889}{5871}$.151	$\frac{2858}{11543}$.248
J	$\frac{601}{1669}$.360	$\frac{422}{2061}$.205	$\frac{528}{3763}$.140	$\frac{1551}{7493}$.207
F	$\frac{439}{1159}$.379	$\frac{290}{1421}$.204	$\frac{267}{2134}$.125	$\frac{996}{4714}$.211
D	$\frac{1389}{3874}$.358	$\frac{474}{2236}$.212	$\frac{520}{3770}$.138	$\frac{2383}{9880}$.241
H	$\frac{885}{1958}$.452	$\frac{334}{1369}$.244	$\frac{441}{2596}$.170	$\frac{1660}{5923}$.280
C	$\frac{683}{1754}$.389	$\frac{492}{2257}$.218	$\frac{607}{4499}$.135	$\frac{1782}{8510}$.209
G	$\frac{490}{1354}$.362	$\frac{347}{1644}$.211	$\frac{464}{3162}$.147	$\frac{1301}{6160}$.211
I	$\frac{326}{989}$.330	$\frac{323}{1455}$.222	$\frac{410}{2983}$.137	$\frac{1059}{5427}$.195
K	$\frac{397}{936}$.424	$\frac{275}{1288}$.214	$\frac{235}{1511}$.156	$\frac{907}{3735}$.243
T	$\frac{713}{1403}$.508	$\frac{291}{1063}$.274	$\frac{231}{1518}$.152	$\frac{1235}{3984}$.310
Total	$\frac{11199}{28172}$.398	$\frac{5561}{25780}$.216	$\frac{6121}{43092}$.142	$\frac{22881}{97044}$.236

Table 6.1: Numbers of single- and two-car accidents in urban areas, classified according to model of car and age of driver.

Driver age group	<div>Single-car accidents</div> <div>Car-car accidents</div>							
	≤ 24		25 - 34		35+		All ages	
Model								
E	$\frac{1646}{1709}$.963	$\frac{1068}{2312}$.462	$\frac{1160}{4057}$.286	$\frac{3874}{8078}$.480
A	$\frac{3381}{3214}$	1.052	$\frac{781}{1449}$.539	$\frac{480}{1547}$.310	$\frac{4642}{6210}$.748
B	$\frac{2024}{1539}$	1.315	$\frac{989}{1292}$.765	$\frac{1504}{3289}$.457	$\frac{4517}{6120}$.738
J	$\frac{707}{774}$.913	$\frac{478}{906}$.528	$\frac{601}{1796}$.335	$\frac{1786}{3476}$.514
F	$\frac{695}{675}$	1.030	$\frac{478}{766}$.624	$\frac{397}{1158}$.343	$\frac{1570}{2599}$.604
D	$\frac{1707}{1539}$	1.109	$\frac{423}{724}$.584	$\frac{430}{1465}$.294	$\frac{2560}{3728}$.687
H	$\frac{1322}{967}$	1.367	$\frac{430}{584}$.736	$\frac{556}{1355}$.410	$\frac{2308}{2906}$.794
C	$\frac{826}{704}$	1.173	$\frac{406}{803}$.506	$\frac{494}{1917}$.258	$\frac{1726}{3424}$.504
G	$\frac{545}{612}$.890	$\frac{282}{635}$.444	$\frac{369}{1499}$.246	$\frac{1196}{2746}$.436
I	$\frac{360}{376}$.957	$\frac{268}{624}$.429	$\frac{364}{1309}$.278	$\frac{992}{2309}$.430
K	$\frac{614}{458}$	1.341	$\frac{340}{566}$.601	$\frac{300}{805}$.373	$\frac{1254}{1829}$.686
T	$\frac{1036}{560}$	1.850	$\frac{308}{368}$.837	$\frac{303}{675}$.449	$\frac{1647}{1603}$	1.027
Total	$\frac{14863}{13127}$	1.132	$\frac{6251}{11029}$.567	$\frac{6958}{20872}$.333	$\frac{28072}{45028}$.623

Table 6.2: Numbers of single- and two-car accidents in rural areas, classified according to model of car and age of driver.

Base level, μ -1.46

Age factors, the ϕ_i :

≤ 24 years	.54
25 - 34 years	-.06
35+ years	-.48

Car factors, the γ_j :

Model E	-.08
A	-.02
B	.09
J	-.06
F	-.08
D	-.08
H	.13
C	-.03
G	-.04
I	-.07
K	.04
T	.20

Values of χ^2 :

fit: 29.1, 22 d.f., not significant ($P > 0.1$)

differences between models: 120.7, 11 d.f., $P < .001$

differences between age groups: 3286.7, 2 d.f., $P < .001$

Table 6.3: Results of analysing the data of table 6.1 (urban accidents)
according to the model $\log_e \frac{1^{N_{ij}}}{2^{N_{ij}}} = \mu + \phi_i + \gamma_j$.
(Remember that $\sum \phi_i = \sum \gamma_j = 0$)

Accuracy of the car factors: their estimated root-mean-variance is 0.03.

Base level, μ - .51

Age factors, the ϕ_i : ≤ 24 years .63

25 - 34 years - .04

35+ years - .59

Car factors, the γ_j : Model E - .18

A - .07

B .25

J - .09

F .00

D - .04

H .21

C - .12

G - .27

I - .21

K .11

T .41

Values of χ^2 :

fit: 61.1, 22 d.f., $P < .001$

differences between models: 538.9, 11 d.f., $P < .001$

differences between age groups: 4142.9, 2 d.f., $P < .001$

Table 6.4: Results of analysing the data of table 6.2 (rural accidents)

according to the model $\log_e \frac{1^{N_{ij}}}{2^{N_{ij}}} = \mu + \phi_i + \gamma_j$.

(Remember that $\Sigma \phi = \Sigma \gamma = 0$).

Accuracy of the car factors: their estimated root-mean-variance is 0.03.

		<u>Urban</u>	<u>Rural</u>
k		.23	.60
A _i	≤ 24 years	1.72	1.87
	25 - 34 years	0.94	0.96
	35+ years	0.62	0.56
B _j	Model E	0.92	0.84
	A	0.98	0.93
	B	1.09	1.28
	J	0.94	0.91
	F	0.92	1.00
	D	0.93	0.96
	H	1.14	1.23
	C	0.97	0.89
	G	0.98	0.76
	I	0.93	0.81
	K	1.04	1.12
	T	1.22	1.51

Table 6.5: The natural antilogarithms of the parameters in tables 6.3 and 6.4. These are the values of k, A_i, and B_j in the equation $\frac{1^{N_{ij}}}{2^{N_{ij}}} = k A_i B_j$.

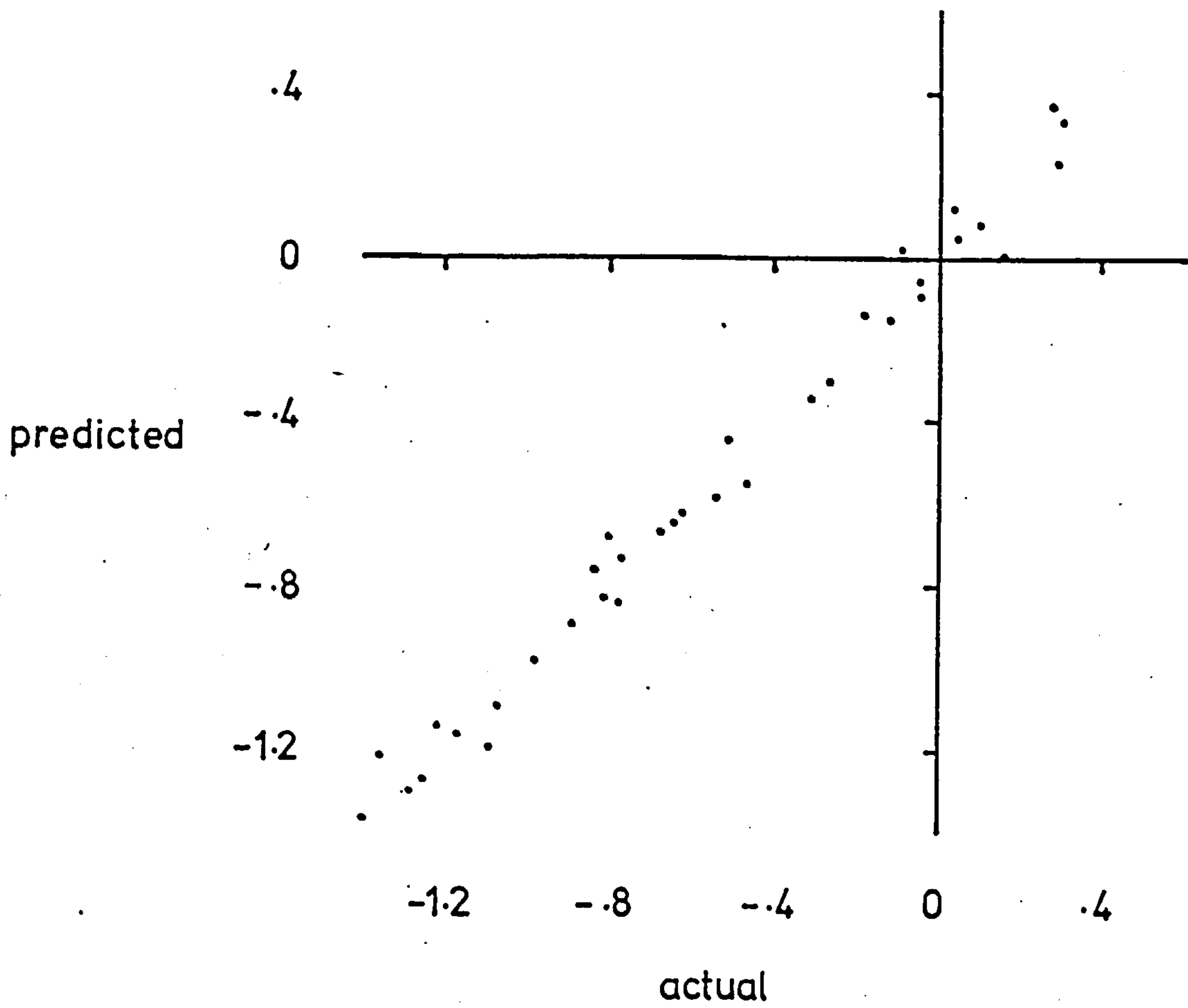


Figure 6.1: for the data of table 6.2, the relation between the actual values of $K \log_e \frac{A}{\pi}$ and the values of $X \beta$ predicted from the best-fit β .

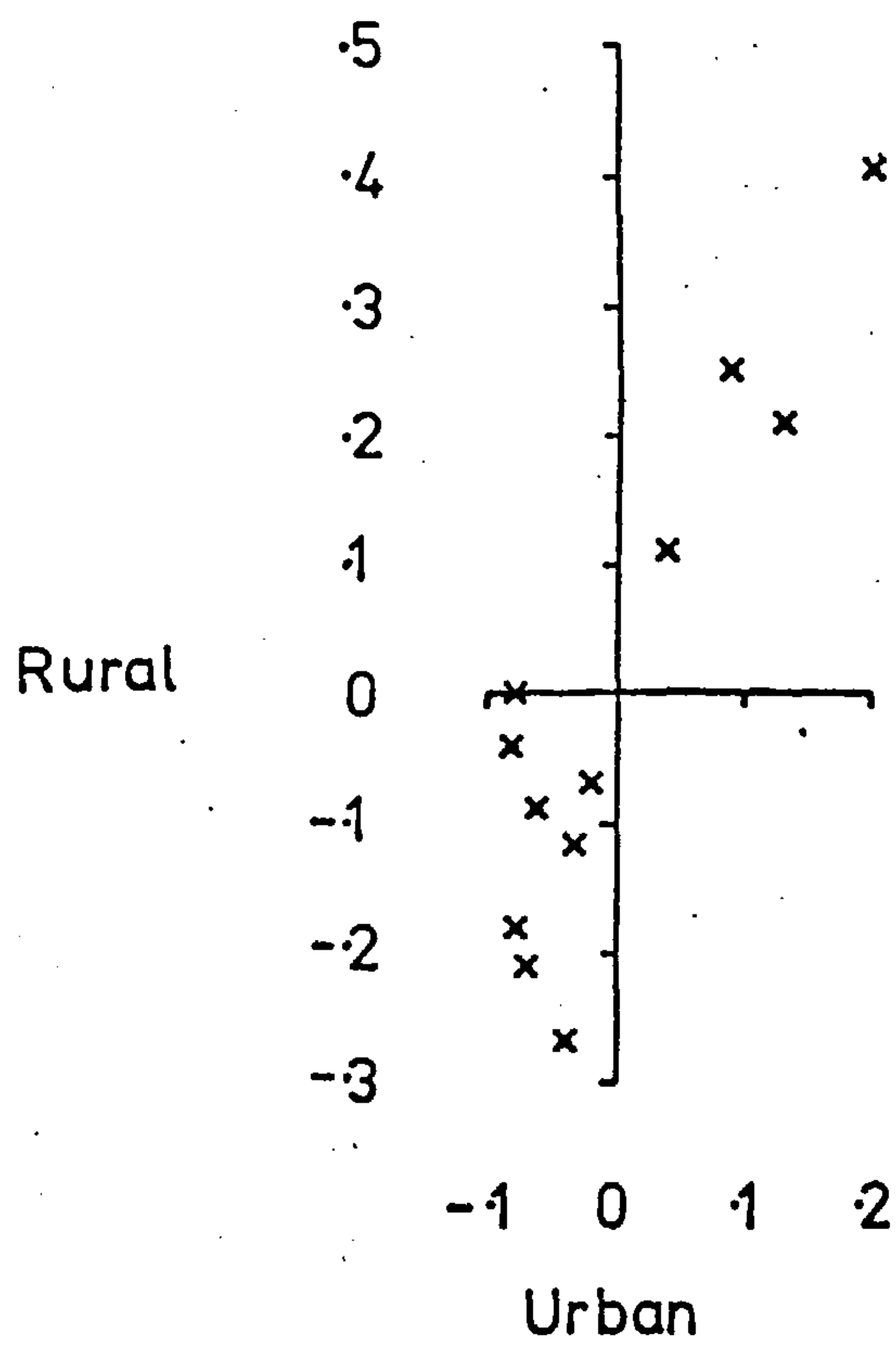


Figure 6.2: the correlation between the car factors for the ratio of single- to two-car accidents found in urban and in rural areas.

6.4 Results: proportion of overturning in single-vehicle accidents

For this analysis, all single-car non-pedestrian accidents in Great Britain in 1969-72 were extracted from national accident tapes and classified according to whether or not overturning was reported to have occurred. Table 6.6 presents the results for the twelve most common models, for male drivers only, and for accidents in urban areas only. This data was input to CATLIN and the results of table 6.8 obtained, using a statistical model of the type described at the end of section 6.2, the two responses being single-car accidents in which overturning did or did not occur respectively. Thus denoting the former by OT_{ij} and the total number of single-car accidents by N_{ij} as before, the model may be expressed as $\log_e \frac{OT_{ij}}{N_{ij}} = \mu + \phi_i + \gamma_j$. The following conclusions may be drawn:

- (i) Our model is statistically acceptable, as the value of χ^2 describing the deviation of the data from the model is only 30.1, which is not statistically significant.
- (ii) Age of driver significantly affects the proportion of overturning in single-car accidents, young drivers being more prone to this than older ones.
- (iii) Model of car significantly affects the proportion of overturning, and the "car factors" which quantify this have been determined for twelve models of car.
- (iv) The influences of car model and of driver age are of roughly similar strengths.

Tables 6.7 and 6.9 give the corresponding results for accidents in rural areas. In this case, χ^2 describing the deviation of the data from the model is much larger, being 135.8. Despite this, we are

inclined to accept the model for the time being for reasons already mentioned, i.e. the value of χ^2 is not very large bearing in mind the large number of cases in the table. Figure 6.3 compares the values of $\sum \hat{g}$ (calculated from the best-fit \hat{g}) with the observed $\sum K \log_e(A \pi)$ and although there is clearly good correlation it is noticeably less good than that in figure 6.1. Also noteworthy in table 6.9 is the smallness of the effect of age group, though it must be remembered that, because of the higher overall level of overturning in rural accidents and the multiplicative effects of the car and age factors with the base level of overturning, the difference in proportion of overturning (rather than the ratio, or difference of logarithms) may be as great as in urban areas. Thus in urban areas the proportion of overturning (averaged over all car models) increases from an overall average of 12.5% ($= e^{-2.08}$) to 17.6% ($= e^{-2.08 + .35}$) when considering drivers less than 25 years old only; and the corresponding figures for rural areas are 43.1% and 47.6%, almost as great an increase in absolute terms, though much less proportionately.

Table 6.10 gives the natural antilogarithms of the parameters listed in tables 6.8 and 6.9: these are the values of k , A_i , and B_j in the equation $\frac{OT_{ij}^N}{1_{ij}^N} = k A_i B_j$.

Thus $k = \exp(\mu)$, $A_i = \exp(\phi_i)$, and $B_j = \exp(\gamma_j)$, and we estimate the proportion of overturning for drivers less than 25 years old in model E in urban areas (for instance) to be $.125 \times 1.41 \times .84 = 0.15$ compared to the actual figure of $200/(1173 + 200) = 0.15$ evident from table 6.6.

A strong correlation between the car factors found for urban and for rural accidents is evident in figure 6.4.

How much difference between the crude proportions of overturning for each model and the corresponding proportions calculated from table 6.10 from which the effect of age of driver has been removed is there? It turns out this is quite small, the root-mean-square difference in overturning being .02 for the urban data (largest difference, .04 for model T) and .01 for the rural data (largest difference, .02 for model D).

Driver age group	Overturning		Non-overturning		Single-vehicle accidents			
	< 24		25 - 34		35+		All ages	
Model								
E	$\frac{200}{1173}$.171	$\frac{94}{863}$.109	$\frac{88}{977}$.090	$\frac{382}{3013}$.127
A	$\frac{342}{2226}$.154	$\frac{62}{660}$.094	$\frac{38}{426}$.089	$\frac{442}{3312}$.134
B	$\frac{160}{1175}$.136	$\frac{63}{571}$.110	$\frac{59}{830}$.071	$\frac{282}{2576}$.110
J	$\frac{85}{516}$.166	$\frac{50}{372}$.134	$\frac{31}{497}$.062	$\frac{166}{1385}$.120
F	$\frac{68}{371}$.183	$\frac{28}{262}$.107	$\frac{27}{240}$.113	$\frac{123}{873}$.141
D	$\frac{284}{1105}$.257	$\frac{73}{401}$.182	$\frac{51}{469}$.109	$\frac{408}{1975}$.207
H	$\frac{192}{693}$.277	$\frac{41}{293}$.140	$\frac{33}{408}$.081	$\frac{266}{1394}$.191
C	$\frac{121}{562}$.215	$\frac{54}{438}$.123	$\frac{66}{541}$.122	$\frac{241}{1541}$.156
G	$\frac{68}{422}$.161	$\frac{37}{310}$.119	$\frac{43}{421}$.102	$\frac{148}{1153}$.128
I	$\frac{47}{279}$.168	$\frac{45}{278}$.162	$\frac{35}{375}$.093	$\frac{127}{932}$.136
K	$\frac{98}{299}$.328	$\frac{39}{236}$.165	$\frac{26}{211}$.123	$\frac{163}{746}$.218
T	$\frac{250}{463}$.540	$\frac{57}{234}$.244	$\frac{32}{199}$.161	$\frac{339}{896}$.378
Total	$\frac{1915}{9284}$.206	$\frac{643}{4918}$.131	$\frac{529}{5594}$.095	$\frac{3087}{19796}$.156

Table 6.6: Numbers of single-car accidents, classified according to whether or not the vehicle overturned and according to model of car and age of driver, in urban areas.

Driver age group	<u>Overturning</u> Non-overturning		Single-vehicle accidents					
	≤ 24		25 - 34		35+		All ages	
<u>Model</u>								
E	$\frac{708}{938}$.755	$\frac{451}{617}$.731	$\frac{349}{811}$.430	$\frac{1508}{2366}$.637
A	$\frac{1269}{2112}$.601	$\frac{285}{496}$.535	$\frac{156}{324}$.481	$\frac{1710}{2932}$.583
B	$\frac{784}{1240}$.632	$\frac{385}{604}$.637	$\frac{554}{950}$.583	$\frac{1723}{2794}$.617
J	$\frac{322}{385}$.836	$\frac{222}{256}$.867	$\frac{249}{352}$.707	$\frac{793}{993}$.799
F	$\frac{349}{346}$	1.009	$\frac{203}{275}$.738	$\frac{170}{227}$.749	$\frac{722}{848}$.851
D	$\frac{837}{870}$.962	$\frac{203}{220}$.923	$\frac{180}{250}$.720	$\frac{1220}{1340}$.910
H	$\frac{677}{290}$	2.334	$\frac{187}{243}$.770	$\frac{254}{302}$.841	$\frac{1118}{835}$	1.339
C	$\frac{398}{464}$.858	$\frac{187}{219}$.854	$\frac{214}{280}$.764	$\frac{799}{963}$.830
G	$\frac{188}{357}$.527	$\frac{104}{178}$.584	$\frac{128}{241}$.531	$\frac{420}{776}$.541
I	$\frac{167}{193}$.865	$\frac{97}{171}$.567	$\frac{142}{222}$.640	$\frac{406}{586}$.693
K	$\frac{333}{281}$	1.185	$\frac{118}{222}$.532	$\frac{97}{203}$.478	$\frac{548}{706}$.776
T	$\frac{626}{410}$	1.527	$\frac{179}{129}$	1.388	$\frac{142}{161}$.882	$\frac{947}{700}$	1.353
Total	$\frac{6658}{7886}$.844	$\frac{2621}{3630}$.722	$\frac{2635}{4323}$.610	$\frac{11914}{15839}$.752

Table 6.7: Numbers of single-car accidents, classified according to whether or not the vehicle overturned and according to model of car and age of driver, in rural areas.

Base level, μ		-2.08
Age factors, the ϕ_i :	≤ 24 years	.35
	25 - 34 years	-.04
	35+ years	-.31
Car factors, the γ_j :	Model E	-.17
	A	-.27
	B	-.33
	J	-.20
	F	-.10
	D	.16
	H	.14
	C	.03
	G	-.14
	I	-.04
	K	.28
	T	.64

Values of χ^2 :

fit: 30.1, 22 d.f., not significant ($P > 0.1$)

differences between models: 325.4, 11 d.f., $P < .001$

differences between age groups: 223.0, 2 d.f., $P < .001$

Table 6.8: Results of analysing the data of table 6.6 (urban accidents)

according to the model $\log_e \frac{O_{ij}^N}{1^N_{ij}} = \mu + \phi_i + \gamma_j$.

(Remember that $\sum \phi_i = \sum \gamma_j = 0$).

Accuracy of the car factors: their estimated root-mean-variance is 0.06.

		<u>Urban</u>	<u>Rural</u>
k		.125	.43
A _i	≤ 24 years	1.41	1.10
	25 - 34 years	0.96	1.01
	35+ years	0.73	0.90
B _j	Model E	0.84	0.90
	A	0.76	0.80
	B	0.72	0.87
	J	0.82	1.02
	F	0.91	1.05
	D	1.17	1.05
	H	1.15	1.36
	C	1.03	1.03
	G	0.87	0.80
	I	0.96	0.95
	K	1.32	1.03
	T	1.90	1.28

Table 6.10: The natural antilogarithms from tables 6.8 and 6.9. These are the values of k, A_i, and B_j in the equation $\frac{OT_{ij}^{N_{ij}}}{1^{N_{ij}}} = k A_i B_j$.

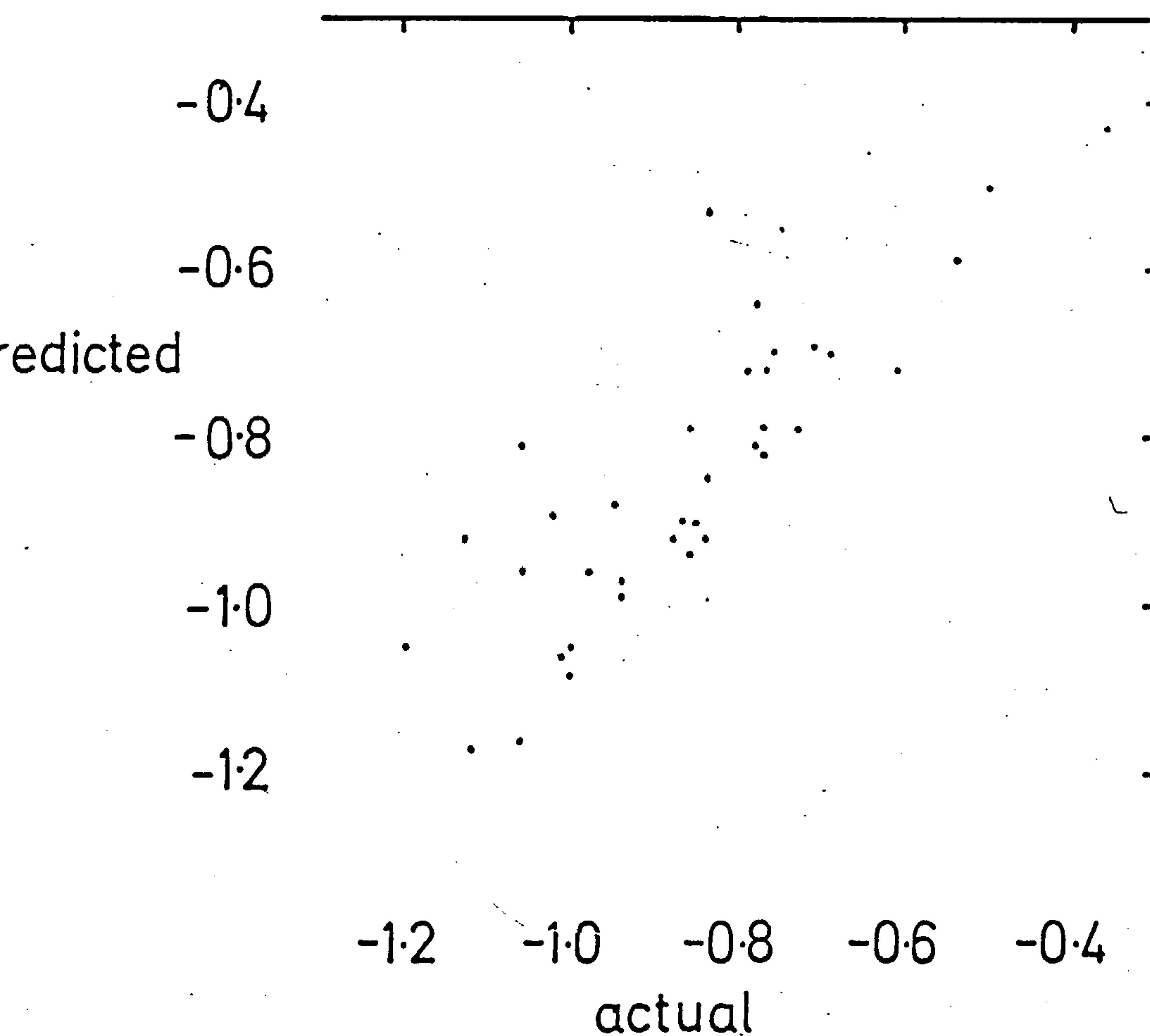


Figure 6.3: for the data of table 6.7, the relation between the actual values of $K \log A \pi$ and the values of $X \beta$ predicted from the best-fit β .

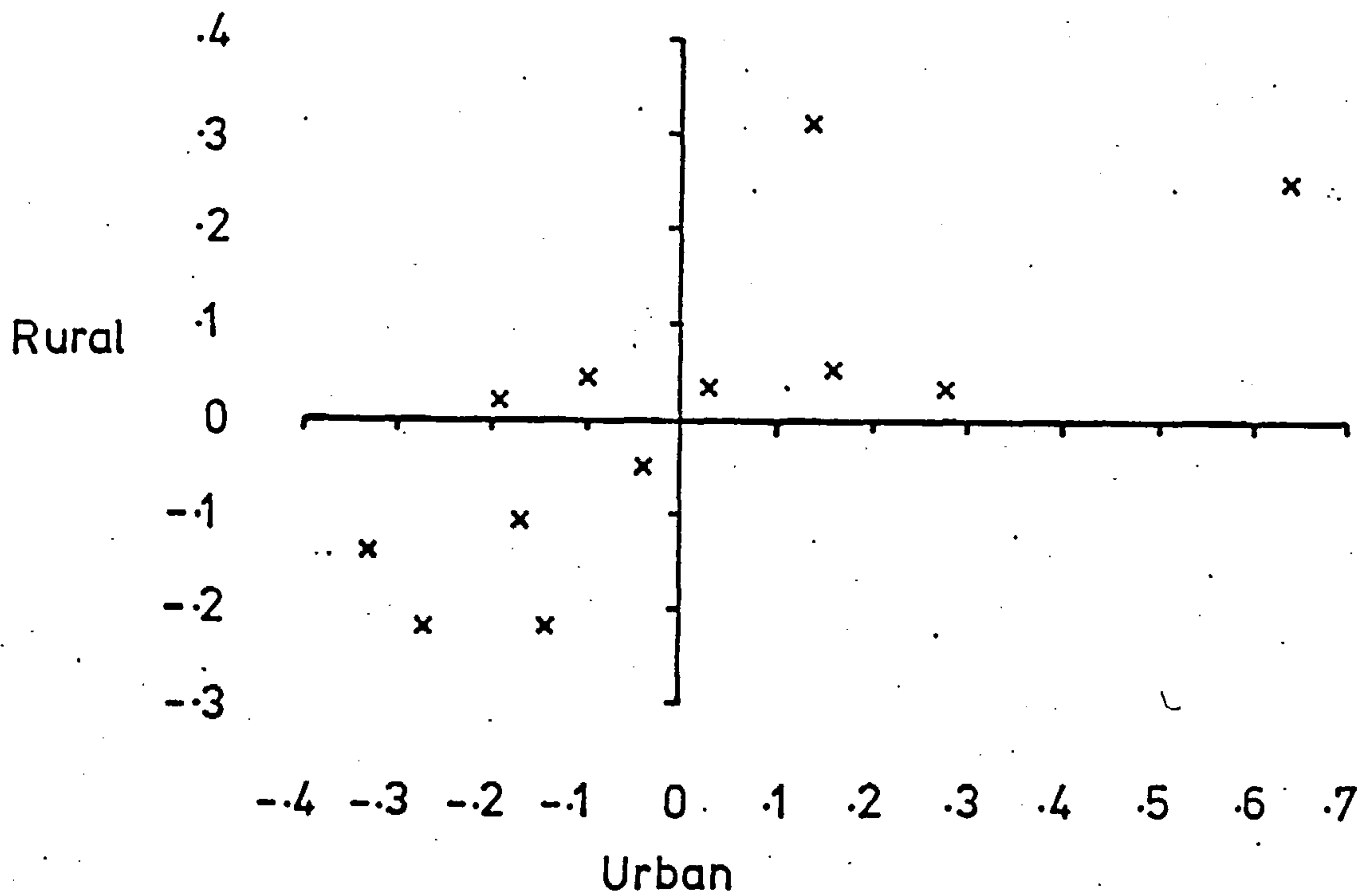


Figure 6.4: the correlation between the car factors for overturning in single-car accidents found in urban and in rural areas.

6.5 Discussion

6.5.1 Do vehicle factors affect the type of accident amongst all age groups of driver?

In the previous sections it has been shown that models of car differ in their relative involvements in single- and two-car accidents and in their frequency of overturning in single-vehicle accidents. Furthermore, the effects in urban and rural areas are correlated. It may reasonably be asked whether this is true for all ages of driver, since it is possible that only for inexperienced drivers do vehicle factors influence the type of accident. Therefore, for each of the three age groups separately, a correlation was performed between the ratio of single- to two-car accidents in urban and in rural areas. This was also done for the ratio of overturning to non-overturning single-car accidents. Table 6.11 shows the correlation coefficients which were found. As all are positive, and five are statistically significant, we reject the above suggestion that only for young drivers do vehicle factors have an effect.

<u>Driver age group</u>	<u>Variable correlated</u>	
	<u>single-car two-car</u>	<u>overturning non-overturning</u> <u>single-car accidents</u>
≤ 24 years	.92**	.60*
25 - 34 years	.75**	.72**
35+ years	.58*	.33

* significant at the 5% level

** significant at the 1% level

Table 6.11: Correlations between results in urban and rural areas for two variables and three age groups separately.

6.5.2 Results for female drivers

All results presented up till now have applied to male drivers only. The results for female drivers are very similar, and will not be presented in detail in order to save space. They are summarised in table 6.12. The following points should be noted:

- (i) The overall ratio of single- to two-car accidents is very similar in both urban and rural areas to the corresponding figure for male drivers.
- (ii) The overall proportion of overturning in rural areas is very similar to that for male drivers, but in urban areas is about 25% less.
- (iii) The effect of age on the ratio of single- to two-car accidents is qualitatively similar to that for male drivers but is somewhat less quantitatively.
- (iv) The effect of age on the proportion of overturning is similar both qualitatively and quantitatively to that for male drivers.
- (v) There is some evidence of a positive correlation between these car factors and those calculated from the male driver data. (Since the position of eleven of the points in each of these correlations exactly specify the position of the twelfth, it is not clear how the significance of such a correlation coefficient should be tested. Here, the correlation coefficient has been calculated in the usual way but, to be on the safe side, an extra degree of freedom has been subtracted when determining the level of significance, leaving ten in this case.)

Dependent variable:		<u>single-car</u> <u>two-car</u>		<u>overturning</u> <u>non-overturning</u> single-vehicle accidents	
Locale:		Urban	Rural	Urban	Rural
Base level, μ		-1.47	-.51	-2.34	-.85
Age factors:	≤ 24 years	.25	.41	.33	.03
	25 - 34 years	-.06	-.03	-.05	.05
	35+ years	-.19	-.38	-.28	-.08
χ^2 :	fit	15.6	20.6	17.2	28.5
	between models	64.6**	202.3**	6.0	39.7**
	between age groups	118.9**	348.0**	30.0**	13.1**
Correlation coefficient between car factors calculated from accidents to male drivers and from those to female drivers:		.41	.90**	-.18	.52 \nless

\nless significant at the 10% level

** significant at the 1% level

Table 6.12: Summary of results for accidents to female drivers.

6.5.3 Alternative statistical models for the relative numbers of single- and two-car accidents

Although the results of sections 6.3 and 6.4 are derived from one particular statistical model, others have also been considered and are given in table 6.13, together with the corresponding values of χ^2 measuring the goodness-of-fit. It can be seen that the model we have so far used is the best if we want to use the same one for both urban and rural data. In the case of all the statistical models in table 6.13, the differences between models of car and between age groups of driver were highly significant.

So far in this Chapter we have only been concerned with interpreting our results at their face value - whether or not models of car differ in the relative numbers of single- and two-car accidents they have, or in the proportion of overturning in single-vehicle accidents. We now make some comments on the wider significance of these results.

Jones (1973) has argued that the influence of car design and handling parameters on the accident rate of two-car collisions ($\frac{2N_{ij}}{M_{ij}}$ in the notation of section 6.2) is likely to be weak, i.e.

$2N_{ij} \approx M_{ij} d_i$, and that the number of two-car accidents involving a particular model can be used as a proxy for the mileage driven by that model. The ratio $\frac{1N_{ij}}{2N_{ij}}$ is then a measure of the safety of the j th model when driven by the i th age group of drivers, and when averaged over all age groups (which is effectively what we are doing when we use CATLIN) the result is an overall measure of safety of that model of car. This is the reason for choosing the first model in table 6.13 as being the one of most interest.

Statistical model	Left hand side of equation for CATLIN	χ^2	Urban	Rural
1. $\frac{{}_1N_{ij}}{{}_2N_{ij}} = K A_i B_j$	$\left(\begin{smallmatrix} 1^{-1} & 1^{-1} \\ 1^0 & 1^0 \end{smallmatrix} \right) \log_e(\underline{I}\underline{\pi})$	29.1	61.1**	
2. $\frac{{}_1N_{ij}}{{}_1N_{ij}+{}_2N_{ij}} = K A_i B_j$	$\left(\begin{smallmatrix} 1^0 & 1^0 \\ 1^0 & 1^0 \end{smallmatrix} \right) \underline{\pi}$	25.1	95.8**	
3. $\frac{{}_1N_{ij}}{{}_1N_{ij}+{}_2N_{ij}} = K + A_i + B_j$	$\left(\begin{smallmatrix} 1^0 & 1^0 \\ 1^0 & 1^0 \end{smallmatrix} \right) \underline{\pi}$	47.4*	61.2**	
4. $\frac{{}_2N_{ij}}{{}_1N_{ij}+{}_2N_{ij}} = K A_i B_j$	$\left(\begin{smallmatrix} 0^1 & 0^1 \\ 1^0 & 1^0 \end{smallmatrix} \right) \underline{\pi}$	54.4**	85.3**	

Table 6.13: comparison of the fit of certain other statistical models with the one whose results are given in tables 6.3 and 6.4. The χ^2 statistics have 22 degrees of freedom, and the levels of significance are indicated, * meaning $P < .01$, and **, meaning $P < .001$.

Consideration was also given to taking this a little further along the lines of the induced exposure model of Thorpe (1964) and Haight (1970, 1973). The basic idea of this is that (a) single-car accidents are caused entirely by attributes of the driver-vehicle combination concerned, (b) in each two-car collision there is a "responsible" and an "innocent" party, (c) the rate of occurrence of two-car accidents in which a particular driver-vehicle category is "responsible" is directly related to the rate of occurrence of single-car accidents to that category, and (d) the rate of occurrence of two-car accidents in which a particular driver-vehicle combination is "innocent" is directly related to the exposure of that category. In Haight's (1973) notation, $\eta_{ij} = \rho_{ij} - \sigma_{ij}$ where η_{ij} is the proportionate exposure of category (ij), ρ_{ij} is twice the ratio of the number of involvements of category (ij) in two-car collisions to the total number of vehicles involved in such accidents, and σ_{ij} is the proportionate involvement in single-car accidents. Thus the natural dependent variable to use is $\sigma_{ij}/(\rho_{ij} - \sigma_{ij})$ and it would be more convenient to regard the two types of accident as being "populations" and the driver age/car model categorisation as being the "responses" for the purposes of CATLIN; but we can easily retain the reverse scheme since

$$\frac{\sigma_{ij}}{\rho_{ij} - \sigma_{ij}} = \frac{({}_2N/{}_1N) {}_1N_{ij}}{{}_2N_{ij} - ({}_2N/{}_1N) {}_1N_{ij}}$$

where ${}_2N$ is the total number of two-car accidents (i.e. half the total number of vehicles involved), and ${}_1N$ is the total number of single-car accidents. For rural accidents table 6.2 indicates ${}_2N/{}_1N = 0.8$, so

$$\frac{0.8 {}_1N_{ij}}{{}_2N_{ij} - 0.8 {}_1N_{ij}} = f_i g_j$$

assuming, as before, that the effects of driver age and model of car combine multiplicatively, so that

$$\log_e \frac{1^{N_{ij}}}{2^{N_{ij}} - 0.8 \cdot 1^{N_{ij}}} = \theta + \phi_i + \gamma_j \quad (6.3)$$

This predicts that $2^{N_{ij}}$ is always greater than $0.8 \cdot 1^{N_{ij}}$, and therefore that $\frac{1^{N_{ij}}}{2^{N_{ij}}} < 1.24$. Expressed generally, this means that $\frac{1^{N_{ij}}}{2^{N_{ij}}}$ should always be less than the overall ratio of (number of single-car accidents) to (number of two-car accidents). This can also be seen by considering that the ratio of single- to two-car accidents is at its maximum when the category concerned is much more dangerous than average, in which case virtually all the two-vehicle accidents in which it is involved are its own fault. So it occupies among two-vehicle accidents the same proportion that it does among single-vehicle accidents. So

$$\frac{2^{N_{ij}}}{2^N} = \frac{1^{N_{ij}}}{1^N} \text{ and } \frac{1^{N_{ij}}}{2^{N_{ij}}} = \frac{1^N}{2^N}.$$

However, in table 6.2 there are four cases in which the ratio of single- to two-car accidents is greater than 1.24, and we consequently felt it not worthwhile trying to fit the model of equation (6.3).

Similarly, for the urban data $\frac{2^N}{1^N} = 2.12$, so $\frac{1^{N_{ij}}}{2^{N_{ij}}}$ should always be less than 0.47, whereas in one case it exceeds this.

6.5.4 Alternative statistical models for the proportion of overturning in single-car accidents

When analysing overturning in single-car accidents, it was felt appropriate to use the proportion of overturning as the dependent variable, rather than the ratio of overturning to non-overturning accidents. But other statistical models have also been considered, and the values of χ^2 for the fit of the models to the data are given in table 6.14.

We now note that there is some tendency for the same car models that have a high ratio of single- to two-car accidents also to have a high rate of overturning in single-vehicle accidents (see figure 6.5), thus suggesting that similar features of vehicle design are responsible for both these tendencies. One such feature might be some measure of the speed of the vehicle - available power, or maximum speed, or maximum acceleration. But this correlation may in part also be due to a statistical artefact: if x , y , and z are respectively the number of single-car overturning, single-car non-overturning, and two-car accidents in which a particular model of car is involved, we are saying that $x/(x + y)$ is correlated with $(x + y)/z$. Clearly, if x increases while both y and z remain constant, both these ratios increase and are therefore correlated. Up till now we have been supposing that there is a two-stage process occurring: either a single-car accident occurs or it does not, and if it does, then the car either overturns or it does not. Therefore y cannot remain constant while x increases: it must decrease so that $(x + y)$ remains constant. But to the extent that this two-stage view of single-car accidents is wrong and single-car overturning and single-car non-overturning accidents occur independently, changes in the number of the former will be reflected in both the ratio of single-

to two-car accidents and in the proportion of overturning in single-car accidents.

Statistical model*	χ^2 (22 d.f.)	
	Urban	Rural
1	35.5	160.4
2	30.1	135.8
3	67.9	170.4
4	73.5	175.4

* Table 6.13 specifies these models. N_{ij} in that table is here replaced by OT_{ij} , and ${}^2N_{ij}$ is replaced by $nonOT_{ij}$.

Table 6.14: Comparison of the fit of certain statistical models to the data on overturning in single-car accidents.

6.5.5 Statistical methods for correlating vehicle parameters with accident rate

Jones (1973, 1975) obtained measures of the safety of a number of models of car and performed multiple linear regressions of these on design and handling parameters of the models concerned, such as length, weight, height of centre of gravity, understeer, braking instability, etc., in an attempt to determine the optimum features of design for primary safety. Although multiple regression is a conventional technique for such situations it has the disadvantage that only the linear effect of a particular variable is taken into account. Furthermore, although program packages such as BMD (Dixon, 1971) produce F statistics and associated probability levels, significance testing of

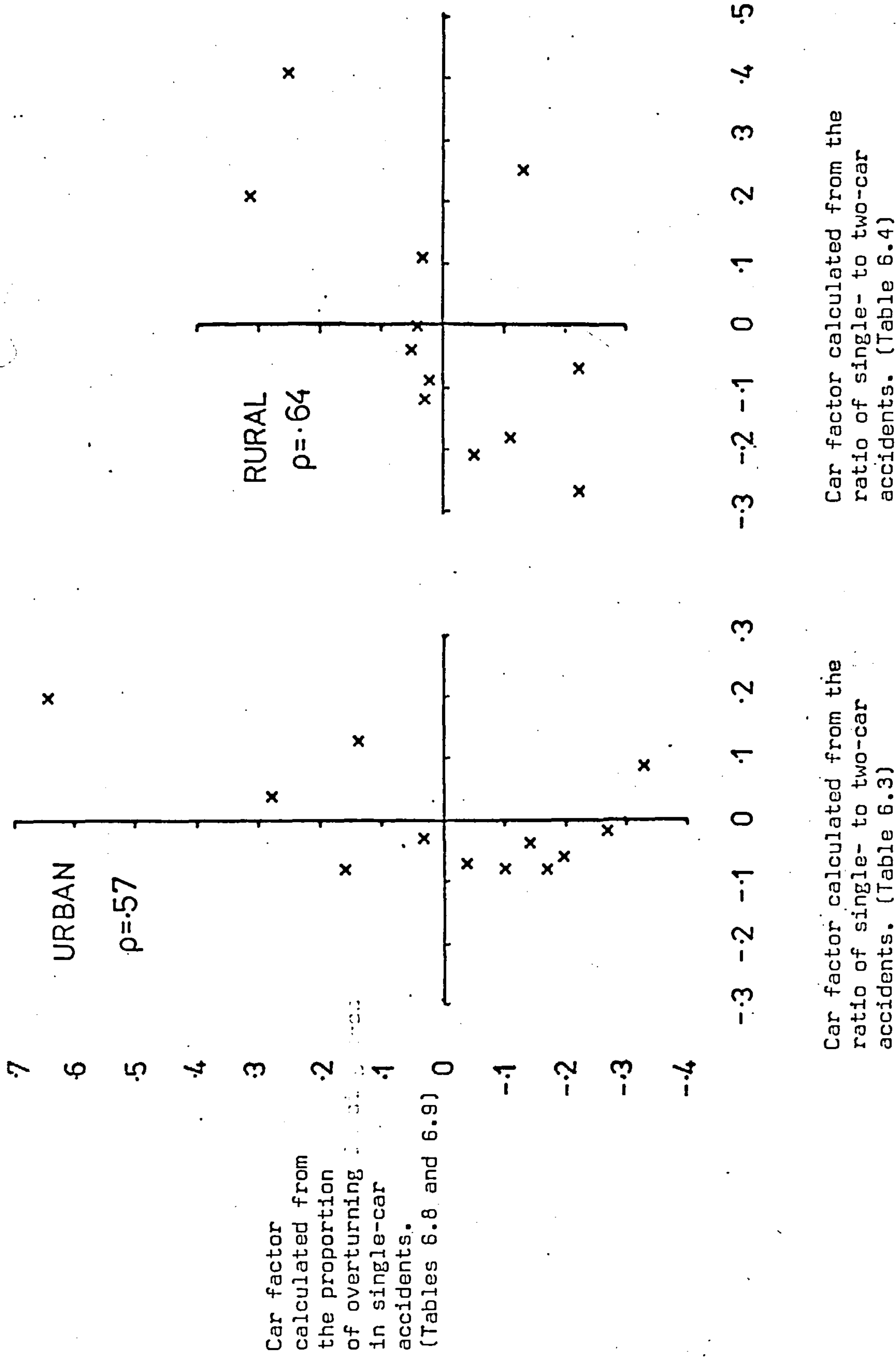


Figure 6.5: showing that the models of car which have a high ratio of single- to two-car accidents also tend to have a high proportion of overturning in single-car accidents.

the regression coefficient is difficult because the variables entered into the equation are those that are the most significant. Thus if there were 20 independent variables, it would be very likely that the most significant of them would be significant at the 5% level. Although some progress has been made towards overcoming this latter difficulty (Forsythe et al, 1973), it was thought worthwhile examining whether a nonparametric procedure could be devised as an alternative to multiple linear regression. The resulting test is presented in general terms in the Appendix (section 6.6) which follows, and in the second example some comments are made specifically with regard to accident rates and vehicle parameters. The point is made that it may be worthwhile carrying out a factor analysis of vehicle parameters, and then relating the accident rate (or relative proportions of different types of accident) to the resulting factors.

6.5.6 Summing-up

In this Chapter we have been concerned with discussing statistical models of the influence of age of driver and model of car on the relative numbers of different types of accident. These have enabled us to derive estimates of the effects of each of these factors that are uncontaminated by the effects of the other. We have shown that the statistical relationships we have proposed give quite a satisfactory explanation of the data (in the sense that χ^2 is quite small compared to the numbers of observations) but there is a statistically significant interaction (in the sense that χ^2 is statistically significant) in several cases. The overall ratio of single- to two-car accidents is about 2½ times greater in rural than in urban areas, and the effect of driver age group is about the same in these two conditions. The effect of model of car is considerably stronger in rural than in urban areas

and there is a strong correlation between the car factors for the two conditions. The overall proportion of overturning in single-car accidents is more than three times higher in rural areas than in urban areas, but the effects (defined in a multiplicative sense) of both driver age and car model are less for the rural accident data. Again, there is a strong correlation between the car effects in urban and in rural areas.

6.6 Appendix: Combining two-tailed rank correlation statistics

6.6.1 The paradigm

The situation considered in this section is as follows: J judges are each presented with N stimuli which are ordered along some dimension (such as large to small, or pleasant to unpleasant) and are asked to rank them. There is reason to believe that some judges will tend to rank the stimuli in one order (1, 2, 3, ... N) while others will order them oppositely (N , $N-1$, $N-2$, ... 1). How can we test whether the judges can detect the ordered nature of the stimuli?

6.6.2 The test

Clearly, for individual judges a two-tailed test of correlation between their judgements and the stimuli can be applied. But if four judges gave correlation coefficients of 0.7, 0.6, -0.7, and -0.8 when observing five stimuli, we might think that this constitutes an overabundance of high correlation coefficients, even though none of them are statistically significant on their own. Although tables have been prepared (Feild and Armenakis, 1974) which give the probability of obtaining m or more differences that are significant at the $P\%$ level ($P = 1$ or 5) in a group of M tests, the inflexible nature of such tables reduces their usefulness: for instance, what is the probability of getting, in a group of twenty tests, one result significant at .001, one at .05, and two at .10? One could certainly look up the case $m = 2$, $M = 20$, in the $P = 5$ table, or $m = 4$, $M = 20$, in the $P = 10$ table, but these would lose the information that one of the results was very much more significant than $P = 5$. Accordingly, the test to be proposed will be more suitable in the particular situation of combining rank

correlation tests.

The test to be proposed is based on the well known rank correlation coefficient of Spearman (1904): if d_{kl} is the difference between the ranks of the k th stimulus and the corresponding judgement of the l th judge, calculate

$$S_l = \sum_k d_{kl}^2 .$$

(S_l is the statistic used to test for correlation between the stimuli and the responses of the l th judge; its distribution for small N is given by Owen (1962).) Under the null hypothesis, S is distributed symmetrically between zero and $\frac{1}{3}(N^3 - N)$, with mean $\frac{1}{6}(N^3 - N)$ and variance $N^2(N+1)^2(N-1)/36$. Lyster (1952) and Page (1963) have shown how Spearman's procedure can be extended to situations where several sets of ranks are each correlated with a criterion ranking, and when each set of ranks is correlated with each other set Kendall's "coefficient of concordance" is applicable (Kendall and Babington Smith, 1939).

We seek a test statistic that is some combination of the S_l 's and is unchanged when any S_l is changed from $E\{S\} - \Delta$ to $E\{S\} + \Delta$, E denoting expectation. An example of such a statistic is

$$Q = \sum_l R_l \quad \text{where} \quad R_l = \min \left\{ S_l, \frac{N^3 - N}{3} - S_l \right\} .$$

In transforming S_l to R_l we are effectively folding the distribution of S about its mean value, so that two values of S equidistant from the mean but on opposite sides of it have the same value of R . Since S has an approximately Normal distribution and the ratio of the mean

absolute deviation to the standard deviation has the value $\sqrt{(2/\pi)}$ for a Normal distribution, it may be shown that the expectation and variance of R are approximately

$$N(N+1)\{N-1-(\frac{2}{\pi}N-\frac{2}{\pi})^{\frac{1}{2}}\}/6$$

and $N^2(N+1)^2(N-1)(\pi-2)/(36\pi)$ respectively.

Q will be small if there are an exceptional number of values of S which are far from the expected value of S , whether they are low, high, or a mixture of low and high. It can now be seen that our problem is to find the distribution of the sum of J independent samples from the distribution of Spearman's S truncated at its mid-point.

6.6.3 Distribution of Q

The distribution of Q under the null hypothesis of random rankings has been enumerated for small values of J and N , and critical values (at the .001, .01, .05, and .10 levels) are given in table 6.15. Since the distribution of Q is discrete, the actual significance levels of the critical values in table 6.15 are less (i.e. more significant) than their nominal levels. It may be noted that

(i) As J becomes large, the distribution of Q tends to Normality, with mean

$$N(N+1)\{N-1-(\frac{2N}{\pi}-\frac{2}{\pi})^{\frac{1}{2}}\}J/6$$

and variance $N^2(N+1)^2(N-1)(\pi-2)J/(36\pi)$.

(ii) As N becomes large, the distribution of S tends to Normality,

Number of judges, J	Number of stimuli, N				
	3	4	5	6	7
2	-	-	0	6	14
	-	0	4	12	26
	-	2	8	20	40
	-	2	10	26	48
3	-	0	4	16	36
	-	2	10	28	54
	0	4	16	38	70
	0	6	20	44	80
4	-	2	12	30	62
	-	4	18	44	82
	0	8	26	58	102
	0	10	30	64	112
5	-	4	18	46	
	0	8	28	60	
	2	12	36	74	
	2	14	40	82	
6	-	8	26		
	0	12	36		
	2	16	46		
	2	18	50		
7	0	10			
	2	16			
	4	20			
	4	24			

Table 6.15: Critical values of the Q statistic. This table gives m such that $\text{Prob}\{Q \leq m\} \leq .001, .01, .05, .10$, for the four rows corresponding to each combination of J and N respectively. A dash means that level of significance cannot be attained for that combination of J and N. Where no entries are given, use the Normal approximation.

so the problem becomes finding the distribution of the sum of J independent samples from a Normal distribution truncated at its mean. Where J is small, this may not be close to Normal. Surprisingly, there does not appear to be a tabulation of this. Francis (1946) came nearest to doing this, but he was interested in the upper tail whereas we are interested in the lower. Fortunately, as will be seen below, even for $J = 2$, $N = 7$, the distribution of Q is quite close to Normal in the critical region.

(iii) Since for $N = 3$ the distribution of R is $(0, 2, 2)$, the exact distribution of Q in this case may be obtained from the binomial expansion of $(\frac{1}{3} + \frac{2}{3})^J$.

The adequacy of the Normal approximation to the distribution of Q has been investigated for the cases $N = 7, J = 2$; $N = 7, J = 4$; $N = 6, J = 5$; $N = 5, J = 6$; $N = 4, J = 7$; $N = 3, J = 9$; and the results given in table 6.16. It can be seen that the approximation is very satisfactory for the 5% and 1% significance levels, so when J and N fall outside the range to which table 6.15 applies, the approximation can be used with confidence.

<u>Case</u>	<u>Nominal significance level</u>			
	<u>.001</u>	<u>.01</u>	<u>.05</u>	<u>.10</u>
N=7, J=2	.0007	.014	.056	.113
N=7, J=4	.0011	.012	.054	.104
N=6, J=5	.0009	.010	.051	.105
N=5, J=6	.0007	.008	.048	.110
N=4, J=7	.0002	.009	.059	.097
N=3, J=9	.0000	.001	.042	.145

Table 6.16: Comparison of nominal and exact significance levels for the Normal approximation. In the body of this table is the exact significance level corresponding to the Normal approximation which has the given nominal significance level.

6.6.4 Example 1

The following example is fictitious: six judges each tasted four brands of tonic water, which differed chiefly in the amount of quinine they contained. The judges ordered the brands according to their preferences, with the results shown in table 6.17. Does the strength of quinine affect their preferences?

Table 6.17 also gives S_1 and R_1 for each judge. For three judges individually (2, 3, and 5) Spearman's S is significant at the 10% level (two-tailed) but a higher level of significance cannot be attained with only four stimuli. If we take into account the preferences of all judges, however, and apply the test proposed above, we find $Q = 12$, which is significant at the 1% level. We conclude that strength of quinine does affect our judges' preferences in a way that is monotonic within judges in the range tested; and from table 6.17 it is clear that

judges 1, 3, 4, and 5 prefer a low level of quinine whereas judges 2, and 6 like a high level.

To use the Normal approximation, we would calculate the expectation of $Q = 4 \times 5 \times 1.618 \times 6 / 6 = 32.36$ and its standard deviation $= \{16 \times 25 \times 3 \times 1.142 \times 6 / (36\pi)\}^{\frac{1}{2}} = 8.53$. Therefore a value of 12 lies $20.36/8.53 = 2.39$ standard deviations below the mean, which corresponds to a one-tailed probability of .008.

Judge	Brand of tonic water (in order of quinine content)				S ₁	R ₁
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>		
1	2	1	3	4	2	2
2	4	3	2	1	20	0
3	1	2	3	4	0	0
4	1	4	2	3	6	6
5	1	2	3	4	0	0
6	3	4	1	2	16	4
						<u>Q = 12</u>

Table 6.17: Data for Example 1.

6.6.5 Example 2

Although the above exposition of the test has been couched in psychological language, naturally applications are not restricted to psychology. Moreover, it is likely that the asymptotic version of the test will be satisfactory even in situations where there is missing data in some of the response rankings.

Although the following example is based on the results of Jones (1975), it is only an extract from his results, and should not be taken as minimising the role of the vehicle in accident causation. Eight models of car are ordered in terms of their accident rate. They are also ordered in terms of certain of their design and handling parameters, such as weight, ratio of height of centre of gravity to track, understeer, etc, with the results given in table 6.18. Is there evidence that these parameters affect the accident rate?

The test in this case enables account to be taken of the fact that in four correlations the chance of finding one that is significant at the 5% level is much greater than 5%. When a parameter is not known for some models, R_1 is calculated after reranking the accident rates to omit the models for which the parameter is unknown. Also the expectation and variance of R_1 will be decreased. We proceed in the same way as usual, adding up the R_1 to get Q . On the null hypothesis, the expectation of Q is 177.7 with standard deviation 31.8. The observed value of Q is very close to the mean, so we do not reject the null hypothesis.

This example has illustrated how the proposed test may be used as an alternative to multiple correlation in some situations, though it should be pointed out that whereas the latter provides a significance level for a combination of variables, our procedure in a sense combines the probability levels from a number of separate tests. Moreover, as was mentioned at the end of section 6.5.5, when stepwise multiple linear regression is used to select a subset of variables, the significance level of the statistics usually calculated is distorted.

An important limitation of the proposed test is that since the null hypothesis states that the response rankings are random and independent of each other, it would be inappropriate to include, for instance, weight and length as two of the car parameters in this example, because they are highly associated. A possible procedure to adopt when many of the independent variables are associated with one another to some degree (as those of Jones (1975) are) would be to perform a preliminary factor analysis on the independent variables - that is, obtain a series of orthogonal vehicle parameters, one of which would perhaps be a measure of size (being derived from weight, length, width, etc.), another would perhaps be a measure of steering performance, another a measure of speed, and so on - and then carry out the test proposed here with the scores of the various models on the new factors being the independent variables to be correlated with the accident rate.

Parameter	Model of car (in order of accident rate)								<u>S₁</u>	<u>R₁</u>	Expectation and variance of R ₁	
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>				
Weight	2	5	8	6	4	3	1	7	86	82	58.7	366.3
Ht. of c.g. ÷ track	2	5	3	1	4	6	7	8	20	20	58.7	366.3
Understeer	3	5	4	1	6	2	-	7	40	40	37.8	189.9
Braking instability	5	-	2	4	1	6	-	3	36	34	22.5	89.0
										Q = 176	177.7	1011.5

Table 6.18: Data for Example 2. (A dash denotes missing data.)

CHAPTER 7: STATISTICAL ASPECTS OF INJURY SEVERITY

7.1 Introduction

This Chapter is concerned with the severity of injury sustained in road accidents. "Severity" is not a term which can easily be unambiguously and quantitatively defined, but it will be shown that it is only necessary for it to be measured on an ordinal scale for interesting statistical relationships to be derived. Neither the problem of how to judge whether one injury is more severe than another, nor the economic valuation of injury, will be discussed, but instead a class of functions which describe how the proportions of people severely injured in two circumstances co-vary as the definition of what constitutes a "severe" injury is made more strict or more lax will be introduced. This will permit predictions to be made about how the percentage of people seriously injured is related to the percentage killed, as some variable, such as age of injured person, which is related to the average severity of injury, is changed. The approach described in this Chapter should be applicable to injuries to each part of the body separately, but only the overall degree of injury will be considered because national accident statistics do not give the nature of the injury.

Although the chief benefit from looking at injury severity in this way is the theoretical insight gained, several examples are given in which this model can be directly applied. The approach described in this Chapter will also be of fundamental importance in the analysis of injury correlations in Chapter 8.

7.2 Theoretical framework

A great number of variables are known to be related to the severity of injury sustained in a road accident. As examples, we can point to the speed of the crash, the design of the vehicle interior, the age of the injured person, and the mass ratio of the vehicles involved. Some of these are discussed in other Chapters. The injury sustained is usually classified trichotomously - fatal, serious, and slight. Intuitively, it is reasonable that those variables which tend to increase the proportion of fatal injuries will also tend to increase the proportion of serious injuries, and it is with the quantification of this relationship with which this Chapter will be concerned.

Assume for the moment that an exact definition of injury severity is available. It will later become apparent that although it is necessary for the purposes of this Chapter that an exact quantitative definition of injury severity measured on a continuous scale should exist, it is not necessary that this definition should be known to the data analyst. In any particular type of accident, there will be a distribution of injury severity, perhaps as in curve A of figure 7.1. In a different type of accident, there will be a different distribution of injury severity, perhaps as in curve B. Assume we divide injuries into two classes, severe and minor. The threshold for this will be some point on the severity axis, marked X in figure 7.1. In the first type of accident, the proportion of cases which are severely injured is the area under curve A to the right of point X (0.100 in this example) and in the second type it is the area under curve B to the right of X (0.218 in this example). Suppose we now change our definition of a "severe" injury, so that the threshold moves to point Y. The proportion of cases considered to be severe will change, as it

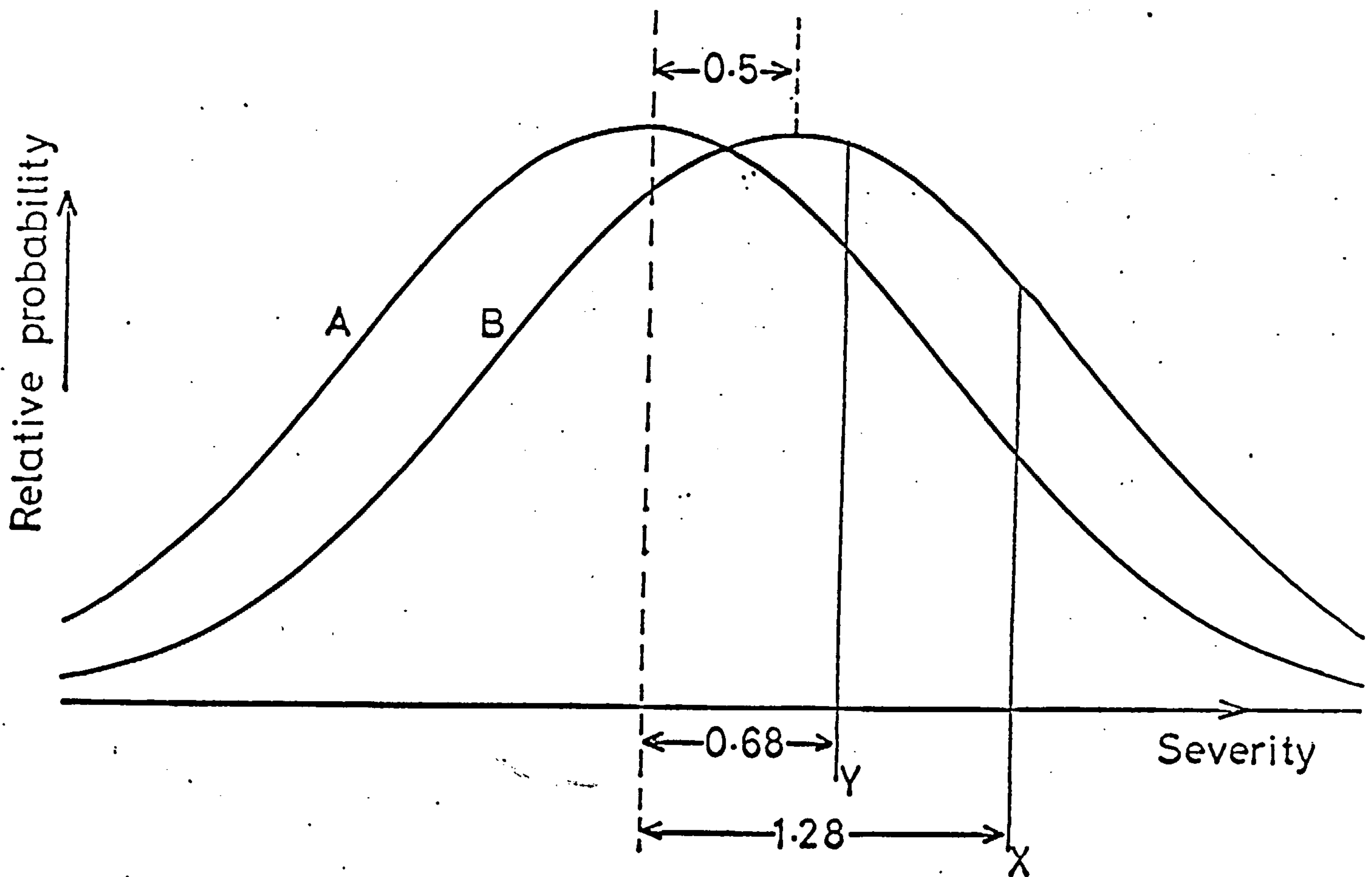


Figure 7.1: Representing the distribution of severity of injury sustained under two conditions, A and B. The distances marked are in units of the standard deviation.

is now the area under the appropriate curve to the right of point Y. 259
It is 0.248 for curve A and 0.429 for curve B in this example. Let us construct a graph - figure 7.2 - on the horizontal axis of which is the proportion severely injured in the first type of accident and on the vertical axis of which is the proportion severely injured in the second type of accident; then corresponding to X being the threshold will be a point at (0.100, 0.218) in figure 7.2, and corresponding to Y will be a point at (0.248, 0.429). If the threshold were between X and Y in figure 7.1, the corresponding point would be between X and Y in figure 7.2; if it were to the right of X in figure 7.1, the corresponding point in figure 7.2 would be to the left and below X; and a definition of injury more lax than Y would be to the right and above Y in figure 7.2. In this way we can draw out a curve in figure 7.2 that corresponds to moving the definition of a "severe" injury from right to left in figure 7.1. It will be convenient to refer to a curve of this type as a Relative Injury Frequency (RIF) curve. The shape of it is defined by the relative shapes of the curves A and B in figure 7.1. If we were prepared to make some assumption about the relative shapes of these curves, we could deduce the shape of the RIF curve. For the example of figure 7.2 it has been assumed that A and B are two Normal distributions having the same variance and separated by half their standard deviation.

It should be noted that the same RIF curve is obtained if the horizontal axis in figure 7.1, severity of injury, is distorted - stretched or compressed - in any monotonic way. This is because the ratio of the heights of the two curves B and A - which equals the slope of the RIF curve - remains the same.

If we had data on the severities of injury in two conditions,

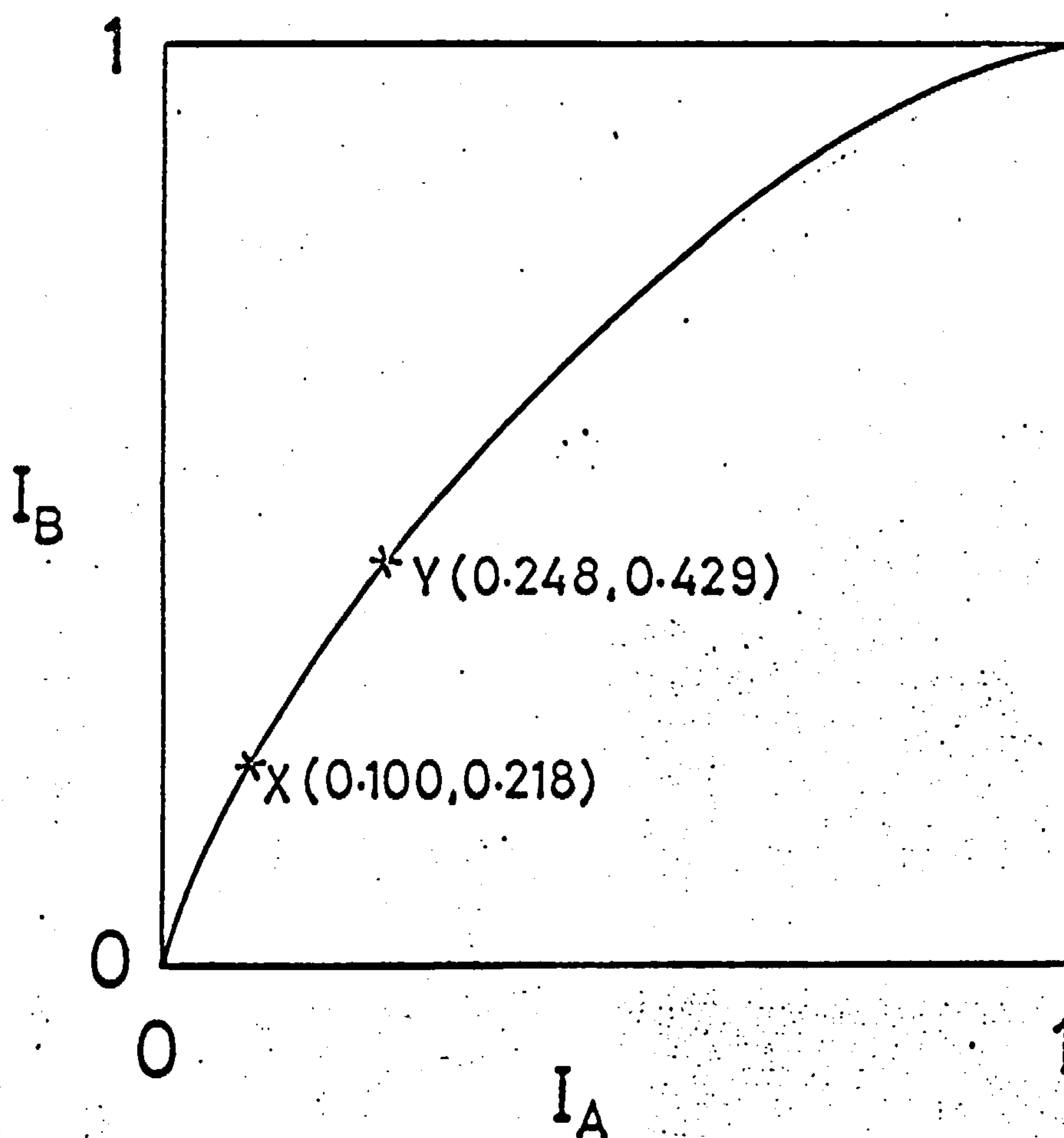


Figure 7.2: Relation between the proportion of cases severely injured in conditions A and B, as the definition of 'severe' is altered.
(An example of a Relative Injury Frequency curve.)

with, for example, five categories of severity, we could plot this graph empirically. That is, if in condition A there were proportions $p_1, p_2, p_3, p_4,$ and p_5 with minor, moderate, serious, critical, and fatal injuries respectively, and for condition B the corresponding proportions were $q_1, q_2, q_3, q_4,$ and q_5 , then with the notation

$$P_i = \sum_{j=i}^5 p_j \quad \text{and} \quad Q_i = \sum_{j=i}^5 q_j, \quad i = 1 \text{ to } 5,$$

we would plot the points $(P_5, Q_5), (P_4, Q_4), (P_3, Q_3), (P_2, Q_2),$ and (P_1, Q_1) , the last of which equals $(1, 1)$.

It is worth recapping the above with some mathematical notation. Let s be the severity of injury, and $f_A(s)$ and $f_B(s)$ be the two functions A and B in figure 7.1. Let

$$I_A(s) = \int_s^{\infty} f_A(t) dt \quad \text{and} \quad I_B(s) = \int_s^{\infty} f_B(t) dt.$$

Then the RIF curve consists of plotting I_B against I_A as s changes.

The slope of the RIF curve is the ratio of the heights of curves A and B because

$$\frac{dI_B}{dI_A} = \frac{dI_B}{ds} \div \frac{dI_A}{ds} = \frac{f_B(s)}{f_A(s)}.$$

Thus if we found empirically that the RIF curve had the shape corresponding to f_A and f_B being two Normal distributions, we would still not be able to conclude that injury severity was in each case really Normally distributed (except in the sense that severity could be defined so that it was Normally distributed in both cases) - all we could say is that injury severities in the two accident types were related in the same way as are two Normal distributions with different

means. To illustrate this, consider figure 7.3. In part A of this figure are two Normal curves separated by 0.3 of their standard deviation, which give rise to the RIF curve of part B of this figure. But this same RIF curve is also derived from the pair of distributions in part C, one of which is exponential, $y_1(s) = \exp(-s)$, the other of which ($y_2(s)$) was derived from the first in the following way: put

$$I_1(s) = \int_s^{\infty} y_1(t) dt = \exp(-s)$$

so that I_1 is the horizontal axis in part B of the figure. Define $z(s)$ by

$$\int_z^{\infty} N(0, 1) = I_1(s)$$

where $N(0, 1)$ is the standard Normal distribution. Let $v_1(s)$ be the Normal ordinate at z , and $v_2(s)$ be the Normal ordinate at $z - 0.3$.

Then since

$$\frac{v_2(s)}{v_1(s)} = \frac{y_2(s)}{y_1(s)}, \quad y_2(s) \text{ is given by } \frac{v_2 y_1}{v_1}.$$

Similarly in part A of figure 7.4 are two exponential distributions, $\exp(-s)$ and $0.7 \exp(-0.7s)$. They give rise to the RIF curve of part B, which is given by the equation $I_2 = I_1^{0.7}$ (see below for the proof of this relation). The same RIF curve is also given by the pair of distributions in part C, one of which ($y_1(s)$) is Normal, the other of which ($y_2(s)$) is obtained as follows: the ratio $\frac{y_2}{y_1}$ is the slope of the RIF curve at a point corresponding to s . Since $\frac{dI_2}{dI_1} = 0.7 I_1^{-0.3}$, and

$$I_1(s) = \int_s^{\infty} f_A(t) dt,$$

we can obtain I_1 from y_1 by using tables of the Normal integral, and then obtain $y_2 = 0.7 I_1^{-0.3} y_1$.

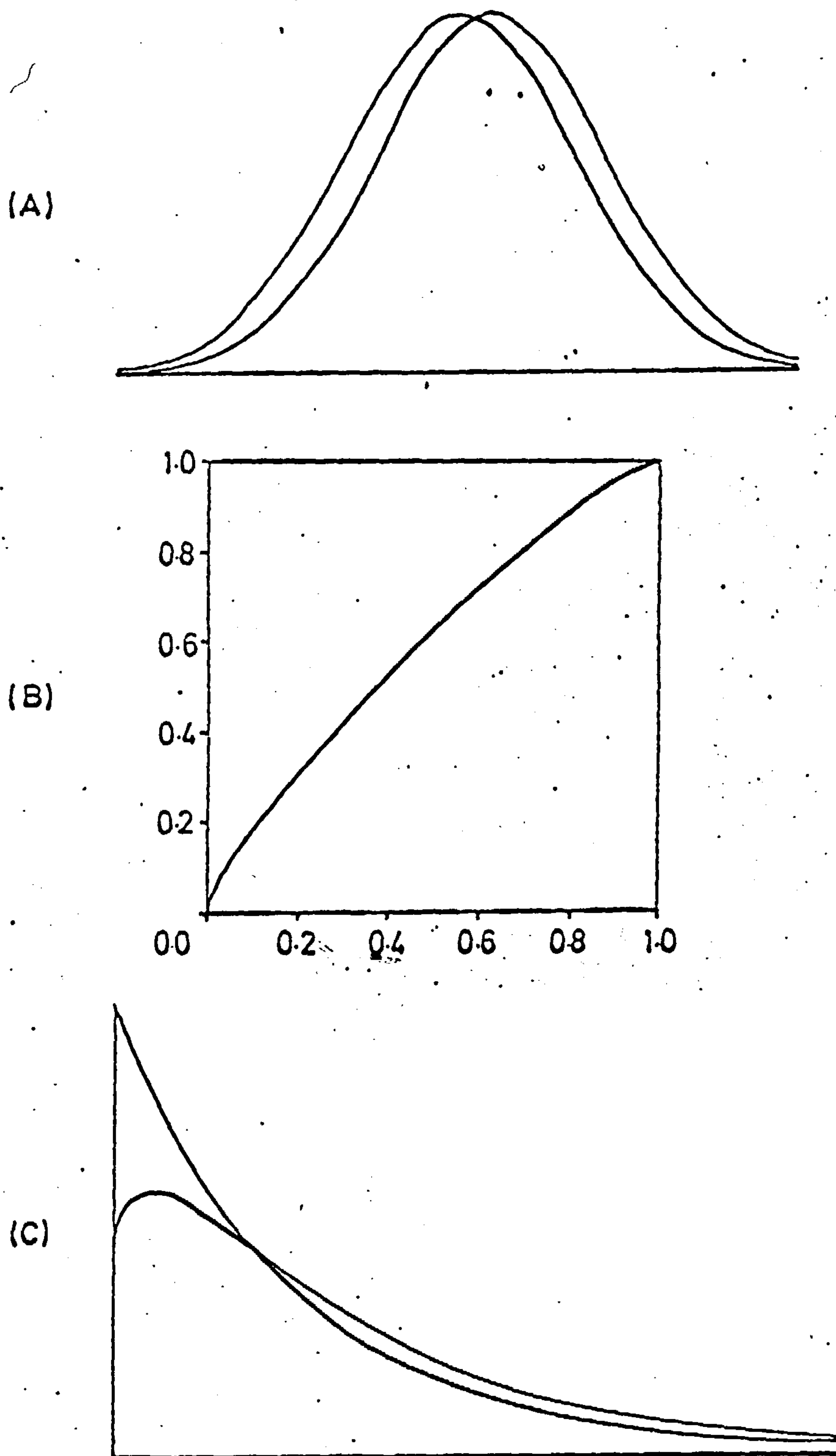


Figure 7.3: the RIF curve in part (B) is implied both by the pair of Normal distributions in part (A) and by the pair of distributions in part (C).

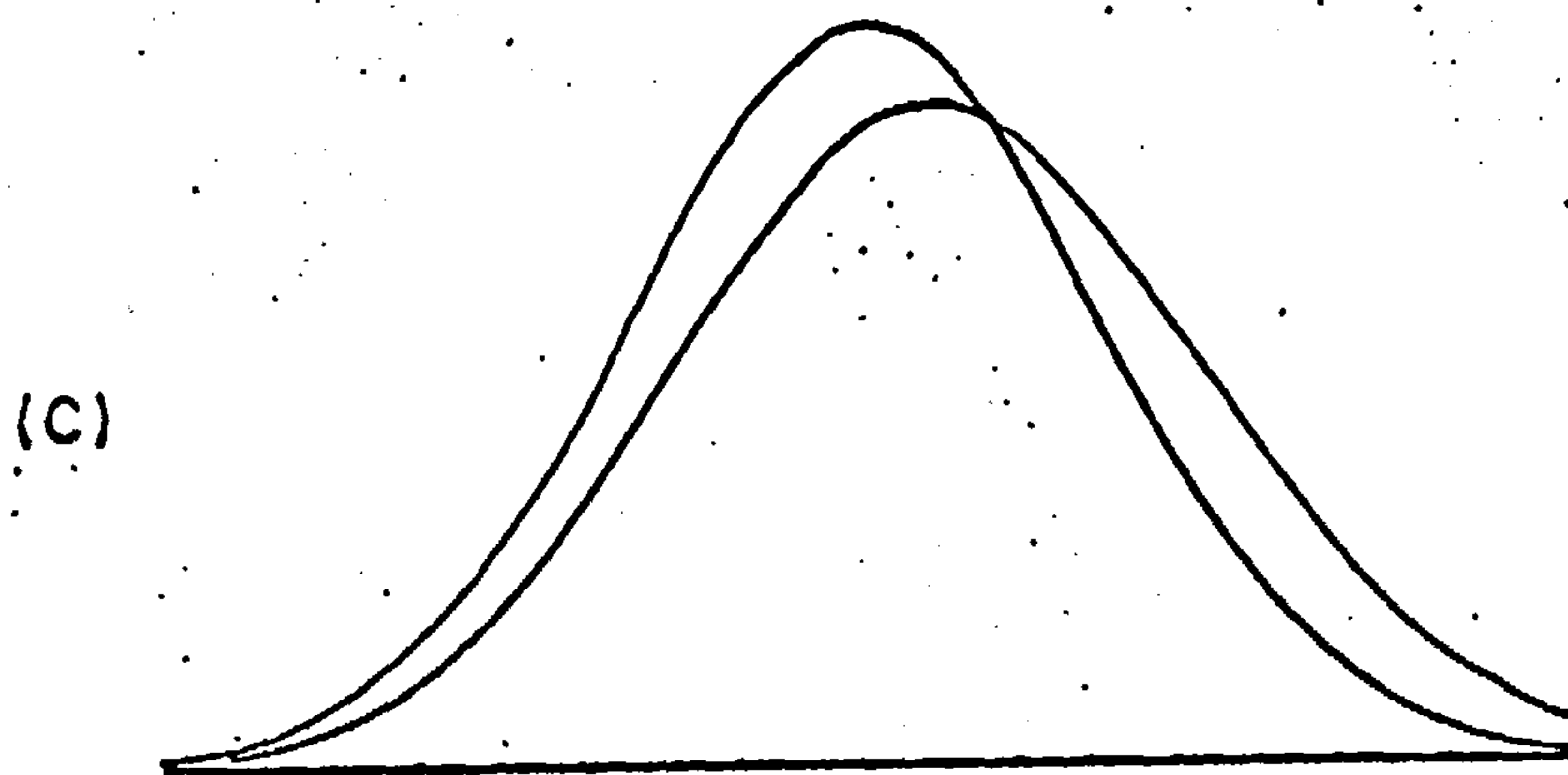
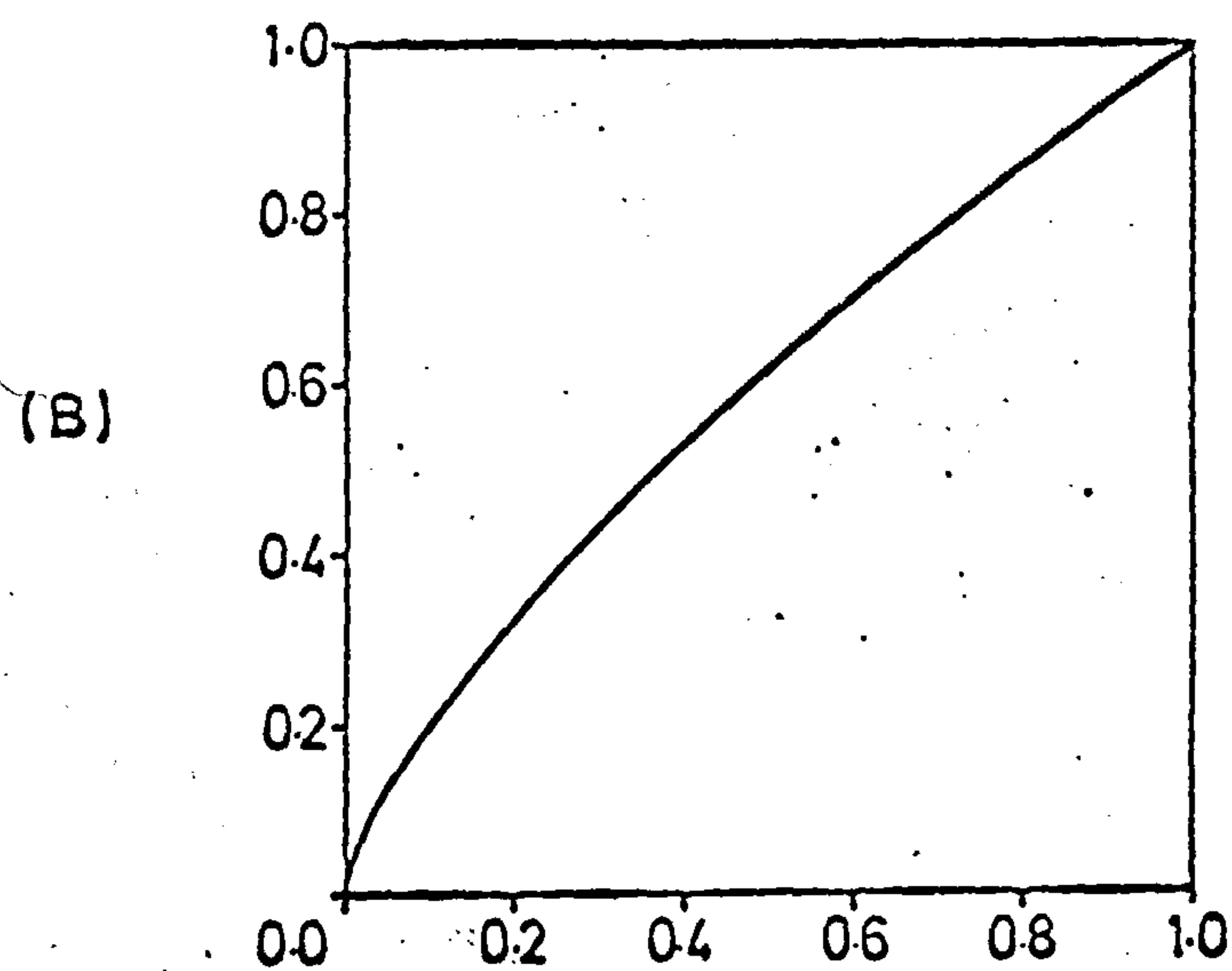
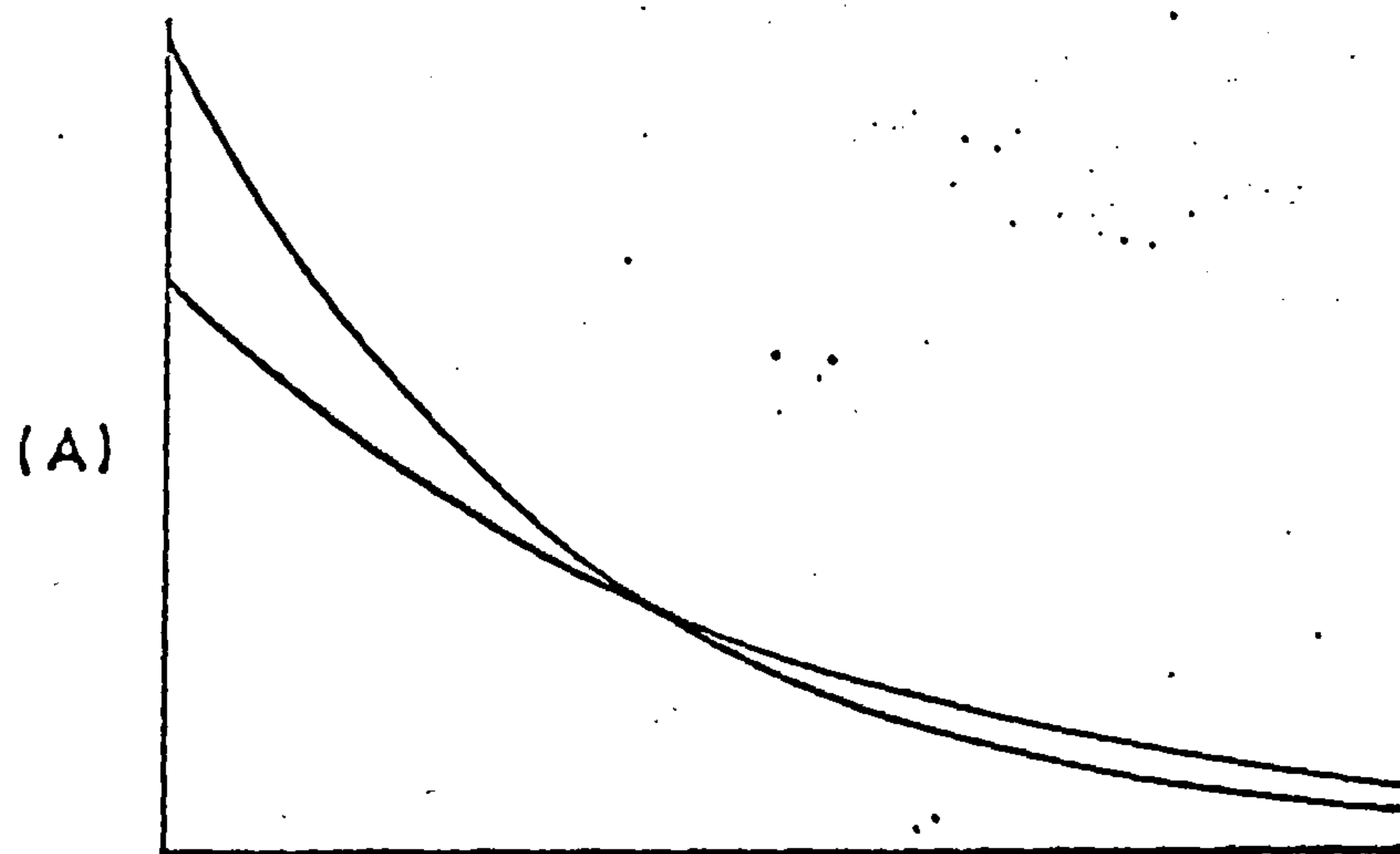


Figure 7.4: the RIF curve in part (B) is implied both by the pair of exponential distributions in part (A) and by the pair of distributions in part (C).

To determine whether the RIF curve does correspond to the situation where f_A and f_B are two Normal distributions, we would plot $z(P_i)$ against $z(Q_i)$, where $z(x)$ is the standardised Normal deviate such that 100x% of cases lie outside the range $(-\infty, z)$ for a Normal distribution of zero mean and unit standard deviation. If this plot is a straight line, we can conclude that the RIF curve does have the postulated shape; furthermore, if it has slope unity, we can conclude that the two Normal distributions have the same variance. We can express this more mathematically as follows: if f_A is Normal with mean μ_A and variance σ_A^2 and f_B is Normal with mean μ_B and variance σ_B^2 , then

$$I_A(s) = \int_s^{\infty} f_A(t) dt = \int_{\frac{s-\mu_A}{\sigma_A}}^{\infty} N(0, 1) .$$

But the z calculated from I_A is defined by

$$I_A = \int_{z_A}^{\infty} N(0, 1) .$$

Therefore,

$$z_A = \frac{s - \mu_A}{\sigma_A} .$$

Similarly,

$$z_B = \frac{s - \mu_B}{\sigma_B} .$$

Eliminating s we obtain

$$z_B = \frac{\sigma_A}{\sigma_B} \cdot z_A + \frac{\mu_A - \mu_B}{\sigma_B}$$

Thus the slope of the line of z_B plotted against z_A is the ratio of the standard deviations of f_A and f_B , and the intercept is the difference between the means, expressed in units of the standard deviation of f_B .

Unfortunately, in those investigations in which injury severity has been classified into five or six categories, the numbers of cases involved have usually been small. So an example in which there are only three categories of injury will have to serve. An unpublished study by Grime and Hutchinson based on British national accident figures shows that in a sample of head-on collisions between two vehicles of equal mass, 3.0% of drivers were killed and 33.8% were seriously injured, the remainder being slightly injured; whereas of drivers of vehicles colliding head-on with vehicles of mass at least ten times the mass of their own vehicle, 11.5% were killed and 38.0% seriously injured. Calling these populations A and B respectively, we have

$$p_3 = .030, p_2 = .338, p_1 = .030, p_0 = .368;$$

$$q_3 = .115, q_2 = .380, q_1 = .115, q_0 = .495.$$

Corresponding z values are 1.89 and 0.34 for population A and 1.20 and 0.01 for population B. If the points (1.89, 1.20) and (0.34, 0.01) are plotted, the straight line formed has slope $\frac{1.19}{1.55} = 0.77$ and intercept -0.25. Thus if injury severity is defined so that its distribution for population B is Normal with zero mean and unit variance, the distribution for population A is approximately Normally distributed with mean -0.25 and standard deviation 0.77.

What can we say about the shape of the RIF curve? (i) Clearly, it must go through (0, 0) and (1, 1). (ii) It must be monotonically increasing. (iii) If the change from one condition to another produces a shift in the location of the distribution of severity without changing its shape, then the curve cannot cross the $I_A = I_B$ line. (If it did, it would mean that one population contained both a greater proportion of very severe injuries and a greater proportion of very minor injuries

than the other population.) (iv) In many cases we would expect the slope of the RIF curve to be monotonically changing also.

These four conditions are all satisfied if f_A and f_B are two Normal distributions of equal variance. But conditions (iii) and (iv) are not true if f_A and f_B are Normal distributions of unequal variance. This is a point against that model, but is not decisive since we are not usually very interested in what happens at the extreme minor-severity end of the scale, and in any case f_A and f_B might be related in approximately this fashion at the severe injury end of the scale, but not over the whole range of severity.

But the simplest function which fulfills these four conditions is the power function, $I_B = I_A^n$. This has an interesting implication for the relationship between f_A and f_B : they are related in the same way that two exponential distributions are. Proof: if $f_A = \lambda \exp(-\lambda s)$ and $f_B = \mu \exp(-\mu s)$, $I_A = \exp(-\lambda s)$ and $I_B = \exp(-\mu s)$, so $I_B = I_A^{\mu/\lambda}$. To determine whether the RIF curve does have this shape, we would plot $\log(P_i)$ against $\log(Q_i)$, and if this is a straight line the slope would give us the exponent n .

So for our previous example, for which $P_3 = .030$, $P_2 = .368$, $Q_3 = .115$, and $Q_2 = .495$, we take the logs of these numbers which are respectively -1.53 , $-.43$, $-.94$, and $-.31$. The points $(-1.54, -.94)$, $(-.43, -.31)$, together with $(0, 0)$ are shown in figure 7.5 and are seen to lie approximately on a straight line of slope $-.65$. Thus the two populations of severities behave approximately in the same way as two exponential distributions whose parameters are in the ratio 3:2.

An important point about this exponential model of injury severity

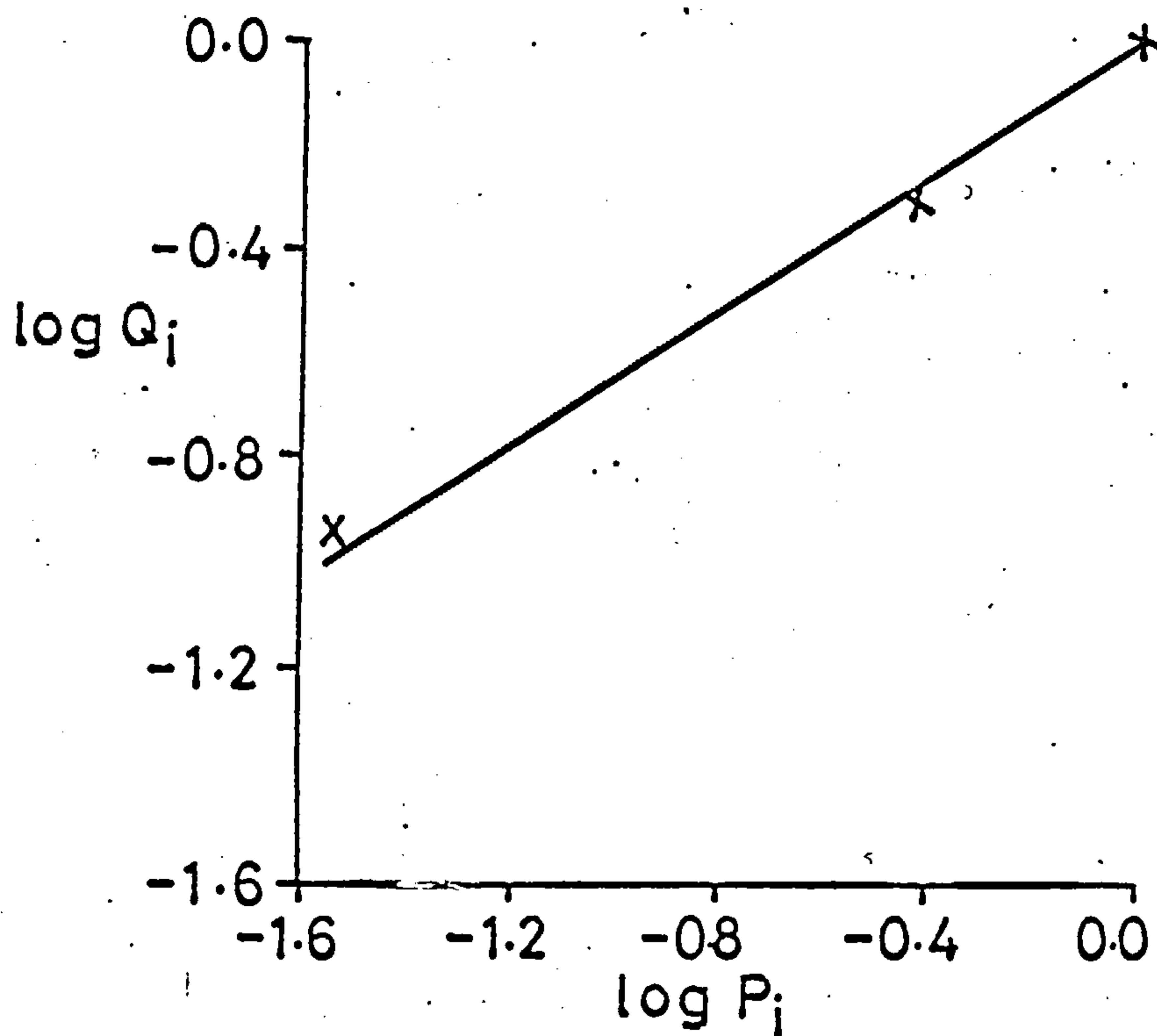


Figure 7.5: An RIF curve(transformed to logarithmic scales) comparing injury to drivers colliding head-on with vehicles of equal mass to their own and to drivers colliding head-on with vehicles of mass at least ten times the mass of their own vehicle.

is that the RIF curve is unaffected if injuries less than a certain threshold are not reported: if $f_A = \lambda \exp(-\lambda s)$ and $f_B = \mu \exp(-\mu s)$; and for $s < t$ injuries are not reported whereas for $s > t$ they are fully reported, for $s > t$ it follows that $I_A = \exp(-\lambda s)/\exp(-\lambda t) = \exp(-\lambda(s-t))$ and $I_B = \exp(-\mu s)/\exp(-\mu t) = \exp(-\mu(s-t)) = I_A^{\mu/\lambda}$. This may also be seen by considering the graph of $\log(P_i)$ against $\log(Q_i)$: this is linear and goes through (0, 0) if the model is correct. If a threshold is introduced, clearly each P_i is multiplied by some constant, and each Q_i is multiplied by some other constant. So each point on the $\log(P_i)$ versus $\log(Q_i)$ graph is moved by a constant vector. So the slope of the line must remain the same. And since the line must still go through (0, 0), it must in fact be the same line.

But with only three points we cannot really tell whether this model is correct or not. And we have already remarked that in those studies where injury severity has been classified in more detail there are usually too few cases in the more severe groups to permit accurate evaluation of the probabilities of them occurring. Following a section briefly outlining some applications of the model described above, an alternative approach to determining the shape of the RIF curve will be developed and applied to British data in which injury is classified as fatal, serious, or slight. In doing this, we will fulfill our promise made earlier to quantify the correlation between the percentage of casualties who are killed and the percentage who are seriously injured.

7.3 Applications

The chief raison d'etre of this Chapter is the insight it gives into what is meant by injury severity, but we now give some particular examples of the usefulness of this approach.

7.3.1 Example 1: Description

When we wish to describe the effectiveness of some device which reduces injury, the numerical value we get will depend strongly on the comparison we make. For instance, fatalities might be reduced from 4% to 2%, thus being halved; or the percentage of fatal and serious injuries together might be reduced from 30% to 20%, a reduction of one third; or we might give a score to each injury category and find a reduction of 1.5 units (which will depend on the categories used and the scores given). The theory proposed in this Chapter enables an objective assessment of changes in injury severity to be made, provided we are prepared to assume some particular functional form for the RIF curve: if it is exponential, a parameter of $\log(.02)/\log(.04) = 1.22$ is implied in our first case, and one of $\log(.20)/\log(.30) = 1.34$ in the second one. If we have injuries classified into several groups in the two conditions, we simply construct an empirical RIF curve and estimate the parameter of the type of curve we select: the definitions of the categories of injury are irrelevant, since if different definitions were used merely the position of the empirical points along the curve would be altered, not the shape of the curve.

7.3.2 Prediction of the effectiveness of an injury reduction measure in one country from a knowledge of its effect elsewhere

Some new device might be invented and introduced in one country, where it is found to reduce the proportion of severe (serious + fatal) injuries to those using it from the normal level of (say) 40% to a level of 25%. What effect would it be expected to have in another country where, because of different conditions or a different definition of a "serious" injury (again including fatalities), the normal level of such injuries is only 20%? If it has been found that the device reduces injuries in the way described by our exponential model, or we are prepared to assume this, then the expected level of serious injuries to users of the device in the second country is $.2^n$, where n is given by $.4^n = .25$. Therefore $n = 1.51$, $.2^n = .09$, and we expect the device to lower the proportion of serious injuries to 9%.

Two other approaches might have been used: (a) to say that in the first country the device reduced injury levels by 15%, so we estimate its likely effect in the second country to be a reduction of severe injuries from 20% to 5%; or (b) to say that the proportion of severe injuries was reduced by a factor of $\frac{25}{40}$, so that an estimate of its effect in the second country is to lower severe injuries to $\frac{20 \times 25}{40}\%$, which is 12.5%.

Notice that the estimate from the exponential model lies between these other two estimates. This is always likely to be true: because of the shape of the RIF curve resulting from the exponential model, $I_B = I_A^n$, it will always lie below the line $I_B = KI_A$ (which corresponds to (b) above) provided that I_A is to the left of the intersection of these two lines. And it is likely to also lie above the line

$I_B = I_A - h$ (corresponding to (a) above) for I_A to the left of the intersection between this line and the RIF curve since the slope of the latter, nI_A^{n-1} , is less than 1 when n and I_A are in the range we are usually interested in.

7.3.3 Compensating for initial level of injury severity when comparing the effectiveness of injury reduction in two circumstances

If the effect of seat belts on average was to reduce the proportion of serious and fatal injuries from 30% to 20%, and a study based on a sample of large cars (in which the proportion of serious and fatal injuries is lower than average, whether with or without seat belts) found that the reduction in injury severity was from 22% to 14%, are seat belts more or less effective in large cars than in cars in general? Assuming the exponential model for the RIF curve, in the first case we find the exponent n to be $\log(.2)/\log(.3) = 1.34$, and in the second case to be $\log(.14)/\log(.22) = 1.30$, very similar. Thus this model provides a way of compensating for the initial level of injury severity when evaluating the effectiveness of an injury reduction measure in two different circumstances.

7.3.4 Estimating the reduction in fatalities in a small sample where only serious and slight injuries are accurately known

If it has been found that a number of different devices based on cushioning or restraining a person involved in an accident all behave according to the same model in the way they reduce injury, we might be prepared to assume this is also true of some new device based on the same principle. So if the normal level of injuries is 3% fatal, 30% serious, and 67% slight, and in a small sample of cases where the device

was in operation we observe no fatalities, 15 serious injuries, and 60 slight injuries, the number of fatalities may be a poor predictor of the true probability of death, because of the small numbers involved. Because fatalities are typically valued at many times that of serious injuries, this means that any estimate of the benefits of the device is not likely to be very accurate. But if we assume the device behaves according to our exponential model, we can try to improve the accuracy of our estimate of benefit. First we would write down the likelihood equation for getting the observed numbers of fatal, serious, and slight injuries, with the free parameter n included. Then we differentiate with respect to n and set the result equal to zero. This gives the parameter of the RIF curve which best fits our data. Once this parameter is known, we can estimate the proportions of fatal, serious, and slight injuries with the device in operation to be respectively $.03^n$, $.33^n - .03^n$, and $1 - .33^n$. These estimates can then be used in a cost-benefit analysis of the device.

7.3.5 Estimating missing data

Table 7.1 presents the results of classifying certain two-vehicle accidents according to the injuries of both drivers involved. The number of cases in which both are uninjured is not known because such accidents were not recorded. For some purposes it may be desirable to estimate that number. One way of doing this is to condense the table into table 7.2, and assume the distributions of degree of injury to the driver of the lighter vehicle in the two conditions (other driver injured, other driver uninjured) are related in the same way as are two exponential distributions. Then if N is the number of cases in which both drivers are uninjured, and n is the exponent of the RIF curve, we have the simultaneous equations

		<u>Driver of lighter vehicle</u>			
		<u>Fatal</u>	<u>Serious</u>	<u>Slight</u>	<u>Uninjured</u>
Driver of heavier vehicle	Fatal	2	5	4	1
	Serious	32	131	54	89
	Slight	34	136	194	219
	Uninjured	26	278	710	?

Table 7.1: Injuries to drivers in two-vehicle accidents in which the ratio of the mass of the lighter vehicle to that of the heavier vehicle was in the range 0.40 to 0.59 (head-on accidents in rural areas in Great Britain, 1969-72).

		<u>Driver of lighter vehicle</u>	
		<u>Injured</u>	<u>Uninjured</u>
Driver of heavier vehicle	Fatal + Serious	228	90
	Slight	364	219
	Uninjured	1014	?

Table 7.2: Condensed from table 7.1.

		<u>Driver of lighter vehicle</u>		
		<u>Fatal + Serious</u>	<u>Slight</u>	<u>Uninjured</u>
Driver of heavier vehicle	Injured	170	194	219
	Uninjured	304	710	?

Table 7.3: Condensed from table 7.1.

$$\left(\frac{228}{1606}\right)^n = \frac{90}{309 + N} \quad \text{and} \quad \left(\frac{592}{1606}\right)^n = \frac{309}{309 + N}$$

which have the solution $n = 1.29$, $N = 814$.

Alternatively we could condense the data as in table 7.3. Then making a similar assumption, with N defined as before and m the exponent of the RIF curve in this case, we have

$$\left(\frac{170}{583}\right)^m = \frac{304}{1014 + N} \quad \text{and} \quad \left(\frac{364}{583}\right)^m = \frac{1014}{1014 + N}$$

which have the solution $m = 1.58$, $N = 1122$.

The values of N estimated by the two methods are quite similar.

We should make one reservation however. There is a substantial correlation between the injuries of the two drivers, which arises because of the influence of the speed of collision. This has not been allowed for in the above calculation. Chapter 8 is devoted to the analysis of tables like 7.1 taking this into account.

7.3.6 Compensating for errors

In using a certain data base to investigate the effect of wearing a seat belt on the driver's injury, we might suspect that there was a bias towards reporting drivers as not wearing a belt, even in some cases where they in fact were, and that this bias is less strong in the case of severe accidents (ones in which the driver was killed or seriously injured) than it is in slight accidents. In the raw data, therefore, the effectiveness of seat belts would be underestimated.

Let us assume that table 7.4 gives the probabilities with which the driver was reported as wearing a belt or not, according to whether he really was wearing one or not.

Now, let r be the proportion of drivers involved in accidents who were actually wearing a seat belt; let p be the proportion of non-seat belt wearers who are severely injured; let the RIF curve of wearers against non-wearers be a power function with exponent n ; so the proportion of seat belt wearers who are severely injured is p^n .

Table 7.5 gives the true proportions classified according to seat belt usage and severity of injury. Applying table 7.4 to this we find the following proportions:

$rp^n q_1$	severely injured, actually wearing a seat belt, reported as wearing a seat belt;
$rp^n (1-q_1)$	severely injured, actually wearing a seat belt, reported as not wearing a seat belt;
$r(1-p^n) q_2$	slightly injured, actually wearing a seat belt, reported as wearing a seat belt;

$r(1-p^n)(1-q_2)$ slightly injured, actually wearing a seat belt,
 reported as not wearing a seat belt;
 $(1-r)p$ severely injured, actually not wearing a seat belt,
 reported as not wearing a seat belt;
 $(1-r)(1-p)$ slightly injured, actually not wearing a seat belt,
 reported as not wearing a seat belt.

Consequently the results we observe are as in table 7.6. We might have several empirical tables of this type, which might refer to drivers of vehicles of different masses, for instance. The base level of injury severity, p , would differ from table to table as it depends strongly on vehicle mass. We might be prepared to assume that r , q_1 , and q_2 were the same in each table. If we were also prepared to assume that the effectiveness of a seat belt (as measured by the parameter n) is the same in vehicles of different masses, then if we had T tables of results we would have $3T$ degrees of freedom (since the fourth proportion in each table is 1 minus the sum of the other three) and $T+4$ parameters to be estimated ($p_1, p_2, \dots, p_T, q_1, q_2, r, n$). If we had two tables only, there would be an exact solution. If we had three or more we could fit the parameters to the data by the method of least squares, for instance. This would give us estimates of both the true effectiveness of seat belts and the true proportion of drivers wearing them, as they might be if uncontaminated by reporting errors.

<u>Fact</u>	<u>Reported</u>	
	<u>Wearing</u>	<u>Not wearing</u>
Severe injury:		
Wearing	q_1	$1 - q_1$
Not wearing	0	1
Slight injury:		
Wearing	q_2	$1 - q_2$
Not wearing	0	1

Table 7.4: Probabilities of being reported as wearing a seat belt, according to whether a seat belt was in fact worn or not, and severity of injury sustained.

<u>Fact</u>	<u>Injury</u>	
	<u>Severe</u>	<u>Slight</u>
Wearing	rp^n	$r(1 - p^n)$
Not wearing	$(1 - r)p$	$(1 - r)(1 - p)$

Table 7.5: Relative proportions classified according to seat belt usage and injury severity.

<u>Reported</u>	<u>Injury</u>	
	<u>Severe</u>	<u>Slight</u>
Wearing	$rp^n q_1$	$r(1 - p^n)q_2$
Not wearing	$rp^n(1 - q_1) + (1 - r)p$	$r(1 - p^n)(1 - q_2) + (1 - r)(1 - p)$

Table 7.8: Observed results.

7.4 Empirical evidence

If the result of changing from category to category on one factor is to change one parameter of the distribution of injury severity - for instance, the mean level of a Normal distribution - then by plotting the proportion of cases seriously injured against the proportion killed, with the different points corresponding to the different categories of the factor, we can deduce the shape of the RIF curves.

For instance, if the distributions are exponential, $f(s) = \lambda \exp(-\lambda s)$, and the different categories of the factor have different values of λ , then if we let

$$x = \int_{\alpha}^{\infty} \lambda \exp(-\lambda s) ds = \exp(-\lambda \alpha) = \text{proportion killed,}$$

and

$$y = \int_{\beta}^{\alpha} \lambda \exp(-\lambda s) ds = \exp(-\lambda \beta) - \exp(-\lambda \alpha) = \text{proportion}$$

seriously injured, then $y = x^{\beta/\alpha} - x$.

Therefore if we had crude information about injury in several different circumstances - for example the proportions killed, seriously, and slightly injured in several different age groups - and we found on plotting y against x that $y = x^m - x$, this would suggest that the distributions of injury severity for different ages were each exponential, and therefore that if we had detailed information about injury severity in (any) two age groups, we would find the RIF curve to be a power function.

The function $y = x^m - x$ has one free parameter $m (= \beta/\alpha)$ that represents the difference between the criterion for a fatal injury (α) and the criterion for a serious injury (β) and quantifies the

correlation between fatal and serious injury that would be expected if changing from category to category altered the overall level of injury severity. If the probabilities of fatal and of serious injury were altered independently when changing from category to category then no such correlation would be apparent.

Examples of the curve $y = x^m - x$ are shown in figure 7.6 for $m = .25, .5, \text{ and } .75$. (The horizontal axis and $y = 1 - x$ are the two limits of this curve, corresponding to $m = 1$ and $m = 0$ respectively.) Although it is unlikely in road accidents that we would observe the fall in the proportion of severe injuries that occurs when the proportion of fatalities gets very high, in types of accident whose overall severity is higher - such as aviation disasters - it is possible that the whole range of this curve might be observed. Notice also that since the proportion of slight and serious injuries together is one minus the proportion of fatalities, the proportion of slight injuries is given by the difference between the line $y = 1 - x$ and the curve $y = x^m - x$. Moreover, if we had four categories of injury, with the divisions between them occurring at γ, β , and α , by plotting the lines $y_1 = x^{\beta/\alpha} - x$, $y_2 = x^{\gamma/\alpha} - x$, and $y_3 = 1 - x$, we could directly illustrate how the proportions in each vary with the proportion in the fatal category since they are respectively $x^{\beta/\alpha} - x = y_1$, $x^{\gamma/\alpha} - x^{\beta/\alpha} = y_2 - y_1$, and $1 - x^{\gamma/\alpha} = y_3 - y_2$.

If instead of being exponential, the distributions were Normal, $f(s) = N(\mu, 1)$, with the different categories having different values of μ , then a different and algebraically difficult relation between y and x would exist.

Is there in fact a correlation between the proportion killed and

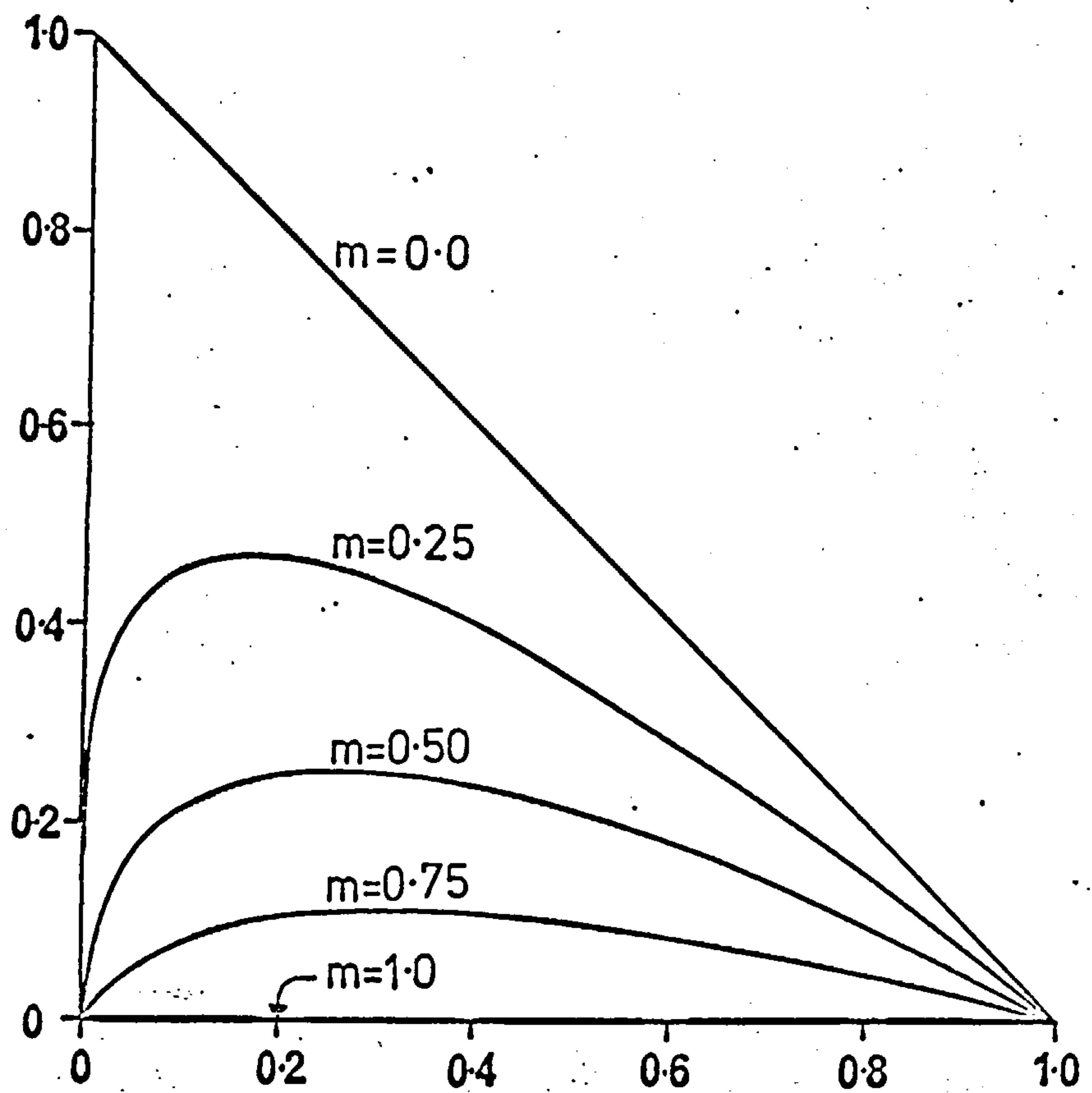


Figure 7.6: examples of curves of the form $y = x^m - x$

the proportion seriously injured? Figure 7.7 indicates that there is. This refers to pedestrian accidents in Great Britain in 1970-71; the different points correspond to different times of day at which the accidents occurred. During daytime, around 3% of pedestrian accidents are fatal and 28% are serious. During the evening and night both these proportions are higher.

The curve marked 'exponential' in figure 7.7 is of the form $y = x^m - x$ and was fitted by least squares. It is in fact $y = x^{.33} - x$ and would arise if the effect of time of day on injury severity was to change the parameter of the exponential distribution, provided that the value of injury severity corresponding to the division between slight and serious injury was 33% of the value corresponding to the division between serious and fatal injury. (See figure 7.8a.)

The curve marked 'Normal' would arise if the effect of time of day on injury severity was to shift the location of a Normal distribution while leaving the variance unchanged. The single parameter that is fitted in this case corresponds to the differences between the criteria for fatal and for serious injury, measured in units of the standard deviation of the Normal distributions. It turns out to be 1.34. (See figure 7.8b.)

Two conclusions may be drawn from figure 7.7: (i) over different times of the day, there is a correlation between the proportion of pedestrians killed and the proportion seriously injured. Hence whatever are the features of time of day that make death more or less likely following a collision between a pedestrian and a motor vehicle, they are the same (or some of them are) as those which cause a greater or a lesser proportion of serious injuries to occur; (ii) the severities

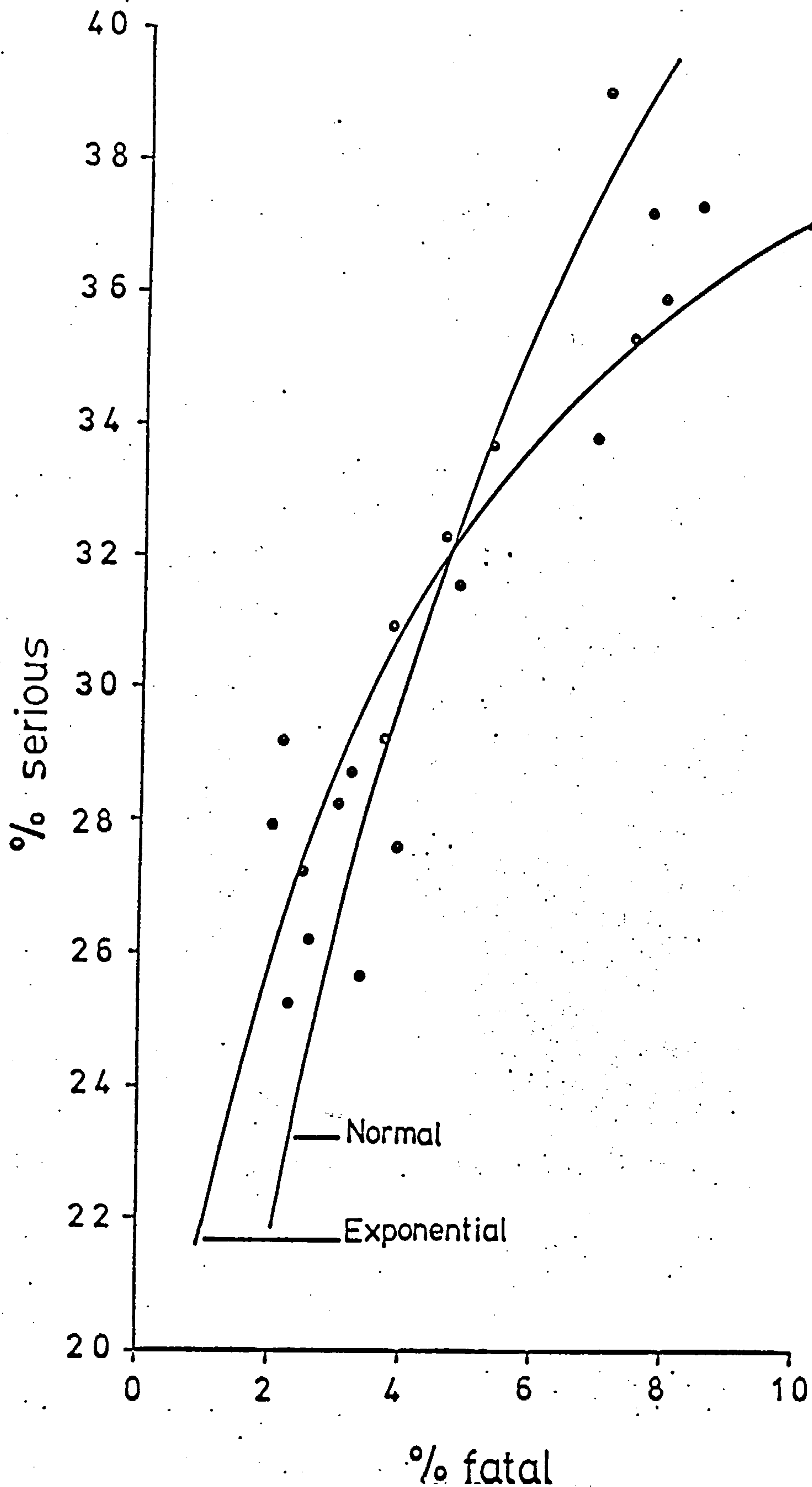


Figure 7.7: The relation between the proportion of pedestrians involved in accidents who are killed and the proportion who are seriously injured, the points corresponding to accidents at different times of the day:

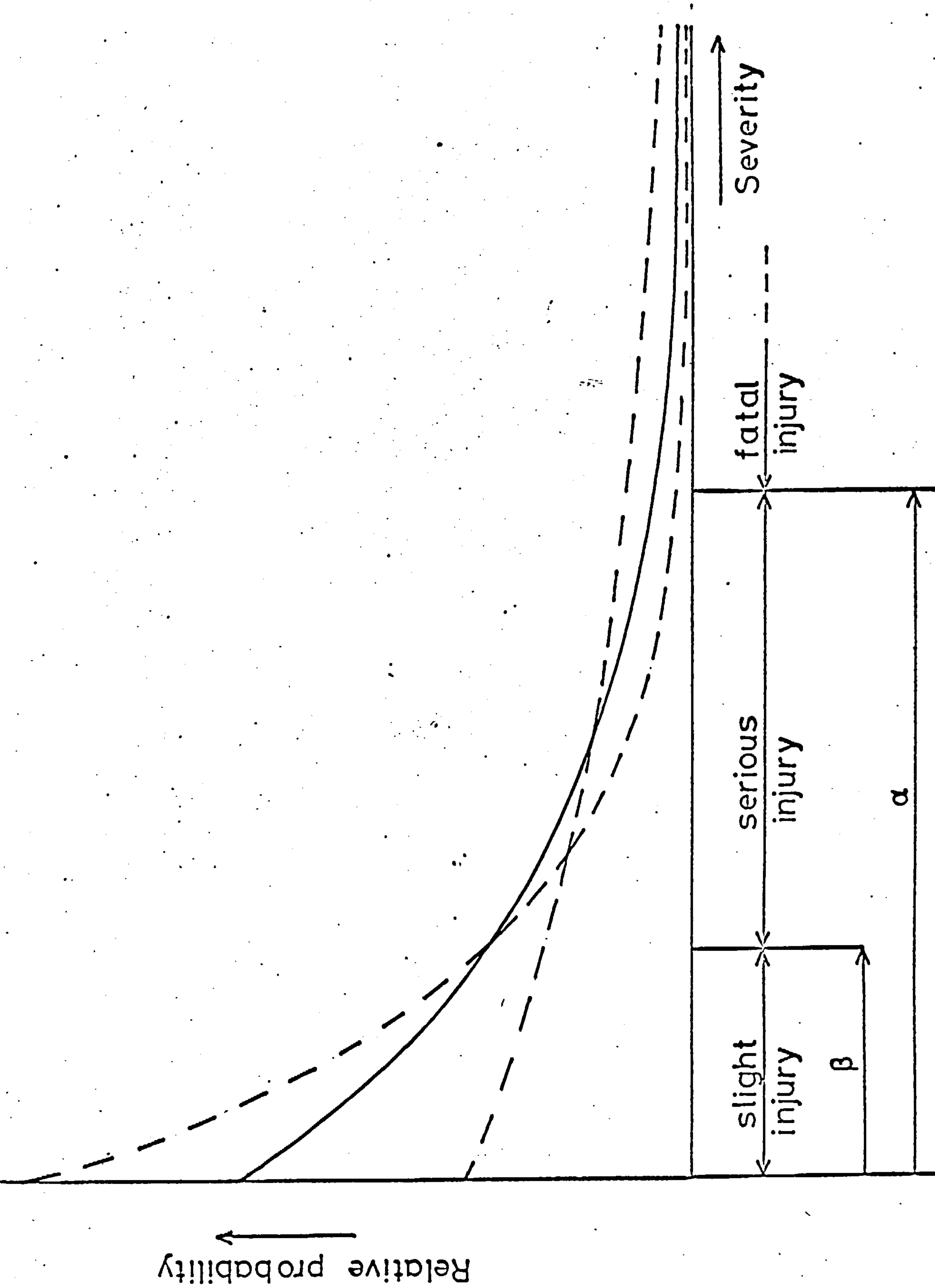


Figure 7.8(a): Illustrations of the definitions of injury corresponding to the exponential model of figure 7.7 ($\beta/\alpha = 0.33$). The three curves shown represent the exponential distributions of injury severity under three hypothetical conditions.

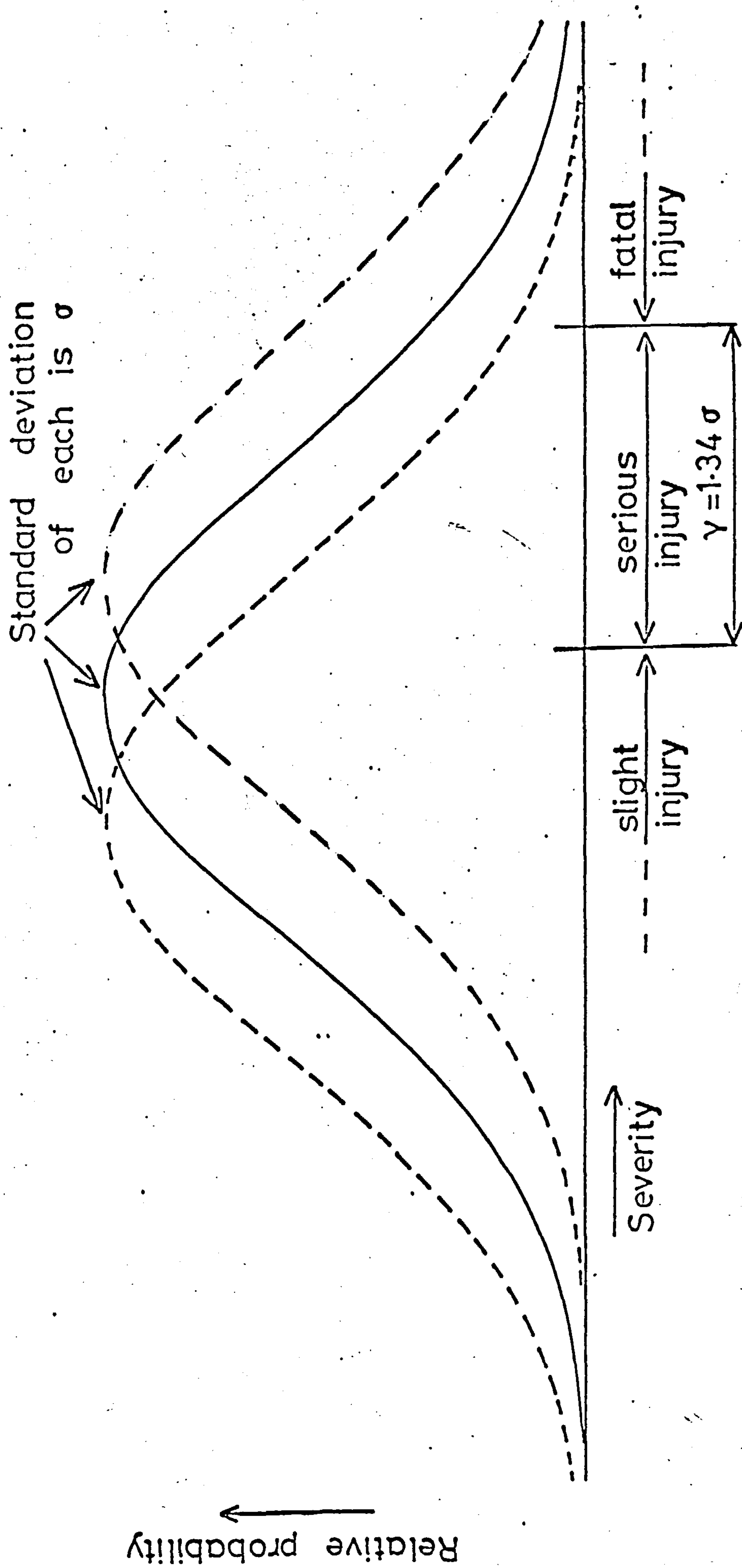


Figure 7.8(b): Illustrations of the definitions of different degrees of injury corresponding to the Normal model of figure 7.7. The three curves shown represent the Normal distributions of injury severity under three hypothetical conditions.

of injuries sustained at different hours of the day are rather better described as being related like different exponential distributions than like different Normal distributions of the same variance. (The sum of squares of deviations from the 'exponential' line in figure 7.7 is about half the sum of squares of deviations from the 'Normal' line.)

It should be noted that each point in figure 7.7 is really the average of a number of separate points, each corresponding to different combinations of all the other categories (besides time of day) that affect injury severity - age of pedestrian, location of accident, type of vehicle, environmental conditions, etc, as discussed in Chapter 2. Considering just two points as contributing to each one in figure 7.7, if the constituent points lie close together on $y = x^{.33} - x$, then so will their average, but if they are far apart, their average will lie off this curve. If, in figure 7.9, points A and C are constituent points, then since any average of them must lie on the straight line connecting them, it must be close to $y = x^{.33} - x$. Whereas if B and D are the contributing points, if their contributions are of the same order of magnitude, their average may lie far from $y = x^{.33} - x$. However, it is unlikely that any point as severe as D contributes anything substantial to any point in figure 7.7. This is a possible explanation for the scatter of the points in figure 7.7. (According to the χ^2 test, they are much too scattered to be derived by chance deviations from $y = x^{.33} - x$.)

Figure 7.10 is similar to figure 7.7 except that the points refer to accidents involving pedestrians of different ages instead of different times of the day. The two curves are the same as those in figure 7.7, that is, the parameters have not been adjusted to fit them to the points. A positive correlation is again apparent, and in this

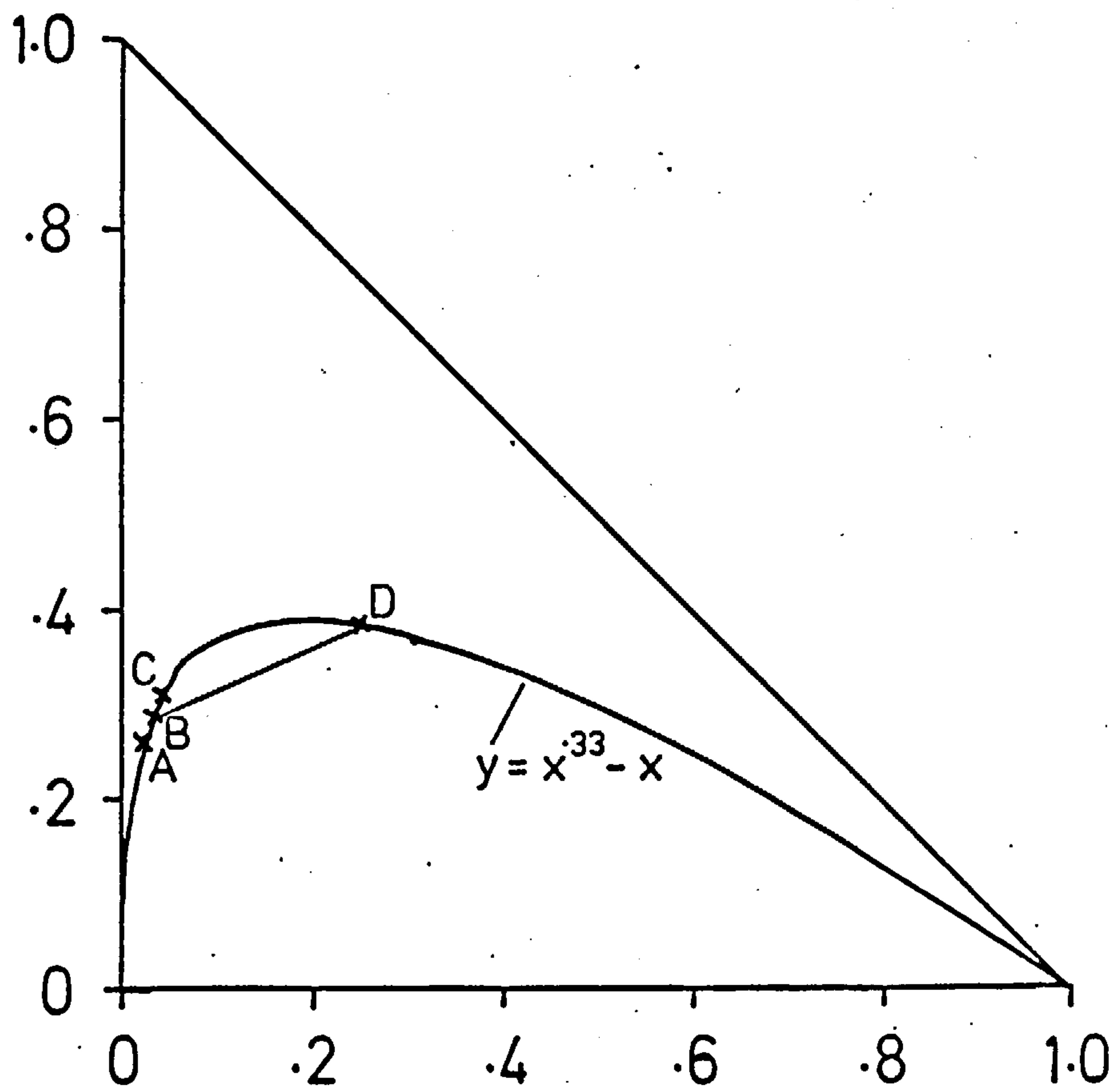


Figure 7.9: illustration of the effect of two points being grouped into one on the position of the observed points derived from $y = x^3 - x$: see text for explanation.

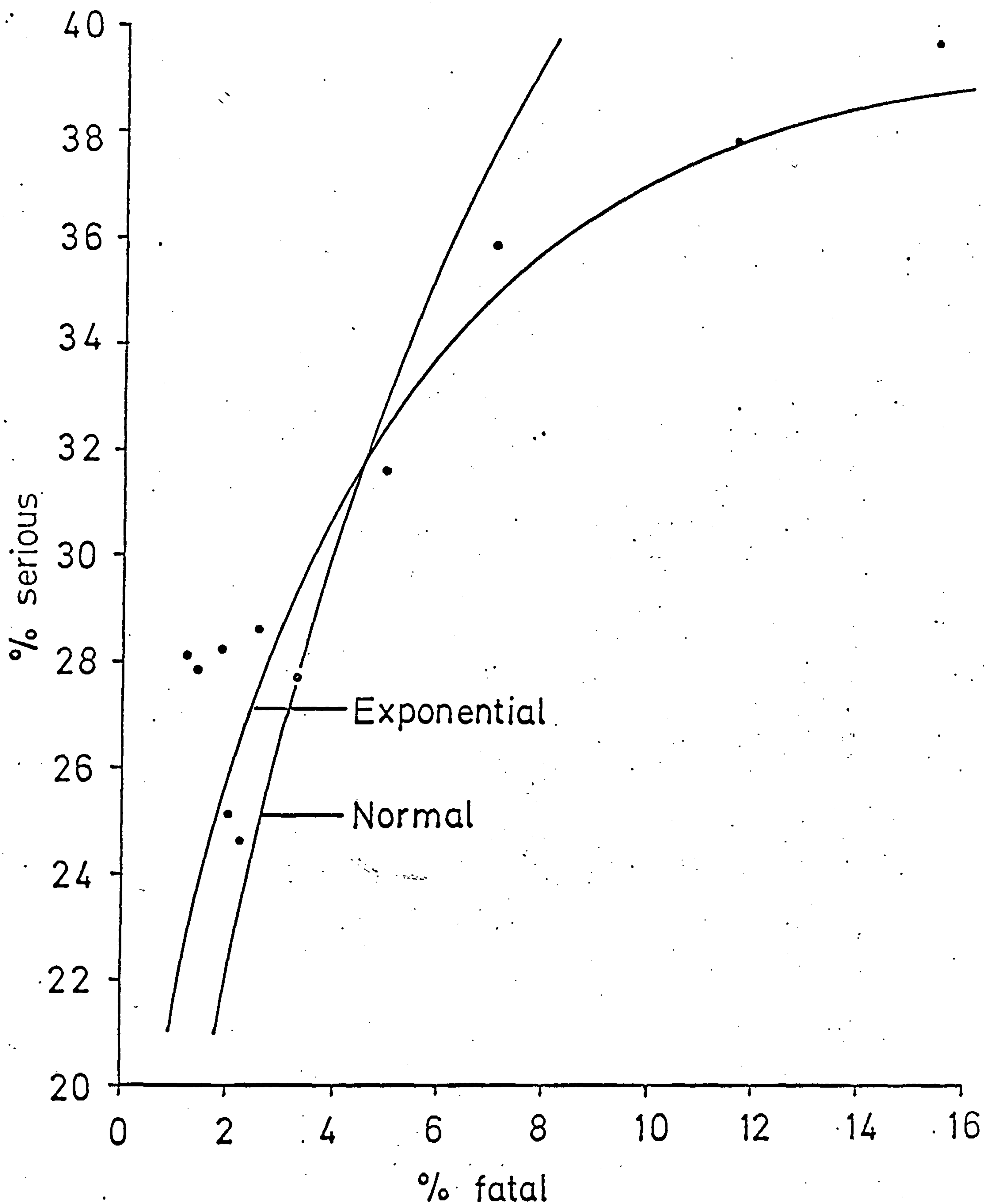


Figure 7.10: The relation between the proportion of pedestrians involved in accidents who are killed and the proportion who are seriously injured, the points corresponding to pedestrians in different age groups. The curves are those fitted to the data of figure 7.7.

case the exponential model is considerably better than the Normal one.

The models of figure 7.7 require that the effect of changing from hour to hour is to change the parameter of an exponential (or Normal) distribution ; and the models of figure 7.10 require that the effect of changing the age of the injured person is similar. For the parameters to be exactly the same in each figure requires an additional assumption, namely that changing from a single hour in figure 7.7 to a single age group in figure 7.10 also is statistically equivalent to changing the parameter of an exponential (or Normal) distribution. It would appear that this assumption is fulfilled in this case; below (figure 7.13) is given one in which it may not be.

Another example is given in figure 7.11: the different points correspond to pedestrian accidents in which the pedestrian is struck by different types of vehicle. In this case there is no correlation between the proportion killed and the proportion seriously injured. Although no definite explanation can be given for this, it may arise because both speed of the vehicle at impact and the trajectory followed by the pedestrian after impact are affected by the type of vehicle and in turn affect the relative probabilities of different degrees of injury. Thus the increased likelihood of death after being struck by a heavy vehicle (compared to that for cars) without a corresponding rise in serious injuries might be due to impacts between pedestrians and heavy vehicles usually being at lower speeds than impacts between pedestrians and cars, thus leading to a reduced severity of injury; but impacts with heavy vehicles are more likely to lead to the pedestrian being run over, and this might produce a rise in the percentage of fatalities that was out of proportion to the rise in serious injuries it produced. It is also noticeable that the points corresponding to impacts with two-

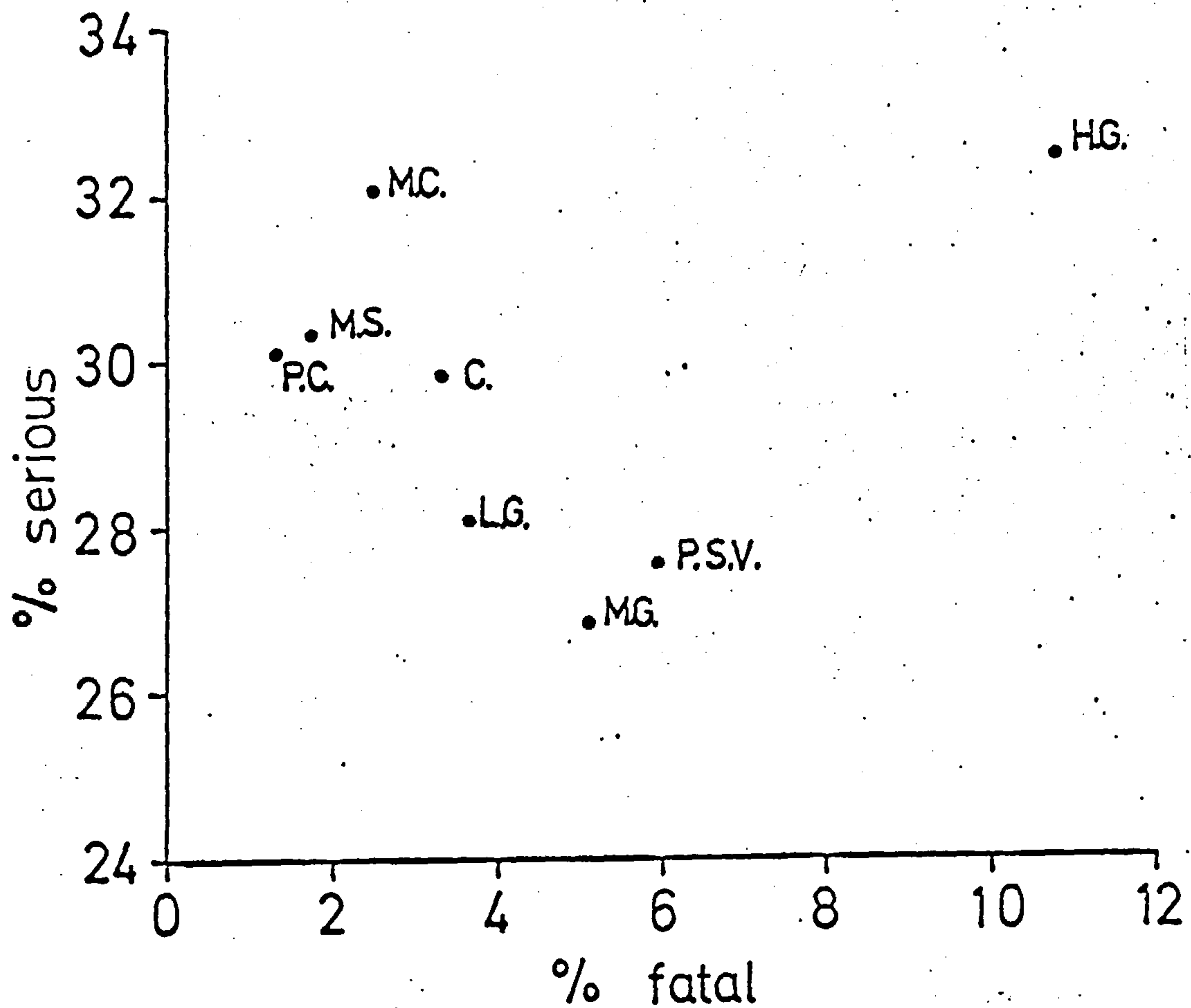


Figure 7.11: The relation between the proportion of pedestrians involved in accidents who are killed and the proportion who are seriously injured, the points corresponding to accidents involving different types of vehicle (C=car, HG=heavy goods vehicle, LG=light goods vehicle, MC=motorcycle, MG=medium goods vehicle, MS=motor scooter, PC=pedal cycle, PSV=public service vehicle).

wheeled vehicles are clustered together at the top left of the figure - the proportion of serious injuries is about the same as for cars, but the proportion of deaths is considerably lower. This might be because the particularly deadly trajectory that the pedestrian can follow when struck by a car in which he cartwheels over the top of the car, striking his head near the base of the windscreen and falling from a height of about six feet, perhaps head first, on to the road, does not happen after impact with a two-wheeled vehicle. In sum, these speculations amount to a suggestion that the effect of trajectory on injury severity may not be equivalent to moving from one exponential distribution to another because the features of a trajectory that tend to lead to death are different from those that make serious (as opposed to slight) injury likely; and that the association with different types of vehicle is the reason for the lack of correlation in figure 7.11.

Another example in which there is a correlation is shown in figure 7.12: this time it is the injuries to drivers involved in collisions between two cars, and the different points correspond to different age groups of driver. Again the curves which were fitted to the data of figure 7.7 are shown: the exponential model again appears to be the better. Notice that it fits well even though the points in that figure cover a rather different range of values than do the points in figure 7.7.

Now consider figure 7.13. The data from which the empirical points there come are injuries to drivers involved in single-vehicle non-pedestrian accidents, and they differ in the age group of the drivers to which they refer - in the youngest group of drivers, 1.5% are killed and 31.0% seriously injured, whereas in the oldest group 6.2% are killed and 37.6% seriously injured. The curves from figure

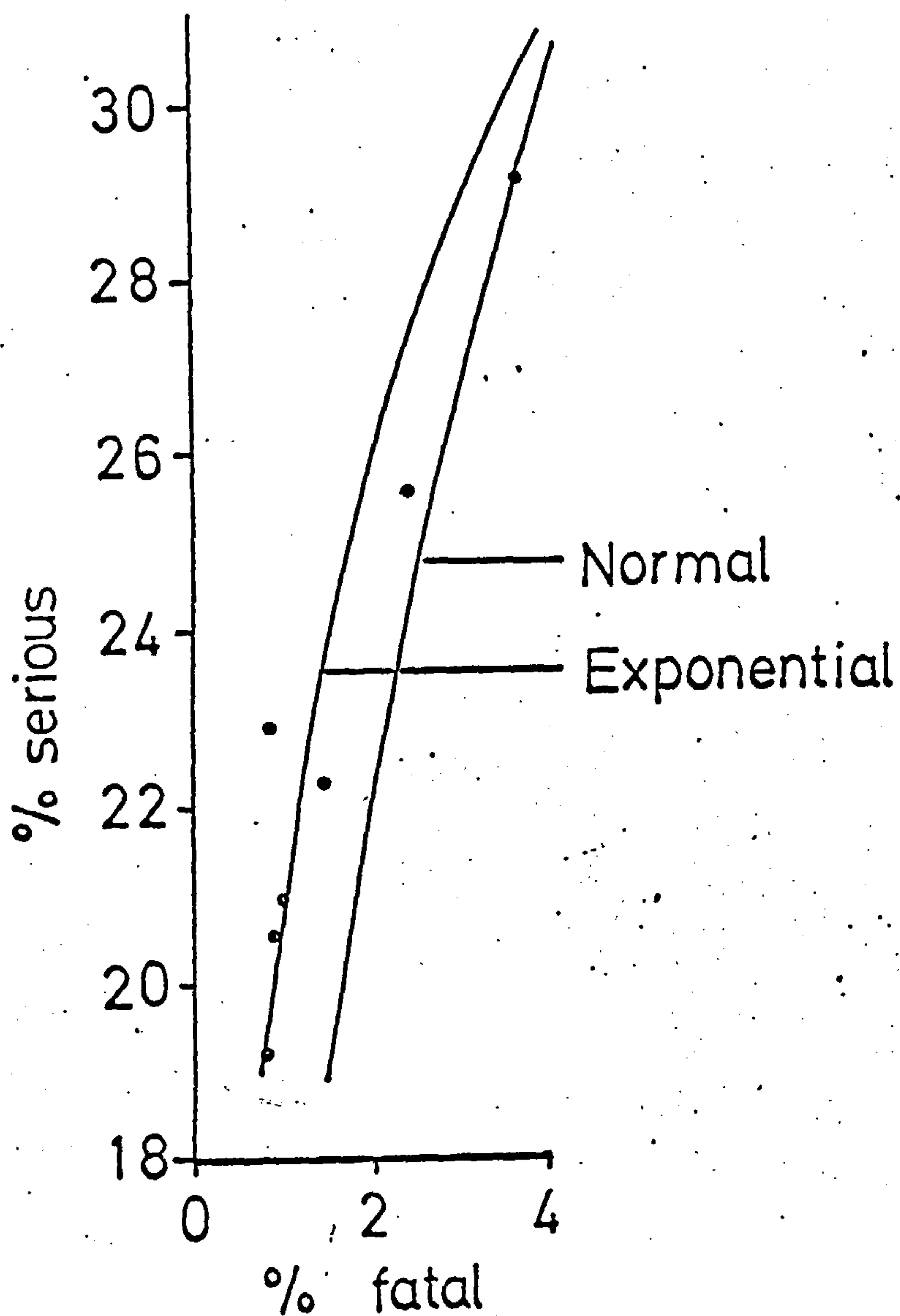


Figure 7.12: The relation between the proportion of car drivers involved in collisions between two cars who are killed and the proportion who are seriously injured, the points corresponding to drivers in different age groups. The curves shown are those fitted to the data of figure 7.7.

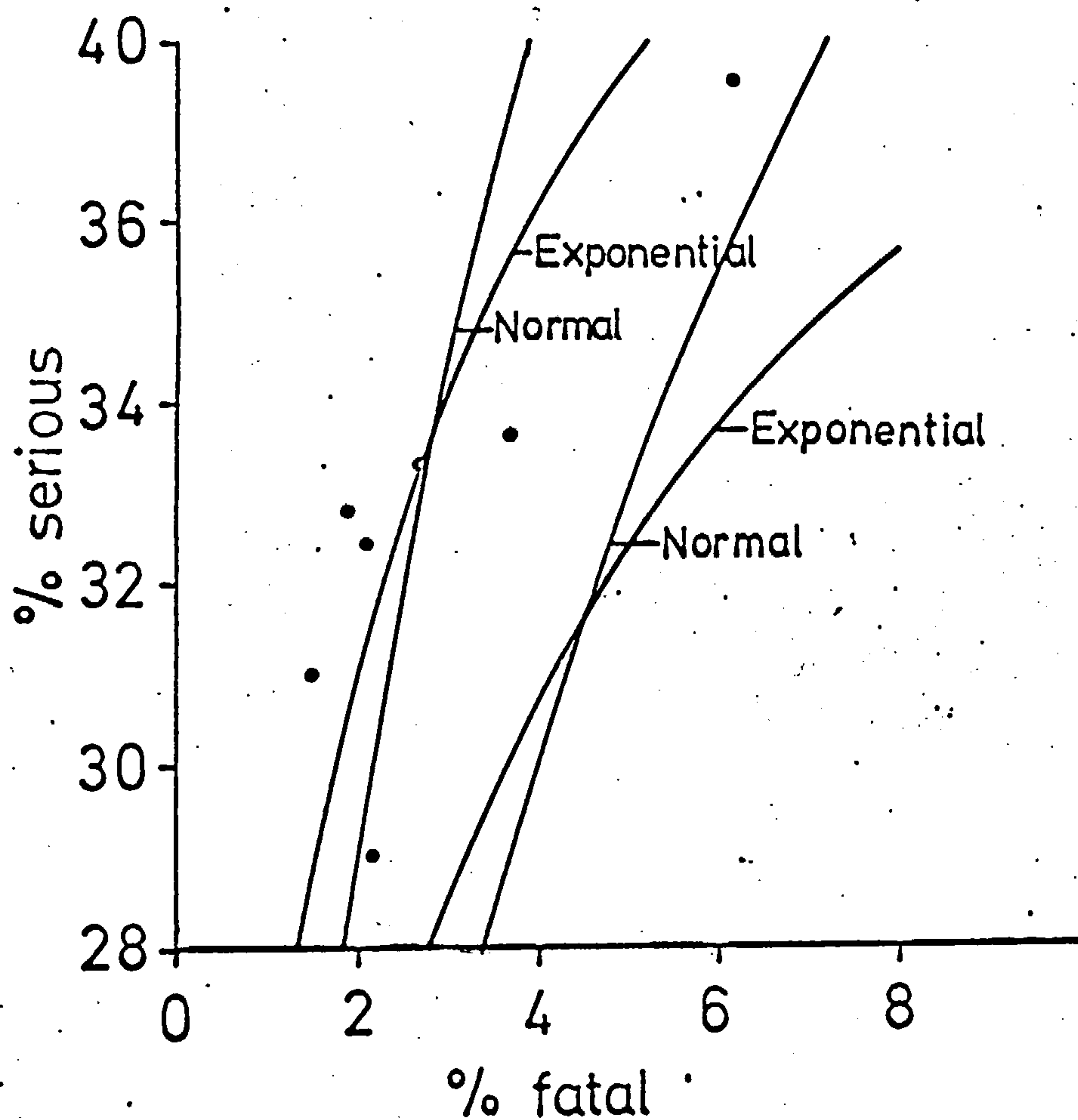


Figure 7.13: The relation between the proportion of car drivers involved in single-vehicle accidents who are killed and the proportion who are seriously injured, the points corresponding to drivers in different age groups. The left-hand pair of curves were fitted to the points shown; the right-hand pair are those shown in figure 7.7.

7.7 are also shown, and in this example both of them fall below all the points. Curves based on exponential and Normal models and fitted to these points are also shown. The exponential curve is the better fit - the sum of squared deviations from it is only one-fifth of that for the Normal curve. Its parameter is .284.

Does this imply that the relative definitions of fatal and serious injuries are different in single-vehicle accidents than they are in pedestrian and in car-car accidents? One explanation is that changing from pedestrian injuries to those of drivers in single-vehicle accidents is not statistically equivalent to changing the parameter of an exponential (or Normal) distribution. Another possible explanation is outlined below.

Assume that in single-vehicle accidents - where there is necessarily only one party involved - there is a category of very slight injuries which go unreported, though they would be reported if other people were involved. Specifically, let there be four categories of injury that can be suffered by a driver in a single-vehicle accident: trivial (not reported), slight, serious, and fatal. Let the three divisions between them occur at γ , β , and α and let the effect of age group on the relative proportions in the four categories be the same as changing the parameter of the exponential distribution. Then the proportions in the four categories will be $1 - \exp(-\lambda\gamma)$, $\exp(-\lambda\gamma) - \exp(-\lambda\beta)$, $\exp(-\lambda\beta) - \exp(-\lambda\alpha)$, and $\exp(-\lambda\alpha)$. So the proportions of those reported that are serious and fatal will be $\exp(-\lambda\beta + \lambda\gamma) - \exp(-\lambda\alpha + \lambda\gamma)$ and $\exp(-\lambda\alpha + \lambda\gamma)$ respectively. Consequently if y is the proportion of serious injuries and x the proportion of fatal ones,

$$y = x^{\frac{\gamma-\beta}{\gamma-\alpha}} - x,$$

and this is the same functional form as the equation we have previously been fitting, $y = x^{\beta/\alpha} - x$.

Now, from the pedestrian data we estimated $\frac{\beta}{\alpha} = .326$, and from the single-vehicle accident data we found a parameter which we now interpret as $\frac{\gamma - \beta}{\gamma - \alpha}$ to be .284. From these we can deduce that $\frac{\gamma}{\beta} = .18$.

Whilst it is certainly not true that only in single-vehicle accidents are there some injuries which go unreported because they are so minor, it is plausible that there are a greater proportion of them than in accidents in which two parties are involved.

More attention has been paid in this section to the exponential model than to the Normal model. This is for two reasons: first, the simplicity of the formulae resulting from the former, whereas the latter does not lead to simple algebraic expressions; second, to the data presented the exponential model appears to give the better fit. But no claim is made at this stage that the exponential model is generally better than the Normal, or that some third model is not better than both. The intention has been to present a new framework for considering injury severity and to compare two particular variations of this framework.

7.5 Further remarks

7.5.1 Comparison of Normal and exponential models at the extremes

Although, because of their general similarity of shape, we would expect it always to be difficult to distinguish between exponential and Normal models, we can point to two places where a distinction between them might be evident:

(i) on the RIF curve, the Normal model is parallel to one of the axes at (1, 1), whereas the exponential model has a finite slope (n).

(ii) On a curve of the type illustrated in figure 7.6, the Normal model has slope -1 at (1, 0), whereas the exponential model has slope $m - 1$.

7.5.2 The Normal and exponential models as special cases of a more general model

The probability density function of a variable with a Gamma distribution is given by

$$y = \frac{x^{c-1} e^{-x/b}}{b^c \Gamma(c)}$$

where $\Gamma(c)$ is the gamma function defined by

$$\Gamma(c) = \int_0^{\infty} e^{-u} u^{c-1} du$$

If $c = 1$, the exponential distribution is obtained. As $c \rightarrow \infty$, the Gamma distribution tends to the Normal distribution.

Therefore a natural generalisation of the Normal and exponential models is to suppose there exists a definition of injury severity such that the different populations of accidents each have a Gamma distribution of severity, with shape parameter c (fixed) but different scale parameters b . Thus curves of the form discussed in section 7.4 will have two parameters, one related to the definitions of the degrees of severity, and one related to the shape of the underlying distributions. A value of 1 of this second parameter will correspond to the exponential model, and a very large value will correspond to the Normal model.

7.5.3 _ Classification errors

When the police are classifying the severity of injury of a casualty, they might find it very easy to determine whether he is killed, and they might find it very easy to decide that he has a very minor injury, but there might be an intermediate range of severity within which the casualty would be classified as "serious" or "slight" at random. Let table 7.7 give the probability of each classification of severity for each given range of actual severity. Then if x = probability of fatal injury and y = probability of serious, $y = p(x^m - x)$. Several graphs of this type are illustrated in figure 7.14 which have been constrained to pass through (.04, .30).

Alternatively, if classification of serious injury is easy, and below a certain severity the classification is random between the "slight" and "serious" categories, and table 7.8 gives the probabilities, the form of the equations is $y = 1 - p + px^m - x$, and several such graphs constrained as above are illustrated in figure 7.15.

The above supposes that the classification errors arise randomly.

Probability	Severity	Classification		
		Fatal	Serious	Slight
$e^{-\lambda\alpha}$	α	1	0	0
$e^{-\lambda\beta} - e^{-\lambda\alpha}$	β	0	p	1-p
$1 - e^{-\lambda\beta}$	0	0	0	1

Table 7.7: Transition matrix giving rise to the curves of figure 7.14.

Probability	Severity	Classification		
		Fatal	Serious	Slight
$e^{-\lambda\alpha}$	α	1	0	0
$e^{-\lambda\beta} - e^{-\lambda\alpha}$	β	0	1	0
$1 - e^{-\lambda\beta}$	0	0	1-p	p

Table 7.8: Transition matrix giving rise to the curves of figure 7.15.

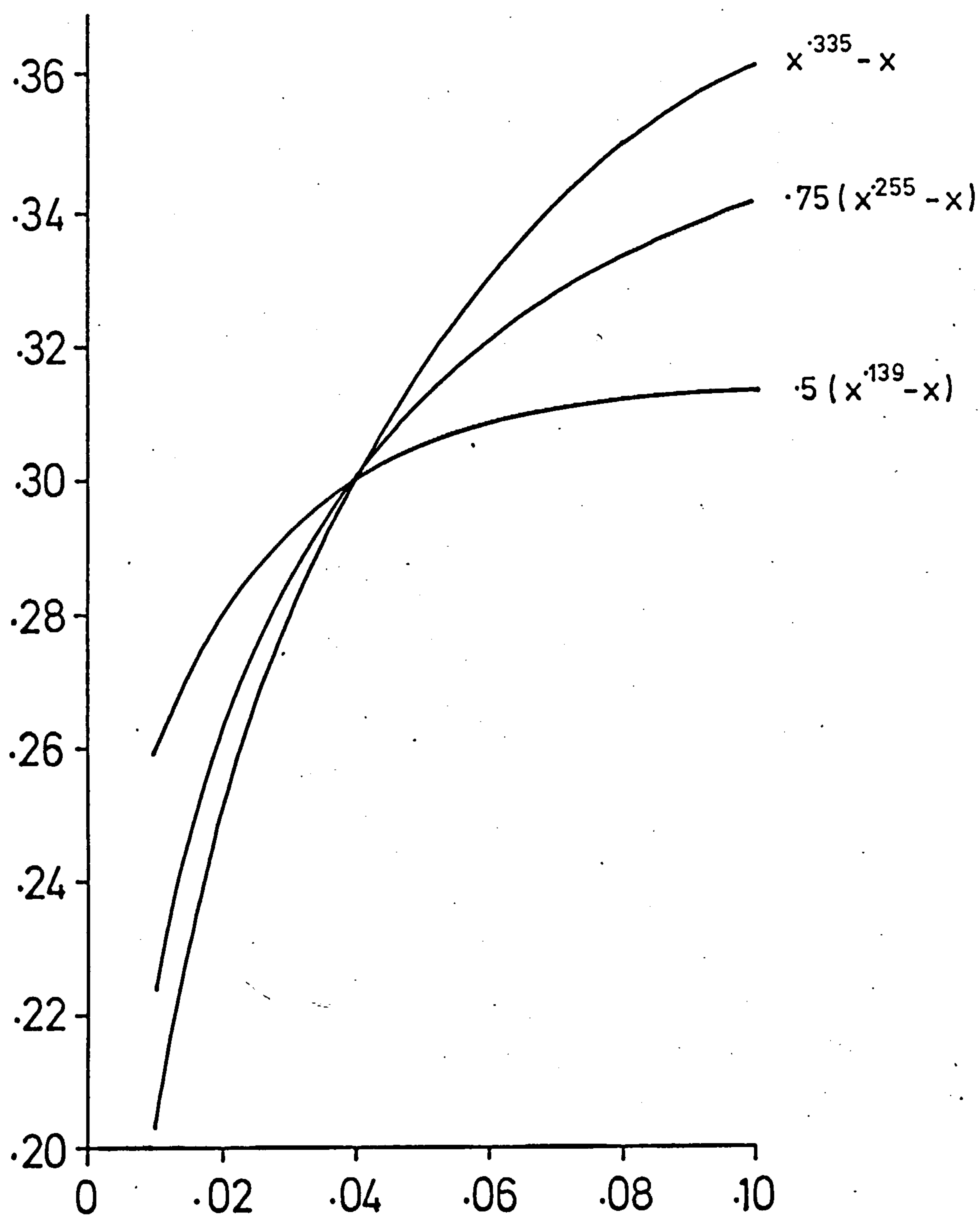


Figure 7.14: curves of the form $y = p(x^m - x)$

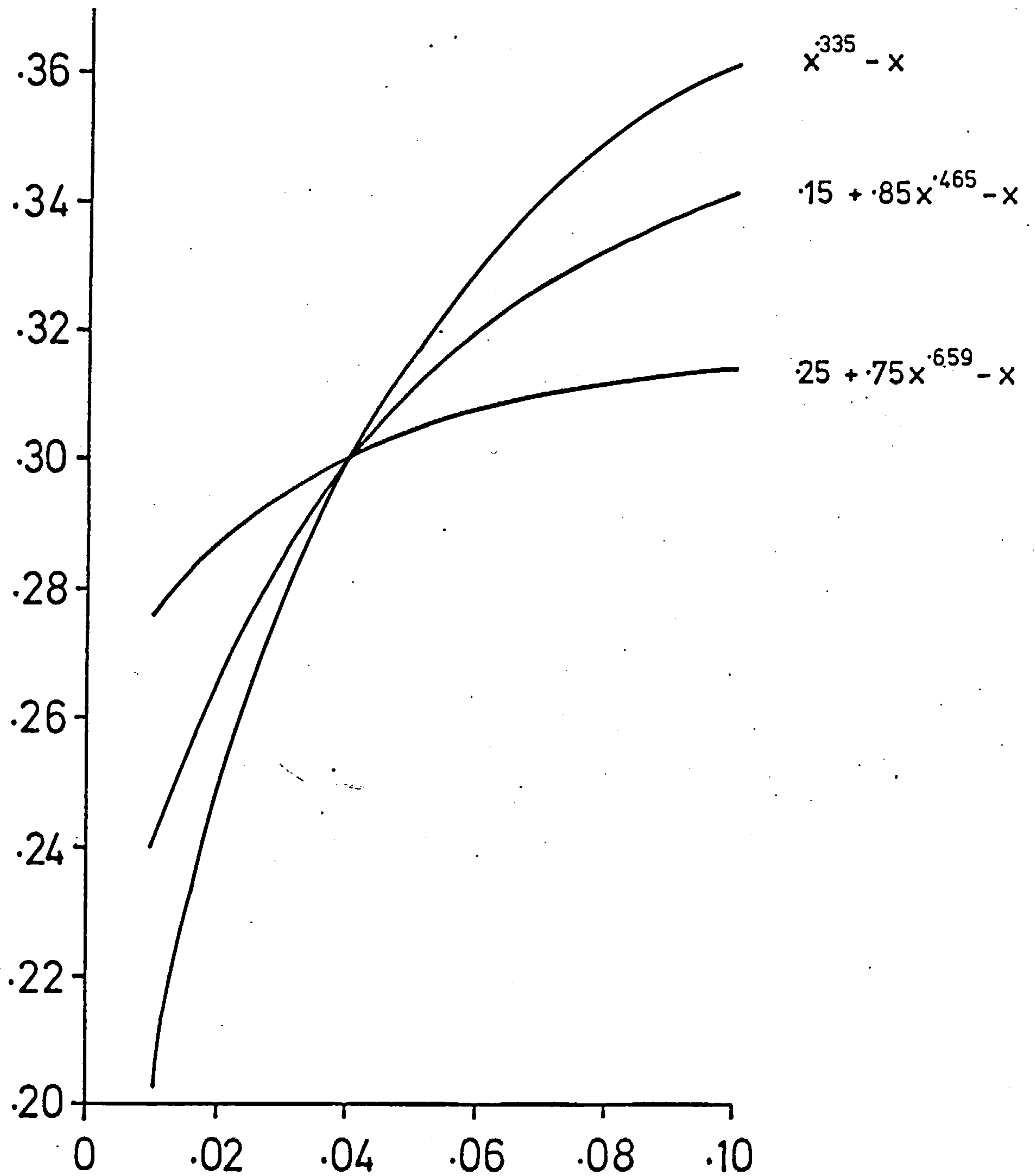


Figure 7.15: curves of the form $y = 1 - p + px^m - x$

If, instead, they are associated with the factor on which the plotted points differ, the resulting graph may reflect the errors rather than the true association between fatal and serious frequencies. Thus Newby (1969) plotted fatalities as a percentage of fatal and serious casualties against fatal and serious as a percentage of the total, with the different points corresponding to different police forces, and found a negative correlation (whereas, on the theory discussed in this Chapter, there should be a positive one since

$$\frac{x}{x^\alpha} = \left(x^\alpha\right)^{\frac{1-\alpha}{\alpha}}).$$

The reason for this is presumably because police forces differ in the precise interpretation they give to the definitions of injury severity, some tending to classify a moderate injury as serious, and some as slight.

7.5.4 Other ways of introducing an extra parameter

Given a scatter of points as in figure 7.7, if there is only one parameter to fit, that will be largely determined by the average position of the points, as already remarked. In order to get the slope of the curve right, an extra parameter will often be needed. The generalisations discussed in sections 7.5.2 and 7.5.3 provide ways of doing this. It has already been said that truncating (at the left) the exponential distributions does not alter the form of the curve, $x^m - x$, but truncation of Normal distributions would do so. Another way of introducing an extra parameter would be to allow the variances of the Normal distributions to vary, though in a way related to the mean level - for instance by assuming the distributions to be log-Normal with standard deviations proportional to the means.

7.5.5 Statistical testing

The RIF curve is the same type of plot used in the Kolmogorov-Smirnov test to determine whether two samples are drawn from the same population. However, the Kolmogorov-Smirnov test is sensitive to general alternatives, and consequently is not very powerful against shift alternatives. Since these will usually be the alternatives of interest, the Mann-Whitney U-test will generally be more appropriate, which has the additional advantage that it may readily be extended to more than two populations (the Kruskal-Wallis test, see section 4.2). If we are prepared to assume the Normal model, the Terry-Hoeffding and van der Waerden tests may be suggested (Walsh, 1965, p.68), and for the exponential model Savage's test (Walsh, 1965, p.71). See also Walsh (1965), p.110-112, Marascuilo (1970), and Grey and Morgan (1972).

The question of testing whether an RIF curve has a particular shape (rather than testing whether it is different from $y = x$) - in other words, a Kolmogorov-Smirnov test where the null hypothesis is not $y = x$ but instead $y = \text{some function of } x$, with one unknown parameter - is apparently unsolved.

Similarly, for curves of the type illustrated in figure 7.6, it would be of interest to test whether the exponential model was significantly better than the Normal model, or vice versa, and to put confidence limits on the parameters calculated. Also, it would be desirable to be able to test whether a significant improvement in fit was obtained when another parameter was introduced by one of the means discussed in sections 7.5.2 to 7.5.4.

7.5.6 Data from other countries

For comparison with figure 7.10, data in which the points represent pedestrians of different ages have been obtained from Sweden (1968-70), France (1969), and Morocco (1970). Figures 7.16 - 7.18 give the results of fitting models of the Normal and exponential types. Correlations between the proportion killed and the proportion seriously injured are once again evident. The parameters found (which would not be expected to be the same as for the British data because of differences in injury definitions) are marked on the figures.

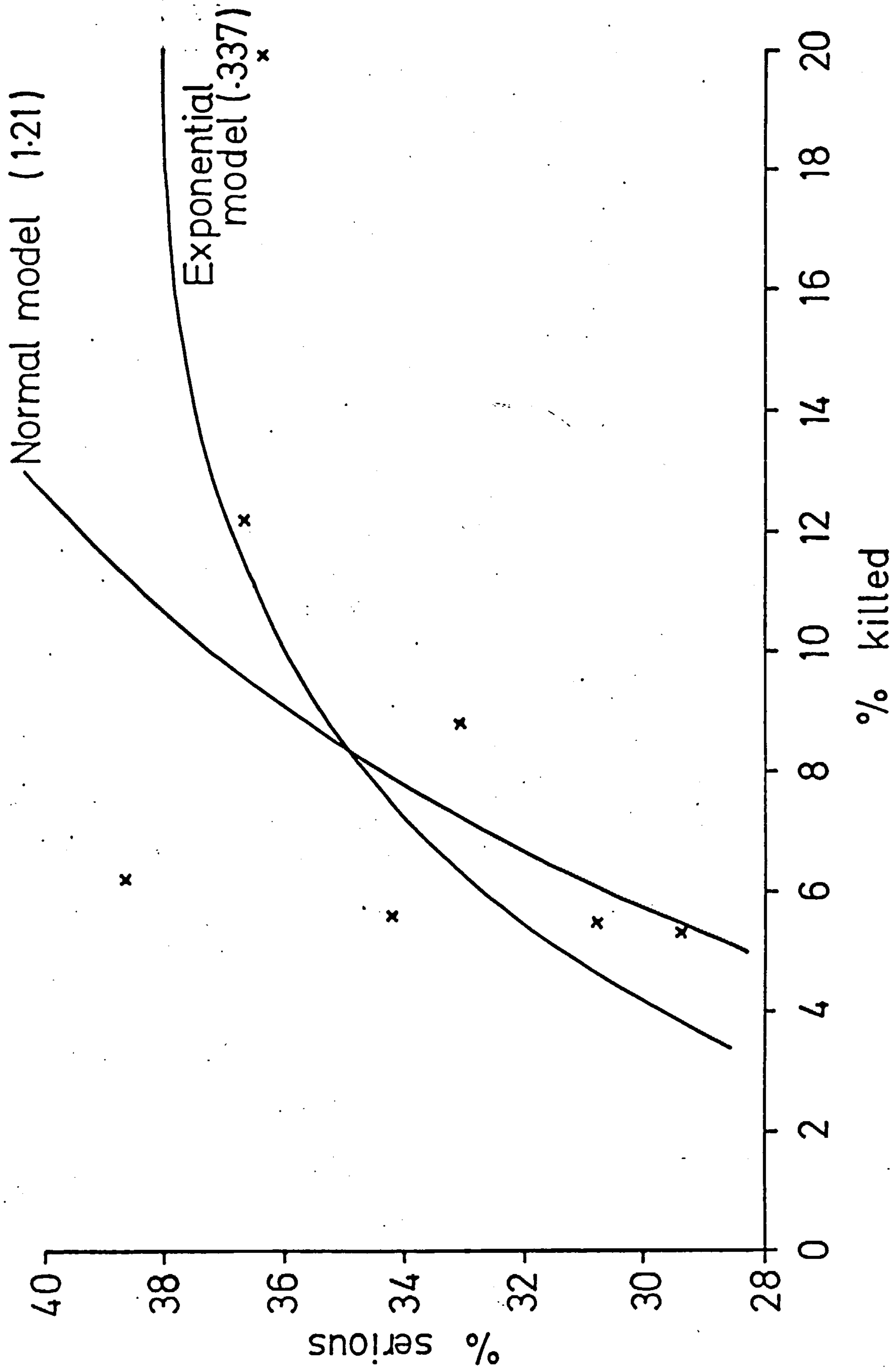


Figure 7.16: correlation between proportion killed and proportion seriously injured, the points corresponding to pedestrians in different age groups (Sweden, 1968-70)

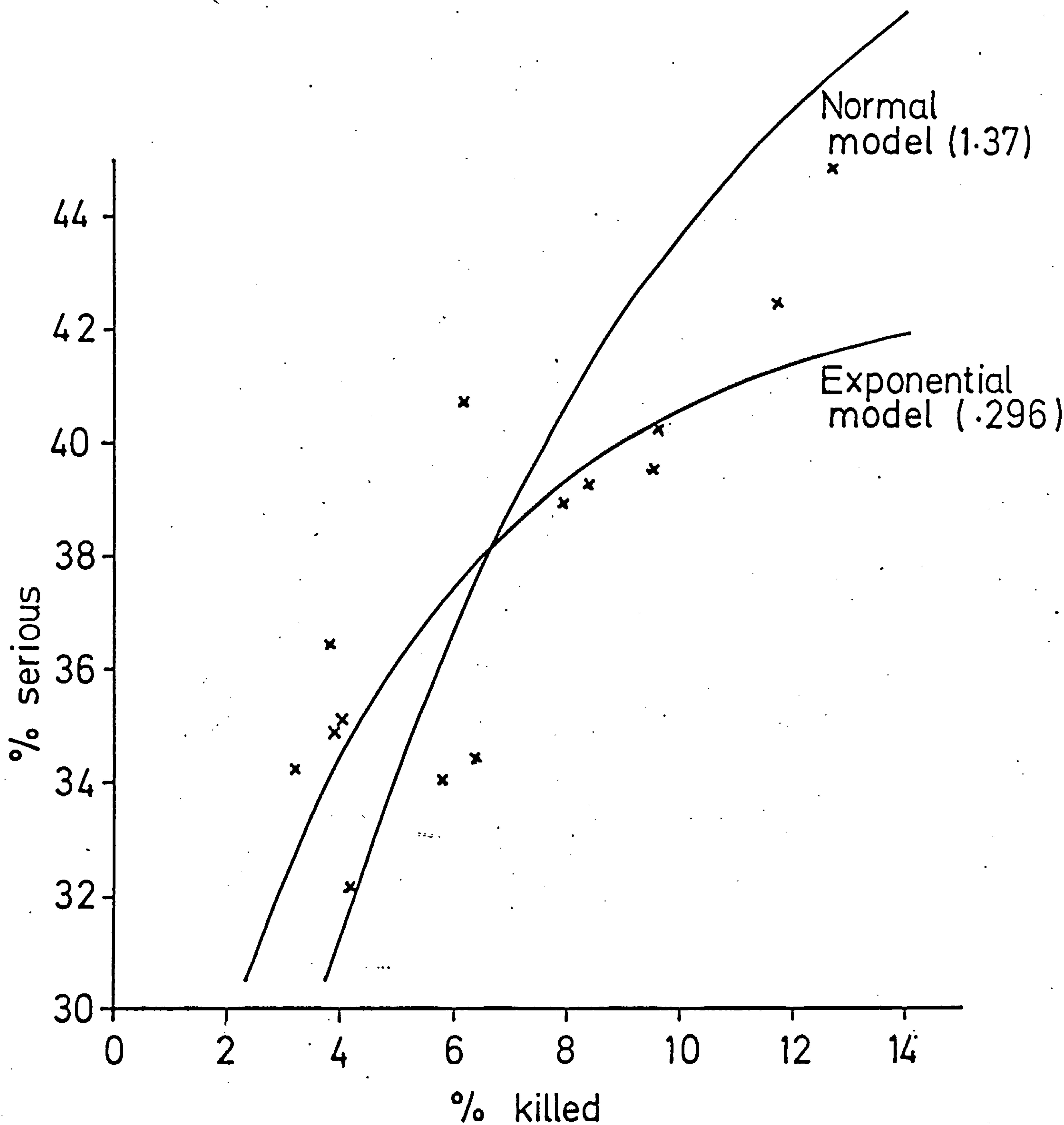


Figure 7.17: correlation between proportion killed and proportion seriously injured, the points corresponding to pedestrians in different age groups (France, 1969)

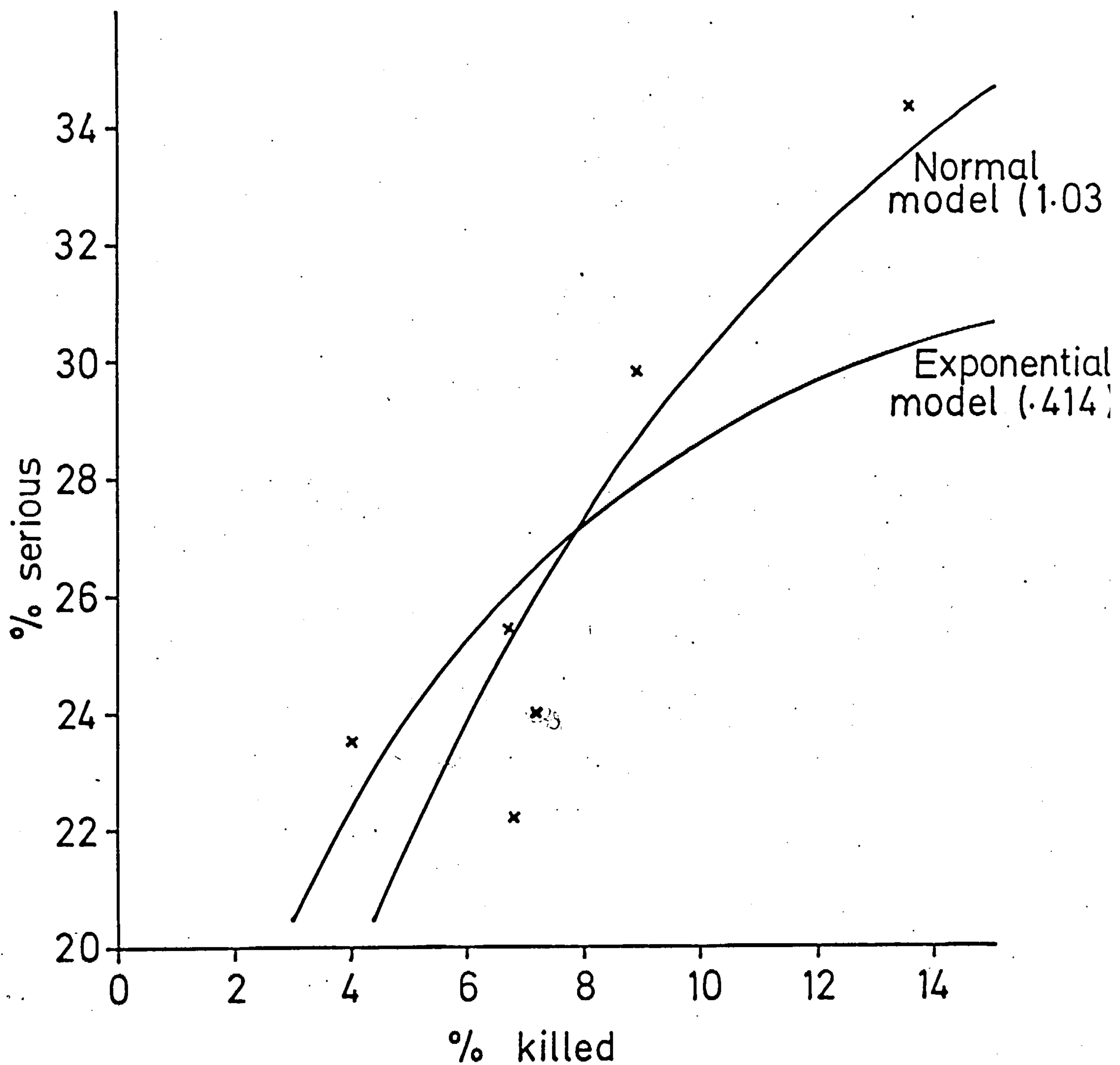


Figure 7.18: correlation between proportion killed and proportion seriously injured, the points corresponding to pedestrians in different age groups (Morocco, 1970)

7.6 Use of an apparent change in the classification of injury as a direct test of our model

A remarkable opportunity for a direct test of the theory discussed in this Chapter is given by the discovery by Satterthwaite (1975) of an apparent change in the classification of injury in one particular region of England (West Yorkshire) between 1968 and 1969. His data shows that there was a fall from 30% to 21% in the proportion of injury accidents classified as serious in this region, whereas in the rest of Great Britain there was a much smaller change in the opposite direction. At the same time the total number of accidents in that region remained much the same, as did the proportion of fatalities (see table 7.9). Mr. Satterthwaite has also provided me with data classifying accidents in West Yorkshire simultaneously according to severity and hour of day, in 1968 and 1969. If hour of day alters injury severity according to our exponential model, and the change in classification was a shift in the threshold along the severity axis from s_1 to s_2 , then the proportion severely injured (serious plus fatal) changes from $\exp(-\lambda_1 s_1)$ to $\exp(-\lambda_1 s_2)$ (i denoting hour of day, and λ_i being the parameter of the exponential distribution for that hour). So if x is the proportion severe in 1968 and y the proportion severe in 1969, $y = x^{s_2/s_1}$, and $\log(y)$ should be proportional to $\log(x)$. Figure 7.19 shows the extent to which this is true. A linear regression performed on the points resulted in an estimate of the intercept that was not significantly different from zero, thus providing support for our model.

	West Yorkshire		Rest of Great Britain	
	1968	1969	1968	1969
Fatal	249 (3.4%)	270 (3.7%)	6013 (2.3%)	6523 (2.6%)
Serious	2090 (29%)	1493 (21%)	69796 (27%)	71956 (28%)
Slight	4892 (68%)	5511 (76%)	180881 (70%)	175930 (69%)

Table 7.9: Showing the change in the proportion of road accidents classified as serious in West Yorkshire between the years 1968 to 1969.

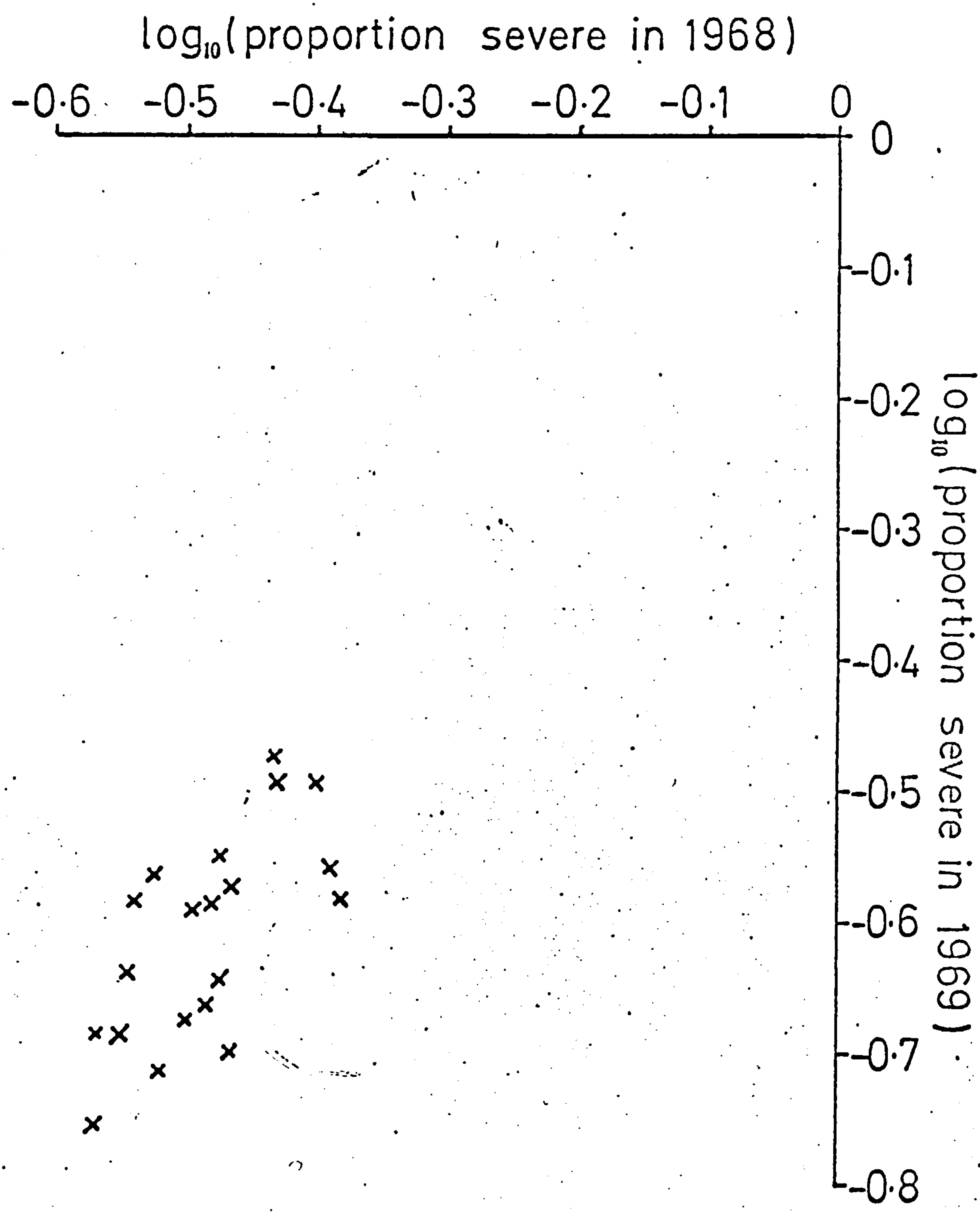


Figure 7.19: proportion of accidents classed as severe (i.e. serious or fatal) in West Yorkshire in 1968 and 1969, the points corresponding to different times of day

CHAPTER 8: INTRA-ACCIDENT CORRELATIONS OF DRIVER INJURY

8.1 Introduction

Consider accidents in which two vehicles collide. We may prepare a table of the injury to the driver of the first vehicle versus the injury to the driver of the second vehicle. When accidents are restricted to those in a particular mass ratio, it is found empirically that there is a positive correlation between the injuries to the two drivers, and it is this to which the title of this Chapter refers. (If account is not taken of the relative masses of the vehicles, the correlation may be obscured or reversed.) This correlation presumably arises because the relative velocity before impact is the same for both drivers in the one accident, but varies considerably between different accidents.

The data analysed in this Chapter was obtained as part of a study of the influence of mass ratio on injury severity. A conventional analysis of this is reported elsewhere (Grime and Hutchinson, 1976). If two vehicles of mass M_1 and M_2 respectively collide head-on at a relative velocity V , the changes in velocity which they undergo are $M_2 V / (M_1 + M_2)$ and $M_1 V / (M_1 + M_2)$. It is to be expected that the degree of injury sustained by an occupant would be strongly related to the change in velocity. To a first approximation, we may assume that V is independent of M_1 and M_2 . In this case some measure of the relative masses will be strongly associated with occupant injury (Grime, 1971; Kihlberg, Harragon, and Campbell, 1964). When we come to plot driver injury against mass ratio, the ratio M_1/M_2 could be used, but instead we shall use (mass of other vehicle)/(sum of masses of the two vehicles), since this is proportional to velocity change.

Tabulations of the injuries to the drivers have been obtained separately for all mass ratios: an example is given in table 8.1. This is restricted to head-on accidents in rural areas: rural areas being those where the speed limit is higher than 40 mph or absent; determination of whether an accident is head-on or not being from details collected by the Stats 19 procedure and recoded as described by Jones (1973, section 5.1.2). This table, as for the whole of this Chapter, refers to accidents in Great Britain in 1969-72. To save space, data for the whole range of mass ratios is presented in a different form, as table 8.2. Because information on non-injury accidents is not collected, the numbers of cases where both drivers escaped injury is not known: the bracketed figure in the final column of table 8.2 is the number of cases where both drivers were uninjured and at least one passenger was injured.

The conventional way of analysing data such as table 8.1 is to forget about the correlation between the injuries to the two drivers and work instead with the row and column marginal totals, calculating, for instance, the ratio of the number of accidents in which the driver of the lighter vehicle died, divided by the total number of injury accidents. This ratio can be obtained for each line of table 8.2, and plotted against mass ratio. While this procedure is perfectly acceptable as a first step, and illustrates clearly the great importance of the factor of mass ratio (as acting through velocity change), there are three ways in which it is somewhat unsatisfactory: (i) the information contained in the correlation of injuries is lost; (ii) the cases in which both drivers are uninjured are taken into account only by leaving them out entirely; (iii) there is a wide range of dependent variables that we could choose as our measure of severity - for instance, fatalities divided by the number of injuries, fatal plus serious

injuries divided by the number of injuries, injured divided by uninjured, and so on.

It was suggested in the previous Chapter, with theoretical and empirical support, that there exists a continuous scale of injury severity, and that the proportion of cases of severity greater than s is an appropriate measure of the average severity of the class of accidents under consideration; and the different measures corresponding to different choices of s are related to each other in a simple way that depends on the assumption made about the underlying distributions of severity. This will enable us to overcome point (iii) in the preceding paragraph: we shall take the probability of being killed (p) as our basic measure of severity; because of the simplicity of the resulting mathematics, we shall assume the exponential model of the previous Chapter, and therefore all alternative measures of severity will be of the form p^n .

To account for the correlation of injuries, we shall approximate the true distribution of relative speeds at impact by only two speeds, each associated with a different average level of injury.

This will enable us to calculate the probability of an observation falling in each of the cells of tables like 8.1, including that cell where both drivers are uninjured. By dividing the remaining probabilities by one minus the probability of both drivers escaping injury, we can get the probabilities we would expect to observe in reality.

Section 8.2 will specify the model in detail; section 8.3 will apply it to head-on accidents in rural areas, showing how different parameters are obtained for different mass ratios; section 8.4 will

extend the analysis to other types of accident; finally, section 8.5 discusses some possible extensions to this method of analysing injury data.

<u>Mass ratio</u>		<u>Driver of lighter vehicle</u>			
<u>.90 - .99</u>		<u>Fat</u>	<u>Ser</u>	<u>Sli</u>	<u>None</u>
Driver of heavier vehicle	Fat	12	34	7	3
	Ser	36	310	143	166
	Sli	10	143	423	496
	None	7	206	595	(698)

Table 8.1: Cross-tabulation of the injuries to the two drivers in head-on accidents in rural areas, mass ratio = .90 - .99.

8.2 Method of analysis

8.2.1 Basic model

Consider crashes in a particular range of mass ratios, say .90 to .99 when expressed as (mass of lighter vehicle)/(mass of heavier vehicle). Let us approximate the true distribution of relative impact speeds by supposing that a proportion f of collisions occur at V_1 and a proportion $(1-f)$ at V_2 (taken to be greater than V_1). Let the probability of the driver of the lighter vehicle dying be p_{1L} and p_{2L} at the two speeds, and the corresponding probabilities for the driver of the heavier vehicle be p_{1H} and p_{2H} (see table 8.3).

Following Chapter 7, we assume that the effect on severity of altering the velocity change at impact is effectively to change the parameter of an exponential distribution, so that the RIF curve when comparing any two velocity changes is a power curve, and therefore if the probability of death is p , then the probability of serious injury is $p^\alpha - p$, of slight injury is $p^\beta - p^\alpha$, and of no injury is $1 - p^\beta$, (where α and β are constants depending on the definitions of the different degrees of injury). Table 8.4 gives the probabilities of each degree of injury to each driver at each of the two speeds.

By taking a weighted average of what happens at each speed, we can calculate the relative frequencies of each cell in a table like 8.1:

the chance of both drivers being killed at an impact speed of V_1 is $p_{1L}p_{1H}$;

the chance of both drivers being killed at an impact speed of V_2 is $p_{2L}p_{2H}$;

therefore the overall chance of both drivers being killed is $f p_{1L} p_{1H} + (1-f) p_{2L} p_{2H}$.

Similarly, the chance of the driver of the lighter vehicle being killed and that of the heavier vehicle being seriously injured is $f p_{1L} (p_{1H}^{\alpha} - p_{1H}) + (1-f) p_{2L} (p_{2H}^{\alpha} - p_{2H})$, and the chance of both drivers being uninjured is $f(1 - p_{1L}^{\beta})(1 - p_{1H}^{\beta}) + (1-f)(1 - p_{2L}^{\beta})(1 - p_{2H}^{\beta})$.

The predicted probabilities of each combination of injuries is given in table 8.5.

We thus have 7 parameters to fit to the first set of data we consider: f , p_{1L} , p_{2L} , p_{1H} , p_{2H} , α , and β . The method of fitting used was to numerically minimise chi-squared, $\sum (O-E)^2/E$ where O is the observed number of cases in a particular cell, E is the predicted number of cases in that cell, and the summation is over the 15 cells for which we have complete data. (The minimisation of certain other functions is considered briefly in section 8.3.2.)

In reality, of course, there is a wide distribution of impact speeds. If

$\phi(v) dv$ = probability of the relative impact velocity being in the range $(v, v+dv)$,

$p(u)$ = probability of death at velocity change u ,

then for masses M_L , M_H ($M_L < M_H$) the first line in table 8.5 would become

$$\int_{-\infty}^{\infty} p\left(\frac{M_H v}{M_L + M_H}\right) p\left(\frac{M_L v}{M_L + M_H}\right) \phi(v) dv$$

and the other lines would be altered similarly.

8.2.2 Extension to other mass ratios

For a different mass ratio to that considered above, the two relative impact speeds will of course produce different velocity changes, which will be reflected in changes to the parameters p_{1L} , p_{2L} , p_{1H} , and p_{2H} . However, α and β will be the same since they only depend on the definitions of the degrees of injury severity. We shall also take f to remain constant, for two reasons: firstly, if the distribution of impact speeds is independent of mass ratio, this is a natural procedure to follow; and, secondly, it makes the interpretation of the resulting parameters so much easier.

8.2.3 The case of equal masses

When the two vehicles concerned are of the same mass, slightly different formulae have to be used, since the two vehicles are no longer distinguishable. Denoting by p_1 and p_2 the probabilities of death at the two speeds (being the same in each vehicle), the chance of both drivers being killed is $f p_1^2 + (1-f)p_2^2$, and the chance of one being killed and the other seriously injured is $2fp_1(p_1^\alpha - p_1) + 2(1-f)p_2(p_2^\alpha - p_2)$, and so on.

Speed:	V_1	V_2
Proportion of impacts:	f	$1-f$
Probability of death:		
lighter vehicle	P_{1L}	P_{2L}
heavier vehicle	P_{1H}	P_{2H}

Table 8.3: Basis of model of correlation.

Vehicle	Speed	Degree of injury to driver			
		Fatal	Serious	Slight	None
Lighter	V_1	P_{1L}	$P_{1L}^\alpha - P_{1L}$	$P_{1L}^\beta - P_{1L}^\alpha$	$1 - P_{1L}^\beta$
Lighter	V_2	P_{2L}	$P_{2L}^\alpha - P_{2L}$	$P_{2L}^\beta - P_{2L}^\alpha$	$1 - P_{2L}^\beta$
Heavier	V_1	P_{1H}	$P_{1H}^\alpha - P_{1H}$	$P_{1H}^\beta - P_{1H}^\alpha$	$1 - P_{1H}^\beta$
Heavier	V_2	P_{2H}	$P_{2H}^\alpha - P_{2H}$	$P_{2H}^\beta - P_{2H}^\alpha$	$1 - P_{2H}^\beta$

Table 8.4: Probabilities of each degree of injury for each combination of vehicle and speed.

	<u>Driver injury</u>		<u>Probability of this outcome</u>
	<u>Lighter vehicle</u>	<u>Heavier vehicle</u>	
1	Fat	Fat	$f p_{1L} p_{1H} + (1-f) p_{2L} p_{2H}$
2	Fat	Ser	$f p_{1L} (p_{1H}^{\alpha} - p_{1H}) + (1-f) p_{2L} (p_{2H}^{\alpha} - p_{2H})$
3	Fat	Sli	$f p_{1L} (p_{1H}^{\beta} - p_{1H}^{\alpha}) + (1-f) p_{2L} (p_{2H}^{\beta} - p_{2H}^{\alpha})$
4	Fat	None	$f p_{1L} (1 - p_{1H}^{\beta}) + (1-f) p_{2L} (1 - p_{2H}^{\beta})$
5	Ser	Fat	$f (p_{1L}^{\alpha} - p_{1L}) p_{1H} + (1-f) (p_{2L}^{\alpha} - p_{2L}) p_{2H}$
6	Ser	Ser	$f (p_{1L}^{\alpha} - p_{1L}) (p_{1H}^{\alpha} - p_{1H}) + (1-f) (p_{2L}^{\alpha} - p_{2L}) (p_{2H}^{\alpha} - p_{2H})$
7	Ser	Sli	$f (p_{1L}^{\alpha} - p_{1L}) (p_{1H}^{\beta} - p_{1H}^{\alpha}) + (1-f) (p_{2L}^{\alpha} - p_{2L}) (p_{2H}^{\beta} - p_{2H}^{\alpha})$
8	Ser	None	$f (p_{1L}^{\alpha} - p_{1L}) (1 - p_{1H}^{\beta}) + (1-f) (p_{2L}^{\alpha} - p_{2L}) (1 - p_{2H}^{\beta})$
9	Sli	Fat	$f (p_{1L}^{\beta} - p_{1L}^{\alpha}) p_{1H} + (1-f) (p_{2L}^{\beta} - p_{2L}^{\alpha}) p_{2H}$
10	Sli	Ser	$f (p_{1L}^{\beta} - p_{1L}^{\alpha}) (p_{1H}^{\alpha} - p_{1H}) + (1-f) (p_{2L}^{\beta} - p_{2L}^{\alpha}) (p_{2H}^{\alpha} - p_{2H})$
11	Sli	Sli	$f (p_{1L}^{\beta} - p_{1L}^{\alpha}) (p_{1H}^{\beta} - p_{1H}^{\alpha}) + (1-f) (p_{2L}^{\beta} - p_{2L}^{\alpha}) (p_{2H}^{\beta} - p_{2H}^{\alpha})$
12	Sli	None	$f (p_{1L}^{\beta} - p_{1L}^{\alpha}) (1 - p_{1H}^{\beta}) + (1-f) (p_{2L}^{\beta} - p_{2L}^{\alpha}) (1 - p_{2H}^{\beta})$
13	None	Fat	$f (1 - p_{1L}^{\beta}) p_{1H} + (1-f) (1 - p_{2L}^{\beta}) p_{2H}$
14	None	Ser	$f (1 - p_{1L}^{\beta}) (p_{1H}^{\alpha} - p_{1H}) + (1-f) (1 - p_{2L}^{\beta}) (p_{2H}^{\alpha} - p_{2H})$
15	None	Sli	$f (1 - p_{1L}^{\beta}) (p_{1H}^{\beta} - p_{1H}^{\alpha}) + (1-f) (1 - p_{2L}^{\beta}) (p_{2H}^{\beta} - p_{2H}^{\alpha})$
16	None	None	$f (1 - p_{1L}^{\beta}) (1 - p_{1H}^{\beta}) + (1-f) (1 - p_{2L}^{\beta}) (1 - p_{2H}^{\beta})$

Table 8.5: Predicted probabilities of each combination of injuries. If the observed frequencies are available only in 15 classes ("both uninjured" having been omitted) the predicted probabilities in rows 1 - 15 above should be divided by one minus the probability in row 16.

8.2.4 Modification for extreme disparities in masses

When the difference between the masses of the two vehicles is large, the probability of the driver of the heavier vehicle being killed becomes very small. To avoid the problem of having too-small an expectation in some of the cells, therefore, at high mass ratios the categories (driver of heavier vehicle killed) and (driver of heavier vehicle seriously injured) are combined. The probability of occurrence of this new category, expressed in terms of the parameters already being used is p_{1H}^{α} or p_{2H}^{α} as appropriate, and so, for instance, the probability of the driver of the lighter vehicle being killed and that of the heavier vehicle being killed or seriously injured is $f p_{1L} p_{1H}^{\alpha} + (1-f) p_{2L} p_{2H}^{\alpha}$. The mass ratios for which such combination of injury categories was performed will be distinguished in the tables of results.

8.3 Results: head-on accidents in rural areas

8.3.1 Data, and results of analysis

The basic data has been given in table 8.2. The full model (7 parameters) was fitted to the data for mass ratios .90 - .99, with the results of table 8.6.

Severity-definition
parameters:

$$\alpha = 0.0961$$

$$\beta = 0.0312$$

Speed:

V_1

V_2

Proportion of impacts: $f = 0.836$ $1-f = 0.164$

Probability of death:

lighter vehicle

$$p_{1L} = .17 \times 10^{-9}$$

$$p_{2L} = 0.124$$

heavier vehicle

$$p_{1H} = .17 \times 10^{-10}$$

$$p_{2H} = 0.106$$

Table 8.6: Results for the 7-parameter model fitted to the data of table 8.1.

Note that the chance of death is smaller in the heavier vehicle than in the lighter one, at both speeds, as it should be. Note also the wide difference in severities of the two speeds: in 84% of cases, there is virtually no chance of being killed, and a 50% chance of escaping without injury, whereas in the remaining 16% the chance of being killed is over 10% and of being at least seriously injured is over 80%. Table 8.7 gives the statistics corresponding to table 8.4 for the estimated values of f , p_{1L} , p_{2L} , p_{1H} , p_{2H} , α , and β . The probability of both drivers being uninjured is $(.836 \times .504 \times .539) + (.164 \times .063 \times .068) =$

<u>Vehicle</u>	<u>Speed</u>	<u>Degree of injury to driver</u>			
		<u>Fatal</u>	<u>Serious</u>	<u>Slight</u>	<u>None</u>
Lighter	V_1	$.17 \times 10^{-9}$.115	.381	.504
Lighter	V_2	.124	.694	.119	.063
Heavier	V_1	$.17 \times 10^{-10}$.092	.369	.539
Heavier	V_2	.106	.700	.126	.068

Table 8.7: Estimated probabilities of each degree of injury for each combination of vehicle and speed, for the data of table 8.1.

This model was then extended to other mass ratios, without changing f , α , and β , with results as in table 8.8. Notice that the chance of death increases with increasing relative velocity change at both speeds. It is obvious from the raw data that the overall likelihood of death increases: but there is nothing in the model to compel these likelihoods to increase at both speeds - one could even decrease provided the other increased sufficiently. That both do increase, therefore, is something of a triumph for the model. Similarly, although it isn't built into the model, the severity at both speeds is greater for the driver of the lighter vehicle than for that of the heavier vehicle. The results for the higher speed are illustrated in figures 8.1 and 8.2. The horizontal axis on both these is (mass of other vehicle)/(mass of own vehicle + mass of other vehicle), i.e. relative velocity change. Each line of table 8.8 is thus represented twice: p_{2L} at (mass of heavier vehicle)/(sum of the vehicle masses), and p_{2H} at (mass of lighter vehicle)/(sum of the vehicle masses). The quantity plotted in figure 8.1 is the chance of being killed, and in figure 8.2 it is this chance raised to the power 0.0961, i.e. the chance

Mass ratio	Estimated likelihood of being killed				χ^2
$\left(\frac{\text{Lighter}}{\text{Heavier}}\right)$	Lighter vehicle		Heavier vehicle		(d.f.)
	V_1	V_2	V_1	V_2	
	(p_{1L})	(p_{2L})	(p_{1H})	(p_{2H})	
1.00	$.21 \times 10^{-9}$.14	*	*	52 (6)
.90 - .99	$.17 \times 10^{-9}$.12	$.17 \times 10^{-10}$.11	17 (7)
.80 - .89	$.11 \times 10^{-8}$.12	$.15 \times 10^{-12}$.085	51 (10)
.70 - .79	$.83 \times 10^{-8}$.16	$.98 \times 10^{-12}$.086	51 (10)
.60 - .69	$.48 \times 10^{-8}$.16	$.23 \times 10^{-14}$.070	163 (10)
.50 - .59	$.73 \times 10^{-8}$.29	$.20 \times 10^{-15}$.021	105 (10)
.40 - .49/	$.10 \times 10^{-8}$.21	$.23 \times 10^{-15}$	$.10 \times 10^{-2}$	70 (6)
.30 - .39/	$.73 \times 10^{-8}$.32	$.11 \times 10^{-21}$	$.15 \times 10^{-3}$	30 (6)
.20 - .29/	$.13 \times 10^{-7}$.24	$.97 \times 10^{-32}$	$.93 \times 10^{-6}$	48 (6)
.10 - .19/	$.12 \times 10^{-8}$.35	$.54 \times 10^{-49}$	$.10 \times 10^{-9}$	108 (6)
.01 - .09/	$.43 \times 10^{-6}$.47	$.61 \times 10^{-48}$	$.92 \times 10^{-10}$	217 (6)

* Both vehicles the same at this mass ratio.

/ For these mass ratios, the abbreviated model in which the fatal and serious categories were combined for the driver of the heavier vehicle was used.

Table 8.8: Results for head-on accidents in rural areas, taking $\alpha = .0961$, $\beta = .0312$, and $f = .836$.

HEAD-ON RURAL

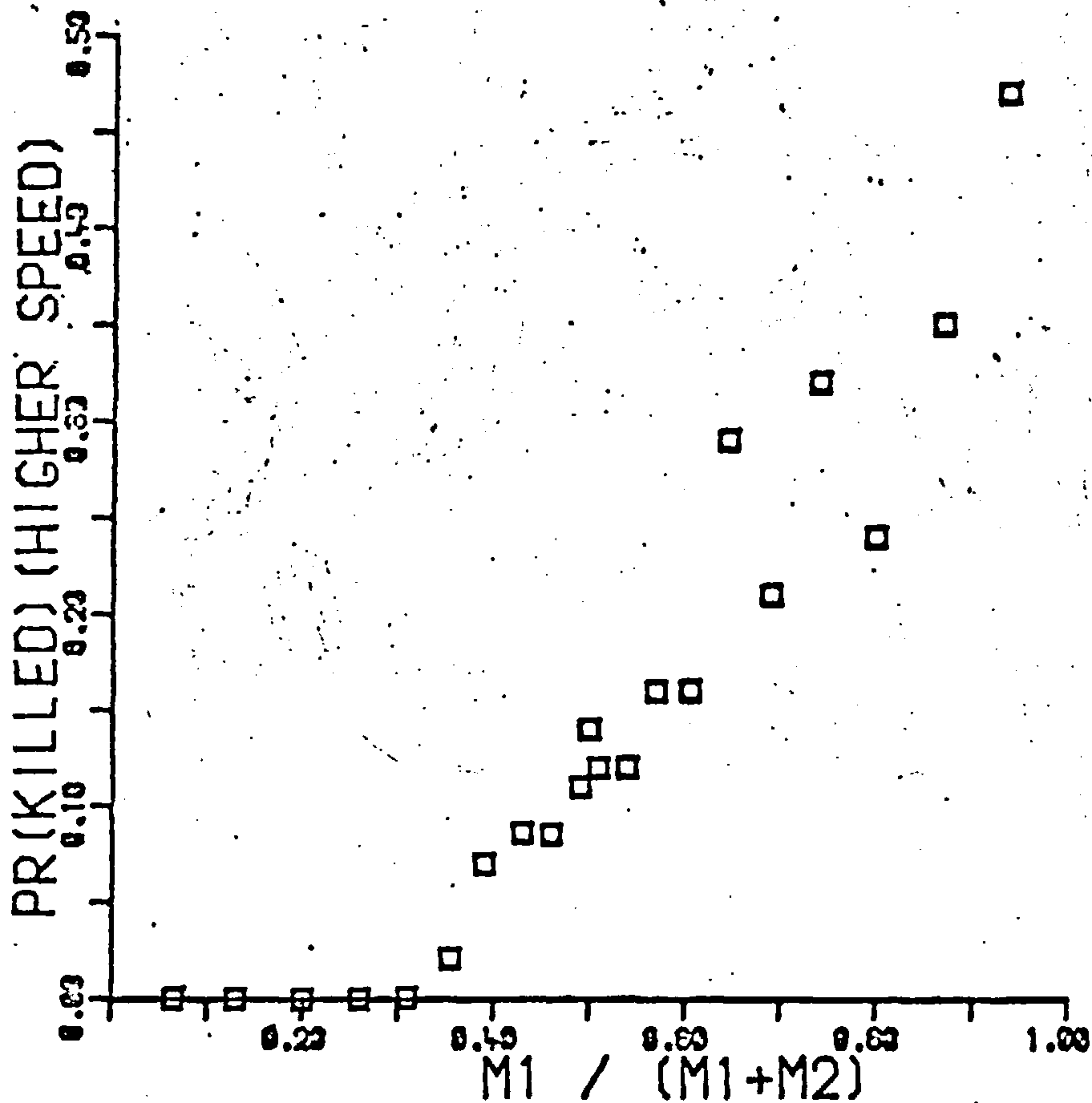


Figure 8.1: Head-on accidents in rural areas: relation of probability of being killed at the higher speed to mass ratio. The horizontal axis is proportional to velocity change, assuming relative velocity of impact is independent of mass ratio.

HEAD-ON RURAL

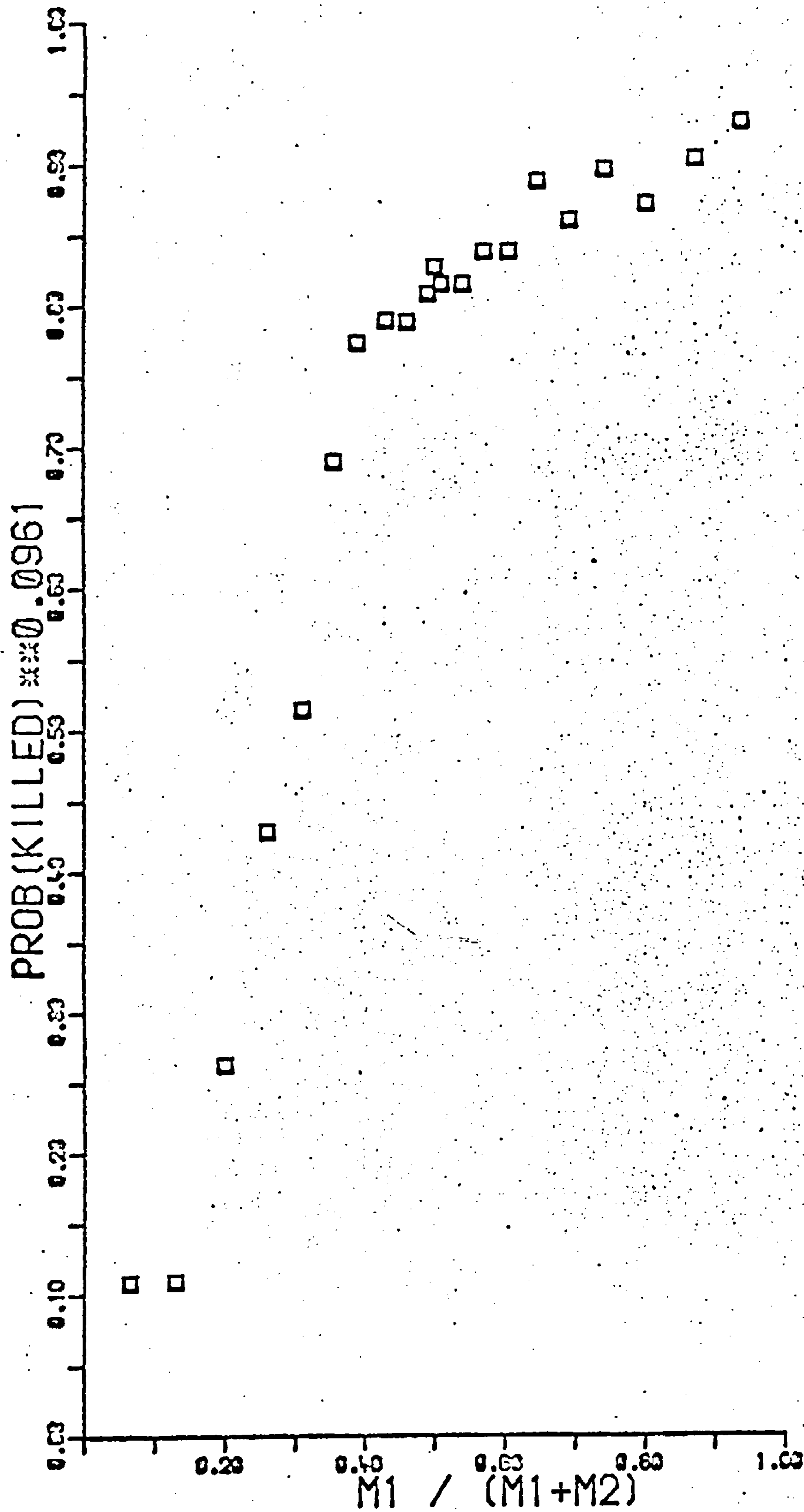


Figure 8.2: As figure 8.1, but with the probability of being either killed or seriously injured replacing the probability of being killed.

of being killed or seriously injured.

In figure 8.3 the measure of severity plotted is the logarithm of the probability of being killed, which enables most of the parameters for the lower speed to be included. The horizontal axis on this figure is also logarithmic. The reason for this is explained below.

One of our basic assumptions is that the probability of death is a function of the change of velocity. If our model was exactly correct, in that only two impact speeds are possible, then the speed change of the heavier vehicle at the higher speed may be similar to that of the lighter vehicle at the lower impact speed. That is, since equality of probabilities of death implies equality of changes in velocity,

$$\frac{M_1}{M_1 + M_2} V_2 = \frac{M'_1}{M'_1 + M'_2} V_1$$

This is equivalent to saying that, for a given value of probability of death,

$$\log \frac{M_1}{M_1 + M_2} - \log \frac{M'_1}{M'_1 + M'_2} = \log V_1 - \log V_2 = \log(V_1/V_2)$$

In figure 8.3 therefore we plot the logarithm of the mass ratio as the horizontal axis. We see that the chance of being killed can be similar at the two speeds - in the range 10^{-6} to 10^{-11} of the chance of being killed. The two lines in figure 8.3 appear approximately parallel in the sense of being separated by a constant horizontal distance, and this distance is an estimate of the logarithm of the ratio of the two speeds.

From figure 8.3 this distance can be seen to be about 0.72 log

HEAD-ON RURAL

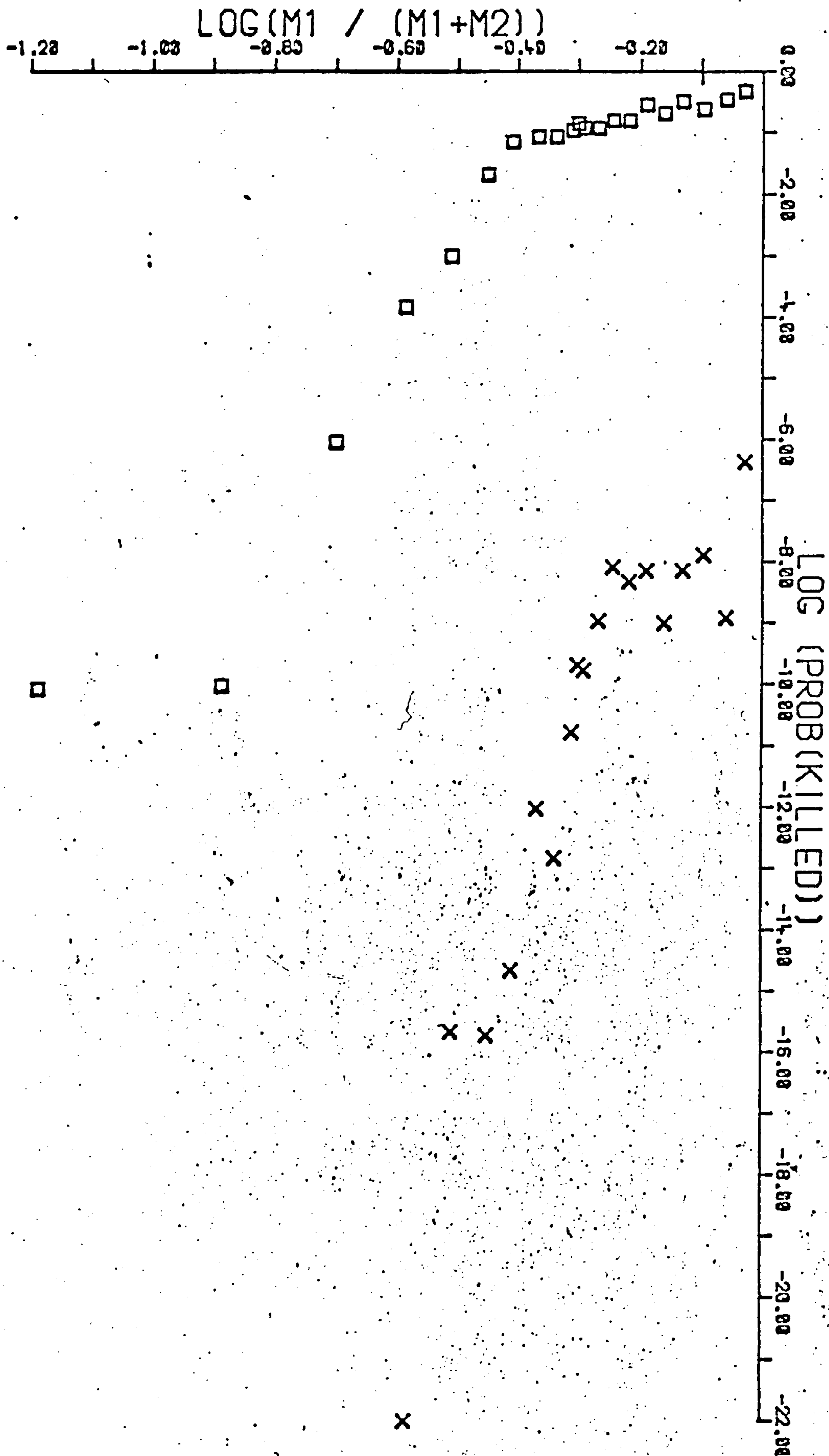


Figure 8.3: As figure 8.1, but with both axes logarithmic, and including both speeds.

units, corresponding to the speeds being in the ratio 1 to 5.25.

(This is further discussed in section 8.5, after the results for other types of accident have been obtained.)

Now, if 84% of collisions occur at speed V_1 , and 16% at $5.25V_1$, the average speed of collisions is $1.68V_1$, and the standard deviation is $1.56V_1$. Therefore the coefficient of variation of collision speeds is $1.56/1.68 = 0.93$. It is remarkable that we have obtained this estimate of the variability of speeds of collision without using any direct data on speeds. There is probably not much point in comparing it with speed distributions obtained from detailed on-the-spot accident studies (such as those of Langwieder (1973) or Mackay (1973)), because these have nearly always been of a selected sample, and ours refers only to head-on accidents in rural areas, but it certainly is not an unreasonable figure. However, it is not clear to what extent this figure is dependent on the particular variant of our basic model that we have chosen to use - the exponential model as opposed to the Normal or one of the other variants discussed in the previous Chapter, and the use of only two speeds rather than a more realistic continuous distribution.

8.3.2 Alternative minimands

We could also have chosen certain other functions to minimise when fitting the parameters to the data. The formula used was $\chi^2 = \sum (O-E)^2/E$. Table 8.9 compares the parameters found when minimising two other χ^2 statistics, $\sum (O-E)^2/O$ and $\sum 2(O) \log_e (O/E)$. All are very similar to those found with the original formula. (There is substantial variation in p_{1L} and p_{1H} , but since these are effectively zero when considering the chance of death, their importance lies in the chances of serious and slight injury that are associated with them; and there is little difference between $(10^{-9}) \cdot 0.0961 = .136$ and $(5 \times 10^{-10}) \cdot 0.0961 = .128$ or between $(10^{-9}) \cdot 0.0312 = .524$ and $(5 \times 10^{-10}) \cdot 0.0312 = .488$.)

Formula of minimand	Minimum χ^2 (7 d.f.)	Parameters						
		α	β	p_{1L}	p_{2L}	p_{1H}	p_{2H}	f
$\sum (O-E)^2/E$	17.0	.0961	.0312	$.17 \times 10^{-9}$.124	$.17 \times 10^{-10}$.106	.836
$\sum (O-E)^2/O$	15.2	.0924	.0300	$.60 \times 10^{-10}$.107	$.52 \times 10^{-11}$.098	.838
$\sum 2(O) \log_e (O/E)$	16.6	.0958	.0311	$.15 \times 10^{-9}$.117	$.15 \times 10^{-10}$.103	.836

Table 8.9: Comparison of parameters found when analysing the data of table 8.1 using alternative minimands.

8.3.3 Comments on goodness-of-fit

Before going on to the corresponding results for other types of accident, some mention should be made of the goodness-of-fit of our model. One criterion of this is χ^2 . In this respect the model clearly fails since so many of the values in table 8.8 are so highly statistically significant. But it should be remembered that the number of observations are very large, and therefore a statistically significant χ^2 could result from quite small imperfections in the model. Moreover, in a sense we don't need χ^2 to tell us that our model is imperfect, since our use of only two impact speeds is obviously unrealistic - the only thing it is less unrealistic than is the conventional omission to model the impact speed variation in any way. χ^2 should always be assessed, therefore, bearing in mind the total number of observations. The ratio of χ^2 to the total number of observations is around 0.06, and in figure 8.4 it is plotted against mass ratio. (The cases mass ratio = 1 (for which the model is different since the two vehicles are indistinguishable) and mass ratio = .90-.99 (to which 7 parameters were fitted) have been omitted from this figure.) It can be seen to be higher when the vehicles are very different in mass (i.e. predominately car/lorry collisions). This may be because the value of f which was obtained from the data for mass ratio .90-.99 is not realistic for car/lorry crashes: in other words, the distribution of impact speeds is not entirely independent of mass ratio, which is plausible.

Another possible way of assessing the model is to plot, for a particular mass ratio, the predicted probabilities of each combination of injuries against the actual probabilities. Figure 8.5 exemplifies this for one of the worst cases - mass ratio .01-.09. In assessing this figure, it should be remembered that the total number of cases here

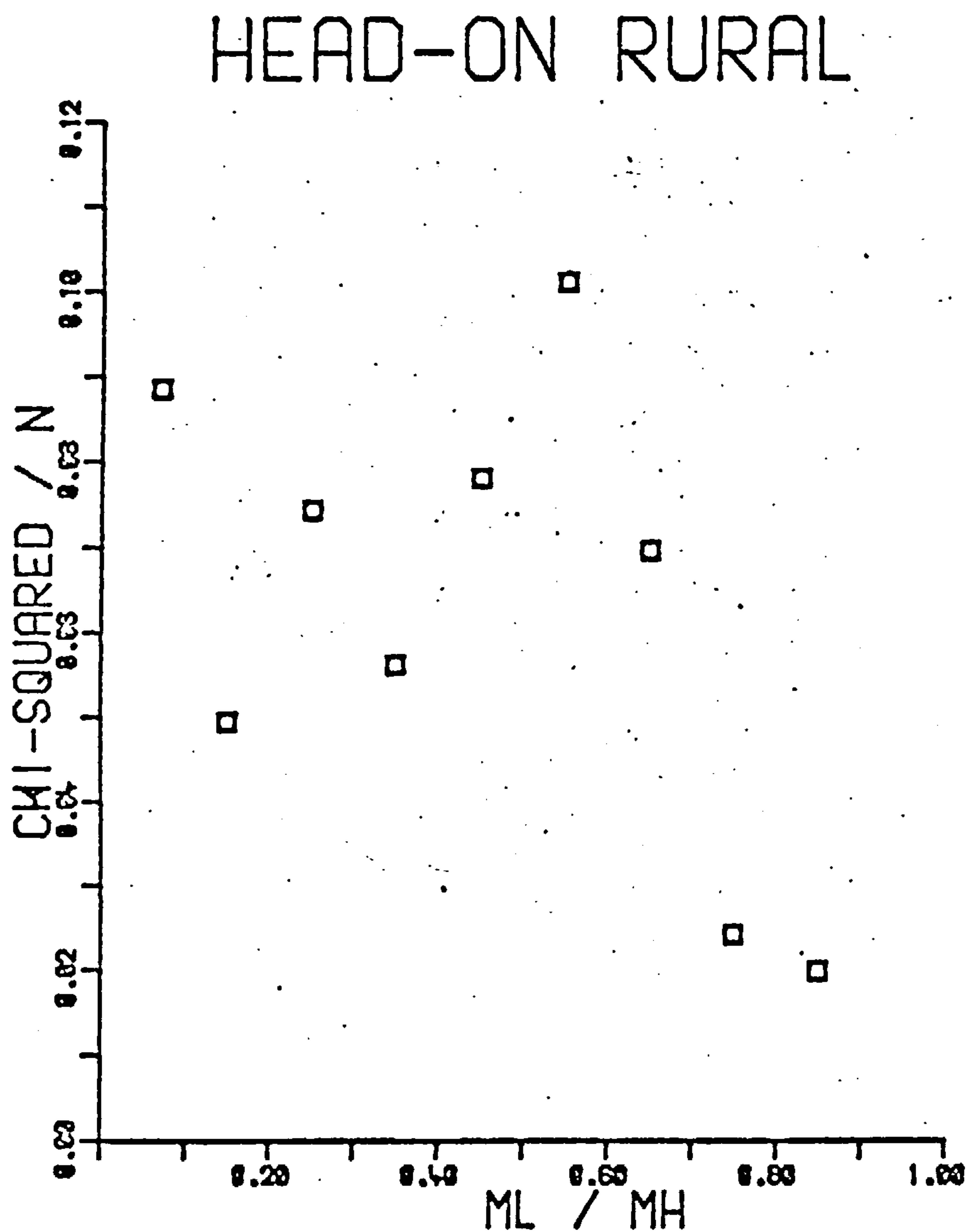


Figure 8.4: Goodness-of-fit of the model as applied to head-on accidents in rural areas: ratio of χ^2 to the total number of cases versus mass ratio.

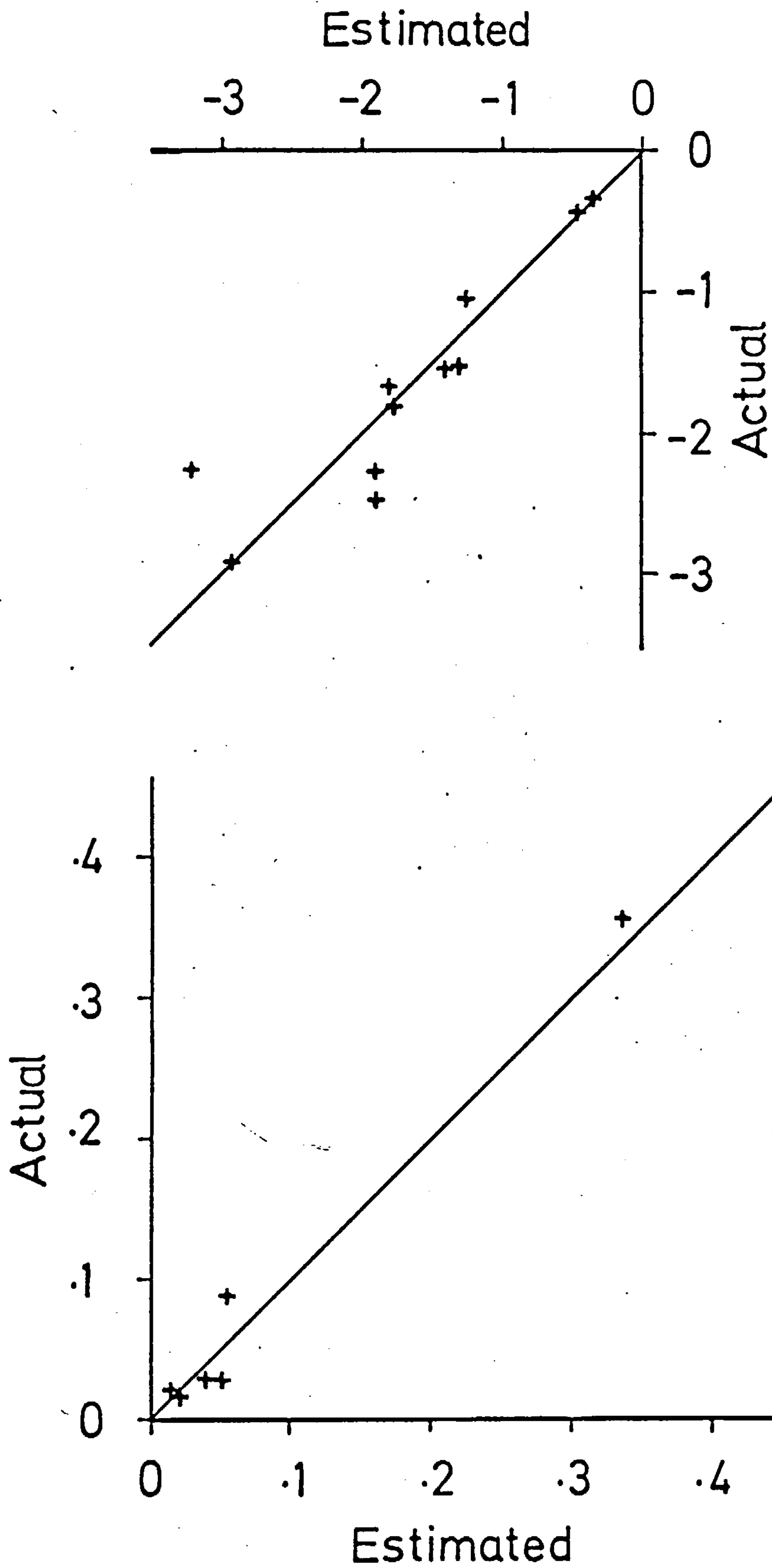


Figure 8.5: Head-on accidents in rural areas, mass ratio .01-.09: correlation of estimated to actual probabilities of each combination of injuries (top, log-log plot; bottom, linear-linear plot).

is about 2500, so that a probability of 0.01 means an expected number of 25, and a probability of 0.001 means an expected number of 2.5 cases.

It is now relevant to make a general criticism of the approach followed in this Chapter: namely, that rather than test its individual elements, the model as a whole has been tested. Although this is inevitable from the nature of the data available, it does have the unfortunate consequence that those parts of it which are inadequate are not known. In as much as we are primarily interested in the determination of the relevant parameters rather than testing a scientific theory of the underlying mechanism of a process (which is what Broadbent (1971, p.460) was chiefly concerned with when making similar comments), the force of this criticism is reduced. And we can begin to give a direct answer, by considering how two alternative models fare. The first of these retains the exponential model but assumes independence between the injuries to the two drivers, i.e. only one impact speed rather than two. Thus, writing p_L and p_H for the probabilities of death in the two vehicles, the chance of both drivers dying is $p_L p_H$, the chance of the driver of the lighter vehicle dying and that of the heavier vehicle being seriously injured is $p_L (p_H^\alpha - p_H)$, and so on. Thus this model has four parameters - p_L , p_H , α , and β - for the first set of data to which it is fitted, and two thereafter, since α and β remain the same. The second alternative drops the exponential model, allowing the probabilities of the four levels of injury to be independent. Calling these p , q , r , and $1-p-q-r$, with subscripts L and H to denote the vehicle, the chance of the driver of the lighter vehicle dying and that of the heavier vehicle being seriously injured is $p_L q_H$, and so on. This model has six parameters for each mass ratio.

These two models have been fitted to the data for mass ratios

.90-.99, and .80-.89, and the values of χ^2 obtained are given below:

exponential model, one speed:	.90 - .99	$\chi^2 = 511$	(four parameters)
	.80 - .89	$\chi^2 = 516$	(two parameters)
unconstrained model, one speed:	.90 - .99	$\chi^2 = 500$	(six parameters)
	.80 - .89	$\chi^2 = 442$	(six parameters)

It is clear that the fit is very little better when the probabilities of the four degrees of injury are unconstrained than when they are connected by the exponential model. Indeed, if the ratio of χ^2 to degrees of freedom is used as the criterion of goodness of fit (analogously to the F-ratio), this is actually better for the constrained model (51.1 instead of 62.5 for mass ratio .90-.99, and 43.0 instead of 55.2 for mass ratio .80-.89).

It is also obvious that having two speeds rather than one very much improves the fit, since χ^2 was 17 (7 degrees of freedom) and 51 (10 degrees of freedom) for these two mass ratios (table 8.8).

8.4 Results: other types of accident

Similar results have been obtained for head-on accidents in urban areas, intersection accidents in rural areas, and intersection accidents in urban areas.

The basic data for head-on accidents in urban areas is given in table 8.10, and the results in table 8.11 and figures 8.6 and 8.7. Parameters α and β remained fixed at the values found for the rural data, since definitions of degrees of injury ought not to be different in town than from country; since the distribution of speeds might well be different, f was fitted to the data for mass ratio .90-.99 as well as the parameters p_{1L} , p_{2L} , p_{1H} , and p_{2H} . f was found to be .894 and was left constant for the other mass ratios. Though there appears to be somewhat more scatter in figures 8.6 and 8.7 than in 8.1 and 8.3, it is clear there is a steadily increasing severity with mass ratio at both speeds, and the driver of the lighter vehicles comes off worse at both speeds.

Turning now to intersection accidents, it should first be noted that there is an inherent asymmetry between the two vehicles that does not occur in head-on accidents: even if they were of equal mass, it might be that the driver suffering a frontal impact is injured to a different degree to the driver suffering a side impact. Furthermore, impacts into the driver's side of the car or into the opposite side may differ. (See figure 8.8.) Unfortunately, it has not proved possible to decide from the national accident tapes which was the striking vehicle, and which the struck, because of the poor coding of the "vehicle damage" variables. However, it was thought worthwhile making an analysis which did not distinguish between them because Grime and

Mass ratio (<u>Lighter</u> <u>Heavier</u>)	Estimated likelihood of being killed				χ^2
	Lighter vehicle		Heavier vehicle		(d.f.)
	V_1 (p_{1L})	V_2 (p_{2L})	V_1 (p_{1H})	V_2 (p_{2H})	
1.00	$.20 \times 10^{-11}$.12	*	*	39 (6)
.90 - .99	$.87 \times 10^{-11}$.075	$.13 \times 10^{-11}$.058	22 (9)
.80 - .89	$.73 \times 10^{-10}$.10	$.51 \times 10^{-13}$.017	36 (10)
.70 - .79	$.59 \times 10^{-9}$.11	$.17 \times 10^{-13}$.071	57 (10)
.60 - .69	$.22 \times 10^{-8}$.21	$.27 \times 10^{-16}$.041	120 (10)
.50 - .59 ⁺	$.83 \times 10^{-9}$.14	$.52 \times 10^{-17}$	$.40 \times 10^{-2}$	31 (6)
.40 - .49 ⁺	$.96 \times 10^{-9}$.22	$.25 \times 10^{-20}$	$.18 \times 10^{-2}$	14 (6)
.30 - .39 ⁺	$.12 \times 10^{-8}$.18	$.17 \times 10^{-21}$	$.50 \times 10^{-5}$	24 (6)
.20 - .29 ⁺	$.69 \times 10^{-9}$.22	$.21 \times 10^{-32}$	$.56 \times 10^{-5}$	43 (6)
.10 - .19 ⁺	$.28 \times 10^{-7}$.36	$.62 \times 10^{-46}$	$.23 \times 10^{-7}$	157 (6)
.01 - .09 ⁺	$.86 \times 10^{-8}$.49	$.69 \times 10^{-57}$	$.24 \times 10^{-9}$	190 (6)

* Both vehicles the same at this mass ratio.

⁺ For these mass ratios, the abbreviated model in which the fatal and serious categories were combined for the driver of the heavier vehicle was used.

Table 8.11: Results for head-on accidents in urban areas, taking $\alpha = .0961$, $\beta = .0312$, and $f = .894$.

HEAD-ON URBAN

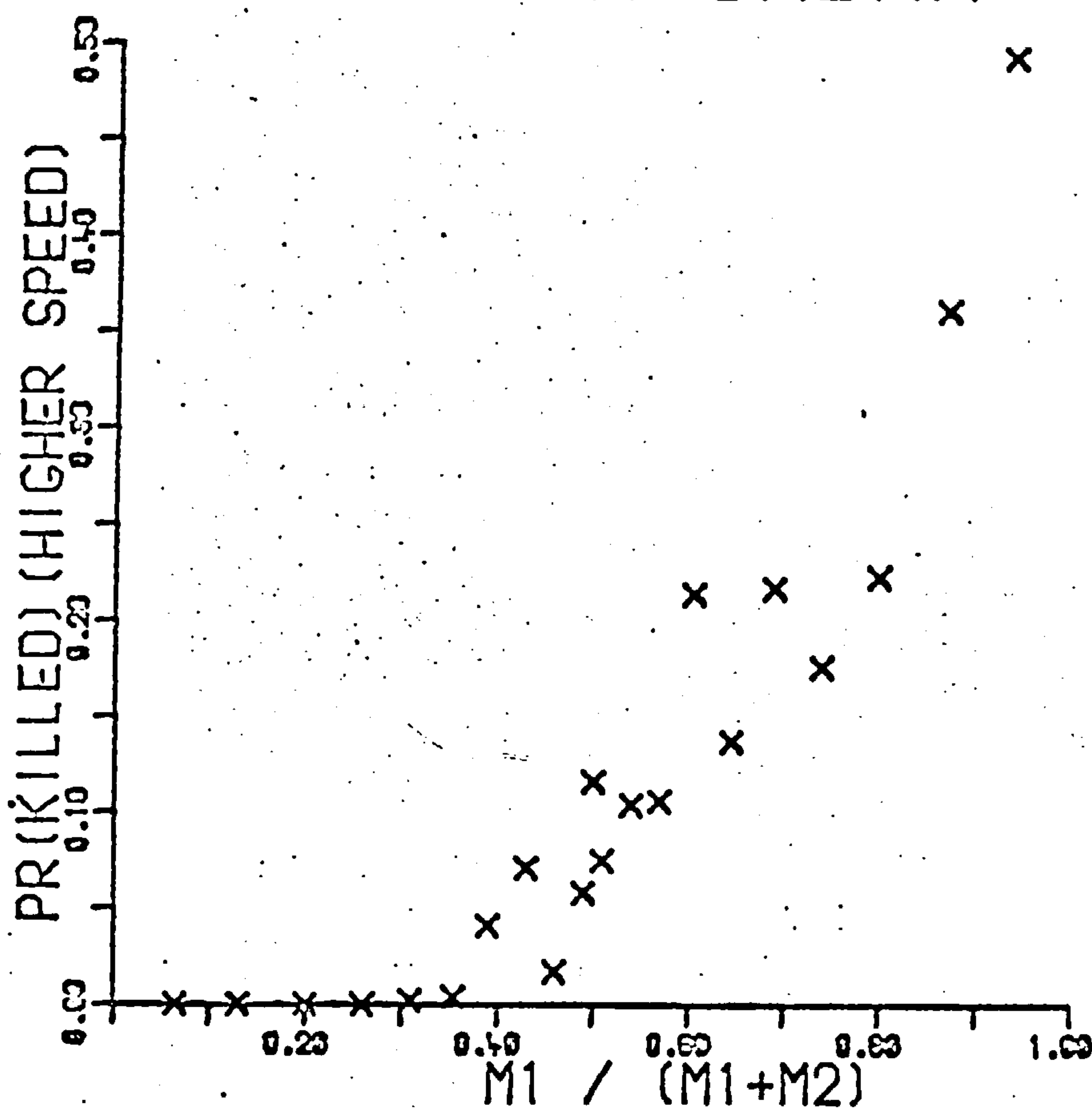


Figure 8.6: As figure 8.1, but for head-on accidents in urban areas.

HEAD-ON URBAN

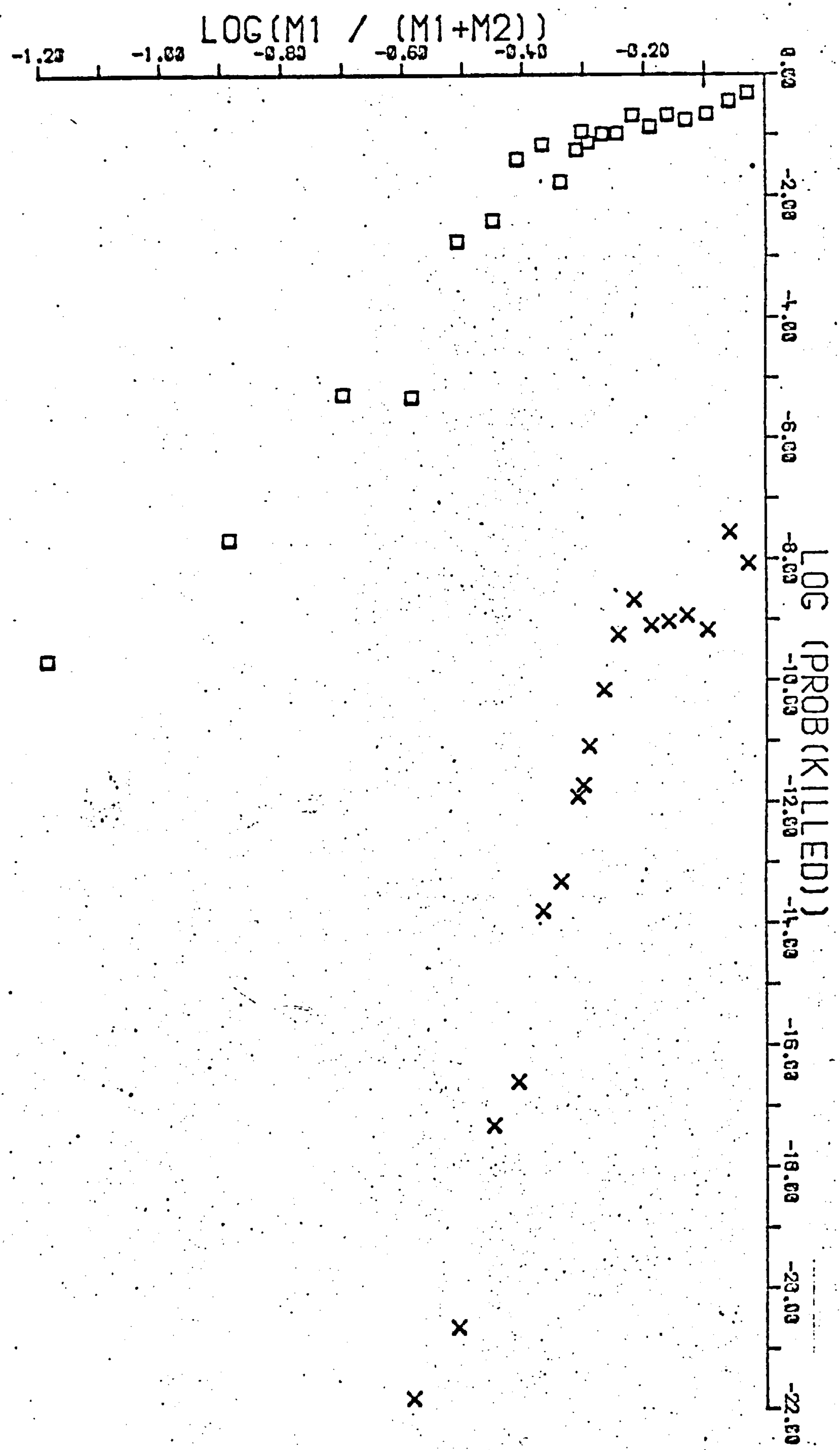
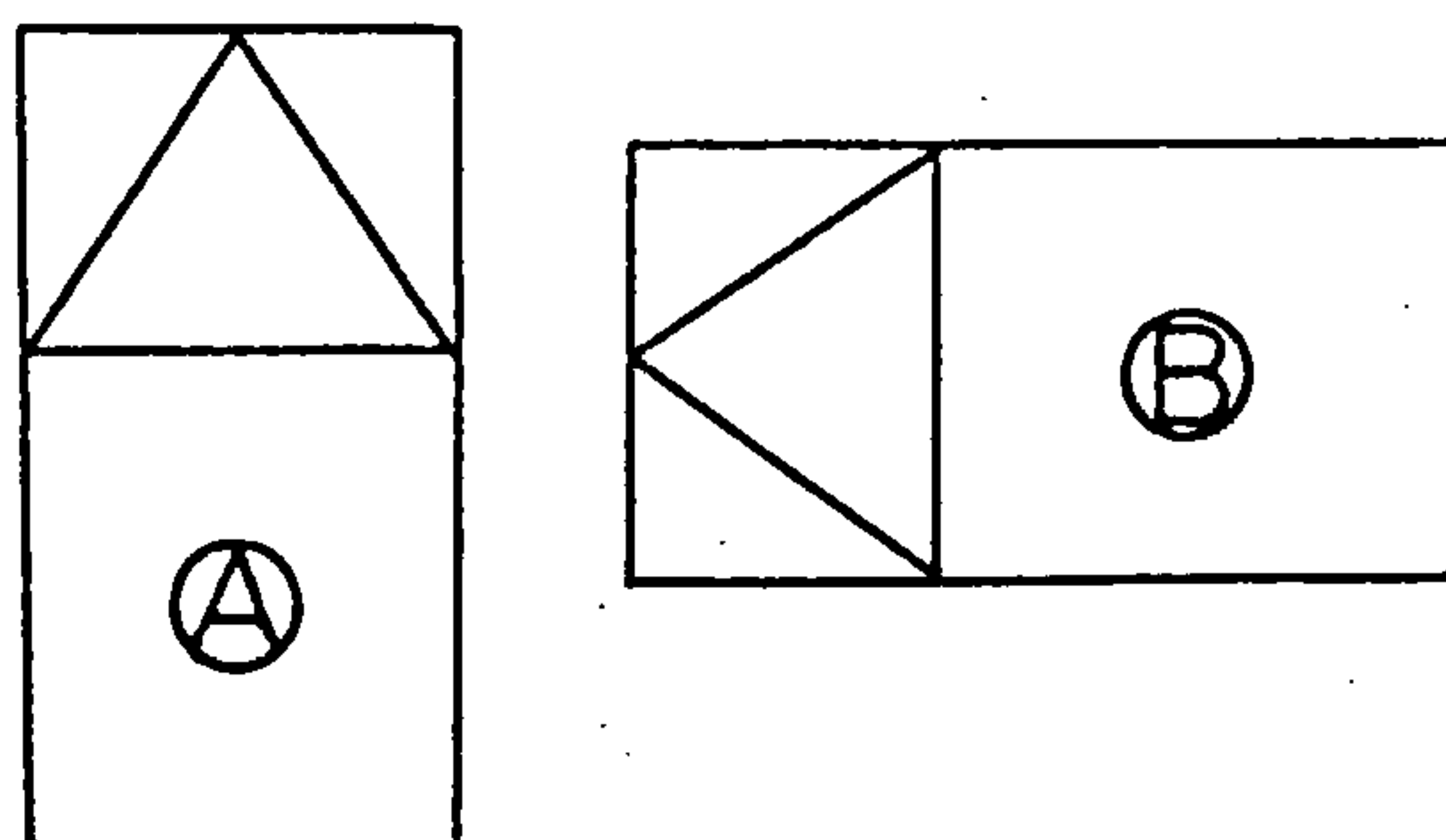


Figure 8.7: As figure 8.3, but for head-on accidents in urban areas.



The driver of the striking car (B) receives a frontal impact, whereas the driver of the struck car (A) receives a side impact. Furthermore, the situation for the driver of the struck car is different in this situation from the one below, where the impact is to the nearside of the car.

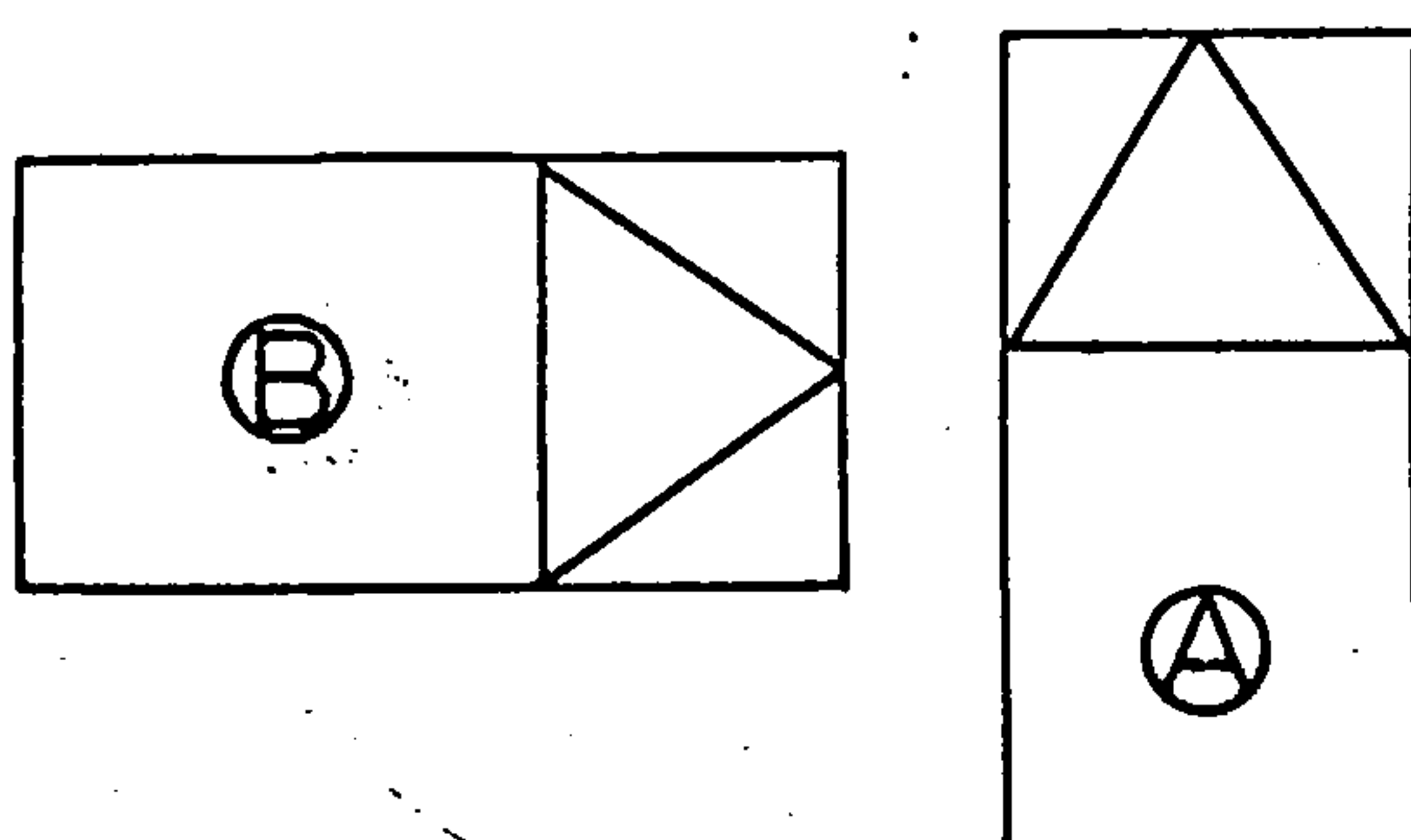


Figure 8.8: illustration of the asymmetry of the intersection accidents.

Jones (1973) found the average severities in the struck and the striking vehicle were very similar. Also, comparing tables 5.10 and 5.12, 42% of drivers suffering frontal impacts had head injury, as opposed to 38% of drivers having side impacts.

The data for intersection accidents in rural areas is given in table 8.12, and the results of the analysis in table 8.13 and figures 8.9 and 8.10. Again parameters α and β remained unaltered, five parameters (including f) were fitted to the data for mass ratio .90-.99, and thereafter f was fixed also.

Tables 8.14 and 8.15 and figures 8.11 and 8.12 give the corresponding figures for intersection accidents in urban areas.

Driver of lighter vehicle: Fat Fat Fat Fat Ser Ser Ser Ser Sli Sli Sli Sli None None None None

Driver of heavier vehicle: Fat Ser Sli None Fat Ser Sli None Fat Ser Sli None Fat Ser Sli None

Mass ratio:

1.00*	0	4	3	4	41	63	82	86	324				(170)
.90 - .99	0	5	5	3	7	154	114	176	6	67	286	485	6 130 431 (628)
.80 - .89	2	7	6	9	5	108	74	217	3	75	311	546	5 80 353 (651)
.70 - .79	2	5	8	6	2	97	92	179	1	44	184	565	1 65 225 (557)
.60 - .69	1	3	10	11	2	63	96	194	5	37	178	567	4 57 205 (444)
.50 - .59	0	3	7	7	2	37	39	128	1	18	81	298	2 31 87 (208)
.40 - .49	0	2	3	5	0	19	30	85	1	12	51	237	0 19 59 (147)
.30 - .39	0	1	2	5	1	14	18	47	0	7	25	141	0 12 27 (77)
.20 - .29	0	0	2	2	0	5	18	55	0	3	20	129	0 5 15 (87)
.10 - .19	0	3	6	34	1	9	19	286	0	2	16	455	0 4 24 (150)
.01 - .09	0	3	9	42	0	4	20	293	0	2	11	423	0 9 14 (147)

* Both vehicles the same at this mass ratio. The number under (Fat, Ser), for instance, is the total number of cases in which one driver was killed and the other seriously injured.

✗ The number of cases in which both drivers were uninjured is that in which one or more passengers were injured.

Table 8.12: Data on driver injuries in intersection accidents in rural areas, in Great Britain, 1969-72.

Mass ratio (<u>Lighter</u>) (<u>Heavier</u>)	Estimated likelihood of being killed				χ^2
	<u>Lighter vehicle</u>		<u>Heavier vehicle</u>		(d.f.)
	V_1 (p_{1L})	V_2 (p_{2L})	V_1 (p_{1H})	V_2 (p_{2H})	
1.00	$.17 \times 10^{-11}$.073	*	*	17 (6)
.90 - .99	$.98 \times 10^{-10}$.056	$.21 \times 10^{-11}$.084	46 (9)
.80 - .89	$.39 \times 10^{-9}$.12	$.35 \times 10^{-13}$.070	84 (10)
.70 - .79	$.36 \times 10^{-9}$.11	$.62 \times 10^{-16}$.025	65 (10)
.60 - .69	$.15 \times 10^{-8}$.17	$.25 \times 10^{-16}$.073	138 (10)
.50 - .59 ⁺	$.27 \times 10^{-8}$.15	$.37 \times 10^{-16}$	$.35 \times 10^{-3}$	67 (6)
.40 - .49 ⁺	$.40 \times 10^{-9}$.10	$.54 \times 10^{-19}$	$.11 \times 10^{-3}$	48 (6)
.30 - .39 ⁺	$.39 \times 10^{-9}$.17	$.13 \times 10^{-18}$	$.15 \times 10^{-3}$	44 (6)
.20 - .29 ⁺	$.20 \times 10^{-8}$.07	$.10 \times 10^{-29}$	$.21 \times 10^{-4}$	17 (6)
.10 - .19 ⁺	$.68 \times 10^{-7}$.31	$.94 \times 10^{-47}$	$.80 \times 10^{-9}$	47 (6)
.01 - .09 ⁺	$.18 \times 10^{-6}$.36	$.12 \times 10^{-53}$	$.49 \times 10^{-6}$	123 (6)

* Both vehicles the same at this mass ratio.

⁺ For these mass ratios, the abbreviated model in which the fatal and serious categories were combined for the driver of the heavier vehicle was used.

Table 8.13: Results for intersection accidents in rural areas, taking $\alpha = .0961$, $\beta = .0312$, and $f = .890$.

INTERSECTION, RURAL

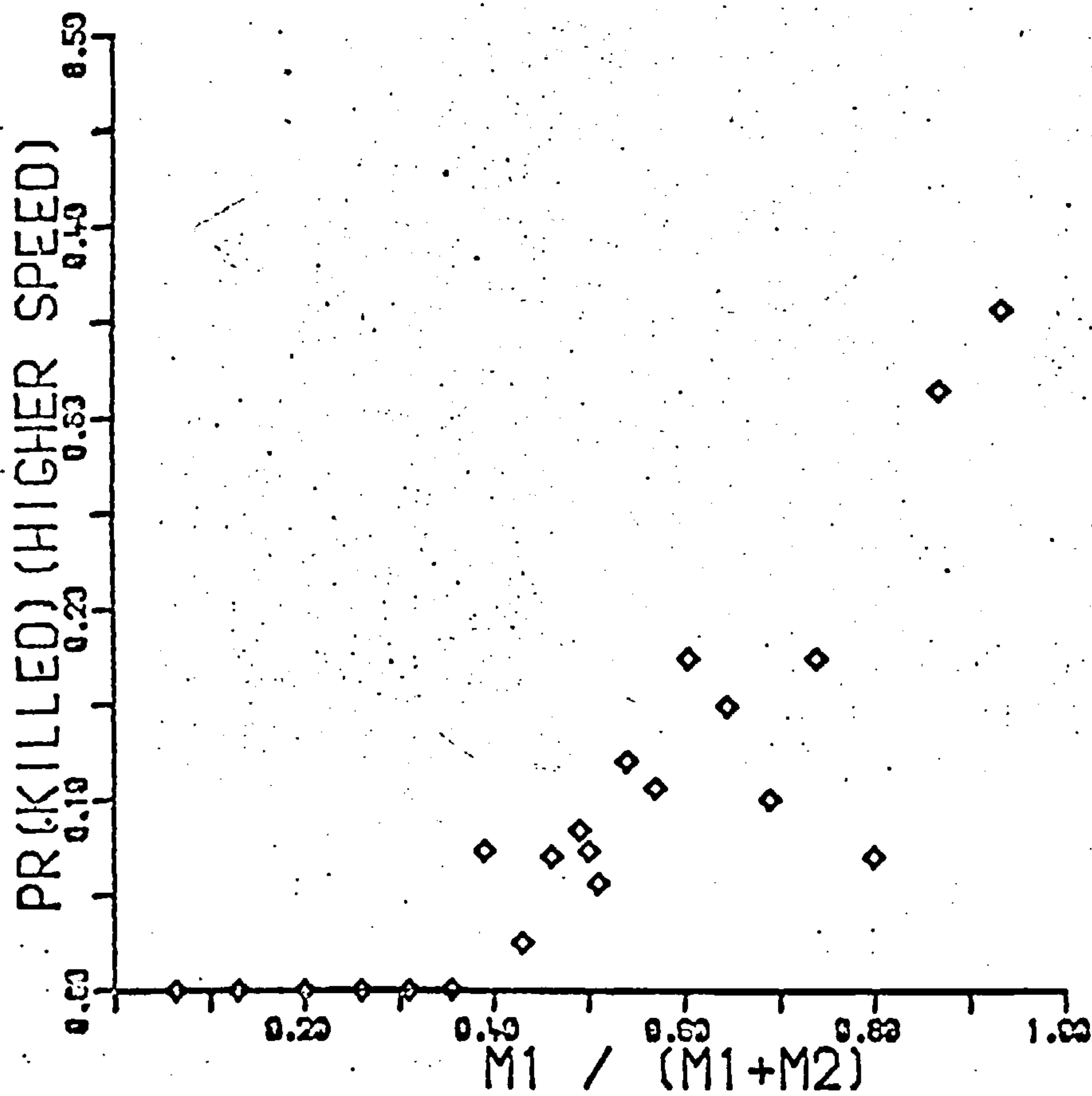


Figure 8.9: As figure 8.1, but for intersection accidents in rural areas.

Driver of lighter vehicle:			Fat Fat Fat Fat			Ser Ser Ser Ser			Slid Slid Slid Slid			None None None None						
Driver of heavier vehicle:			Fat Ser Slid None			Fat Ser Slid None			Fat Ser Slid None			Fat Ser Slid None						
Mass ratio:																		
1.00*			0	0	3	5	32	90	163	363	1220			(1066)				
.90 - .99			0	2	2	8	3	105	167	304	5	143	1069	2355	4	270	1791	(3373)
.80 - .89			0	0	6	8	1	92	198	389	2	142	1017	2548	3	189	1491	(3364)
.70 - .79			1	1	2	7	4	70	146	428	1	95	693	2288	2	123	1038	(2724)
.60 - .69			0	3	5	9	2	64	145	434	3	71	556	2497	1	128	804	(2356)
.50 - .59			0	1	2	5	0	38	71	249	1	27	219	1081	0	54	323	(1058)
.40 - .49			0	3	4	4	0	23	44	157	1	22	140	874	0	36	254	(696)
.30 - .39			0	1	0	1	0	13	19	110	1	3	92	441	0	17	105	(333)
.20 - .29			0	0	2	4	0	10	28	119	1	5	57	453	0	8	63	(251)
.10 - .19			0	1	3	36	0	5	27	436	1	3	48	1202	2	8	69	(765)
.01 - .09			0	0	6	55	0	3	23	475	1	4	41	1341	0	6	63	(970)

* Both vehicles the same at this ratio. The number under (Fat, Ser), for instance, is the total number of cases in which one driver was killed and the other seriously injured.

† The number of cases in which both drivers were uninjured is that in which one or more passengers were injured.

Table 8.14: Data on driver injuries in intersection accidents in urban areas, in Great Britain, 1969-72.

Mass ratio <u>(Lighter)</u> <u>Heavier</u>	Estimated likelihood of being killed				χ^2
	<u>Lighter vehicle</u>		<u>Heavier vehicle</u>		(d.f.)
	V_1 (p_{1L})	V_2 (p_{2L})	V_1 (p_{1H})	V_2 (p_{2H})	
1.00	$.97 \times 10^{-14}$.15	*	*	40 (6)
.90 - .99	$.33 \times 10^{-13}$.12	$.83 \times 10^{-15}$.086	88 (9)
.80 - .89 ⁺	$.33 \times 10^{-12}$.10	$.64 \times 10^{-16}$	$.16 \times 10^{-2}$	79 (6)
.70 - .79 ⁺	$.12 \times 10^{-11}$.092	$.40 \times 10^{-18}$	$.39 \times 10^{-2}$	87 (6)
.60 - .69 ⁺	$.16 \times 10^{-11}$.11	$.66 \times 10^{-20}$	$.26 \times 10^{-2}$	228 (6)
.50 - .59 ⁺	$.19 \times 10^{-10}$.15	$.43 \times 10^{-20}$	$.71 \times 10^{-2}$	107 (6)
.40 - .49 ⁺	$.12 \times 10^{-11}$.17	$.21 \times 10^{-21}$	$.32 \times 10^{-2}$	96 (6)
.30 - .39 ⁺	$.23 \times 10^{-10}$.077	$.14 \times 10^{-22}$.024	37 (6)
.20 - .29 ⁺	$.10 \times 10^{-9}$.23	$.10 \times 10^{-26}$	$.41 \times 10^{-3}$	46 (6)
.10 - .19 ⁺	$.16 \times 10^{-8}$.64	$.59 \times 10^{-32}$	$.18 \times 10^{-11}$	201 (6)
.01 - .09 ⁺	$.16 \times 10^{-8}$.78	$.37 \times 10^{-34}$	$.39 \times 10^{-17}$	194 (6)

* Both vehicles the same at this mass ratio.

⁺ For these mass ratios, the abbreviated model in which the fatal and serious categories were combined for the driver of the heavier vehicle was used.

Table 8.15: Results for intersections accidents in urban areas, taking $\alpha = .0961$, $\beta = .0312$, and $f = .981$.

INTERSECTION URBAN

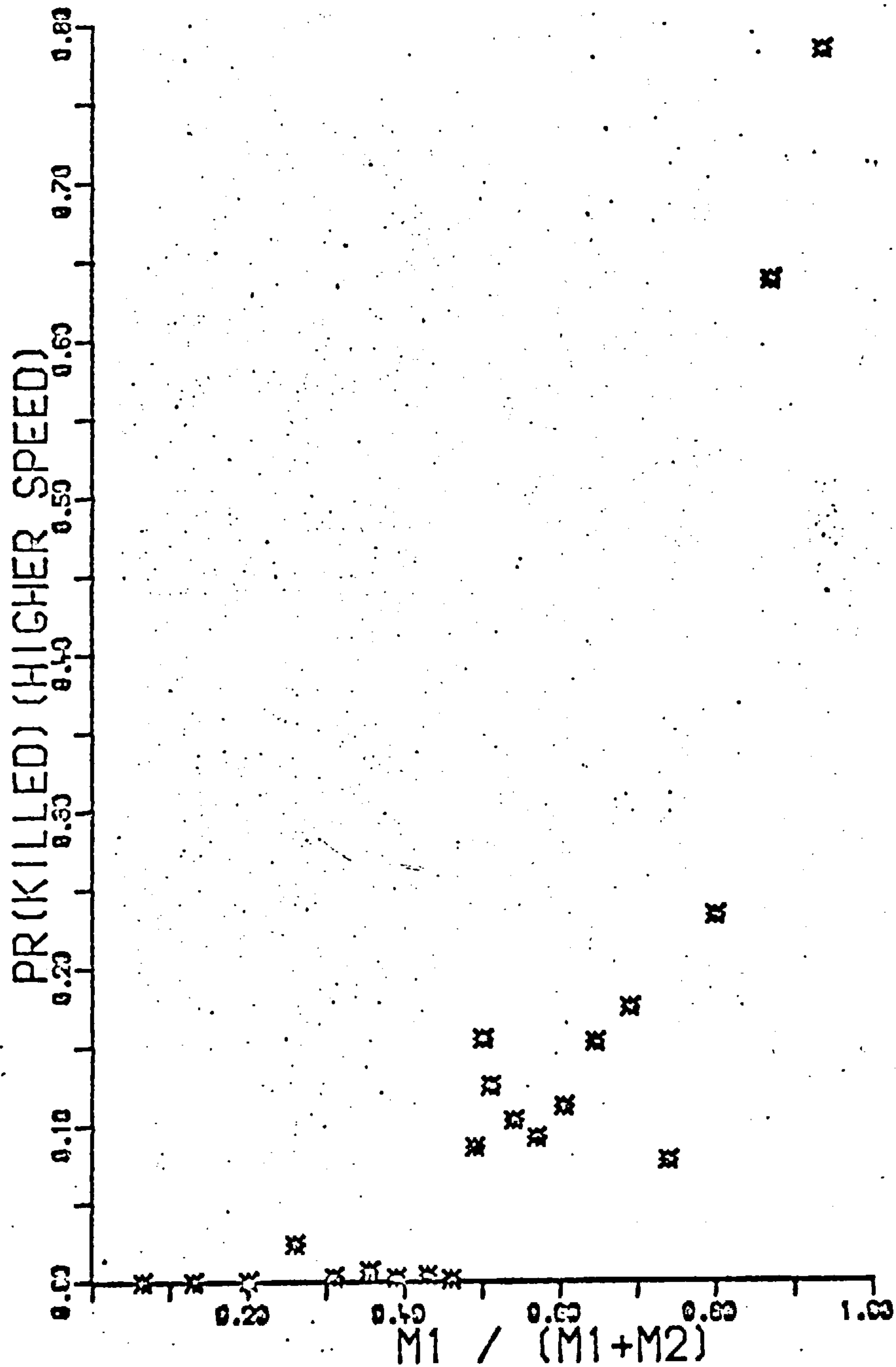
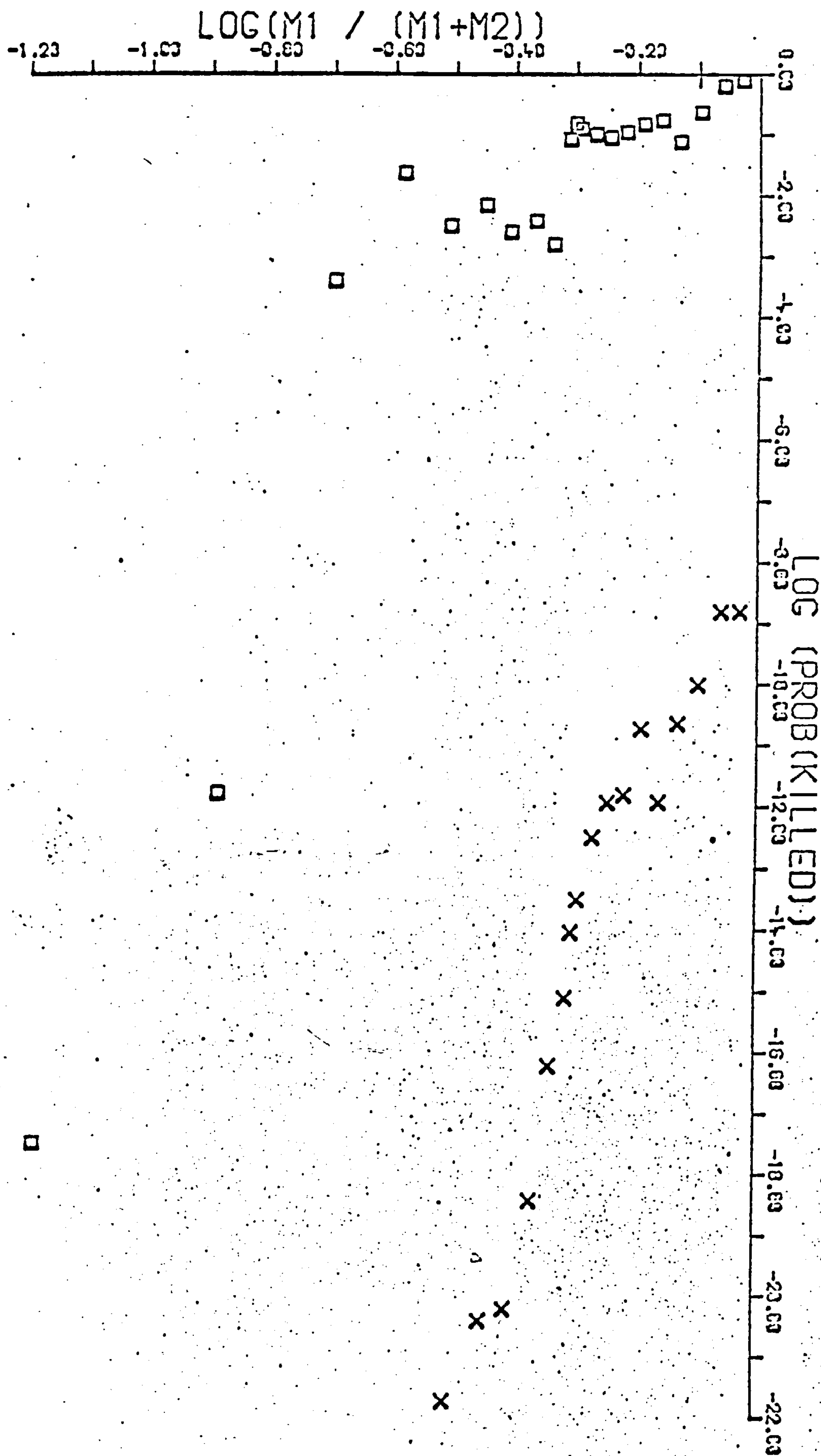


Figure 8.11: As figure 8.1, but for intersection accidents in urban areas.

INTERSECTION URBAN

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8.5 Comparison of types of accident

It is remarkable that the graphs of $\log(\text{probability of being killed})$ versus $\log(\text{mass ratio})$ are approximately parallel (in the sense of section 8.3.1) for both speeds and all four types of accident (see figure 8.13). This enables us to compare the speeds of different types of accident. Now, the average horizontal distance between the points corresponding to different types of accident may be estimated for each speed separately over the whole range of variation, and the horizontal distance between the two speeds for each type of accident separately may also be estimated, but over a fairly small range where the severity in the heavier vehicle at the higher speed overlaps with the severity in the lighter vehicle at the lower speed. The two sets of distances that result are not necessarily compatible because the lines are ^{not} all exactly parallel. The distances illustrated in figure 8.14 have been estimated (by eye) as a reasonable compromise between best estimates of within- and between-accident type variability. (Because of the information about the shape of the curve supplied by other accident types, the variability of rural head-on accidents is not precisely the same as estimated previously in section 8.3.1.)

By selecting one of the speeds as our unit of measurement, we may express the other speeds in terms of this. Taking the average speed of rural head-on accidents to be 1, the others are as shown in table 8.16. This shows that the speeds of rural head-on accidents are both the highest and the most variable of the four types considered, and urban intersection accidents the slowest and the least variable. The results are certainly very reasonable, though for reasons discussed earlier, it is difficult to find appropriate real speed distributions to compare them with.

BOTH

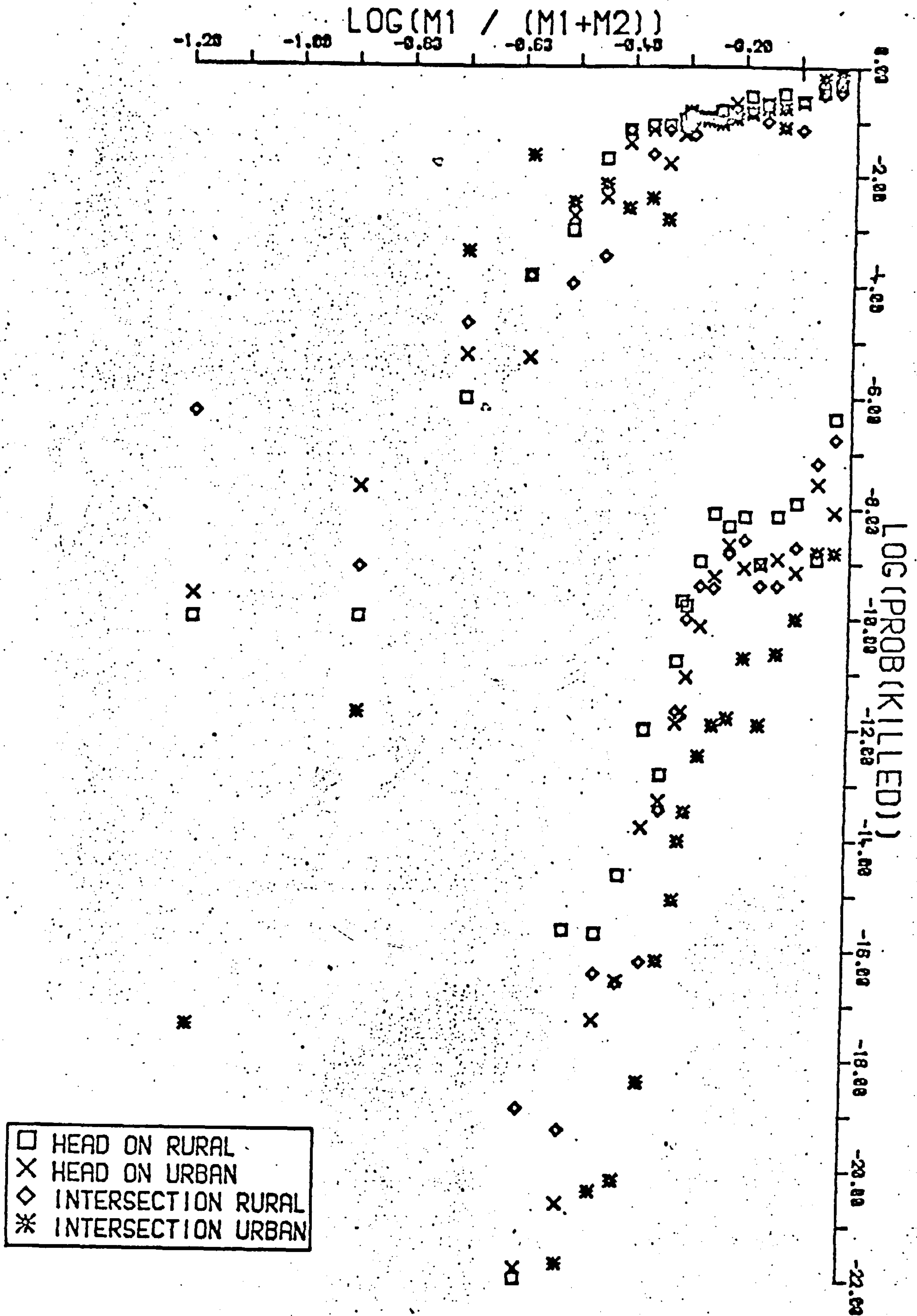
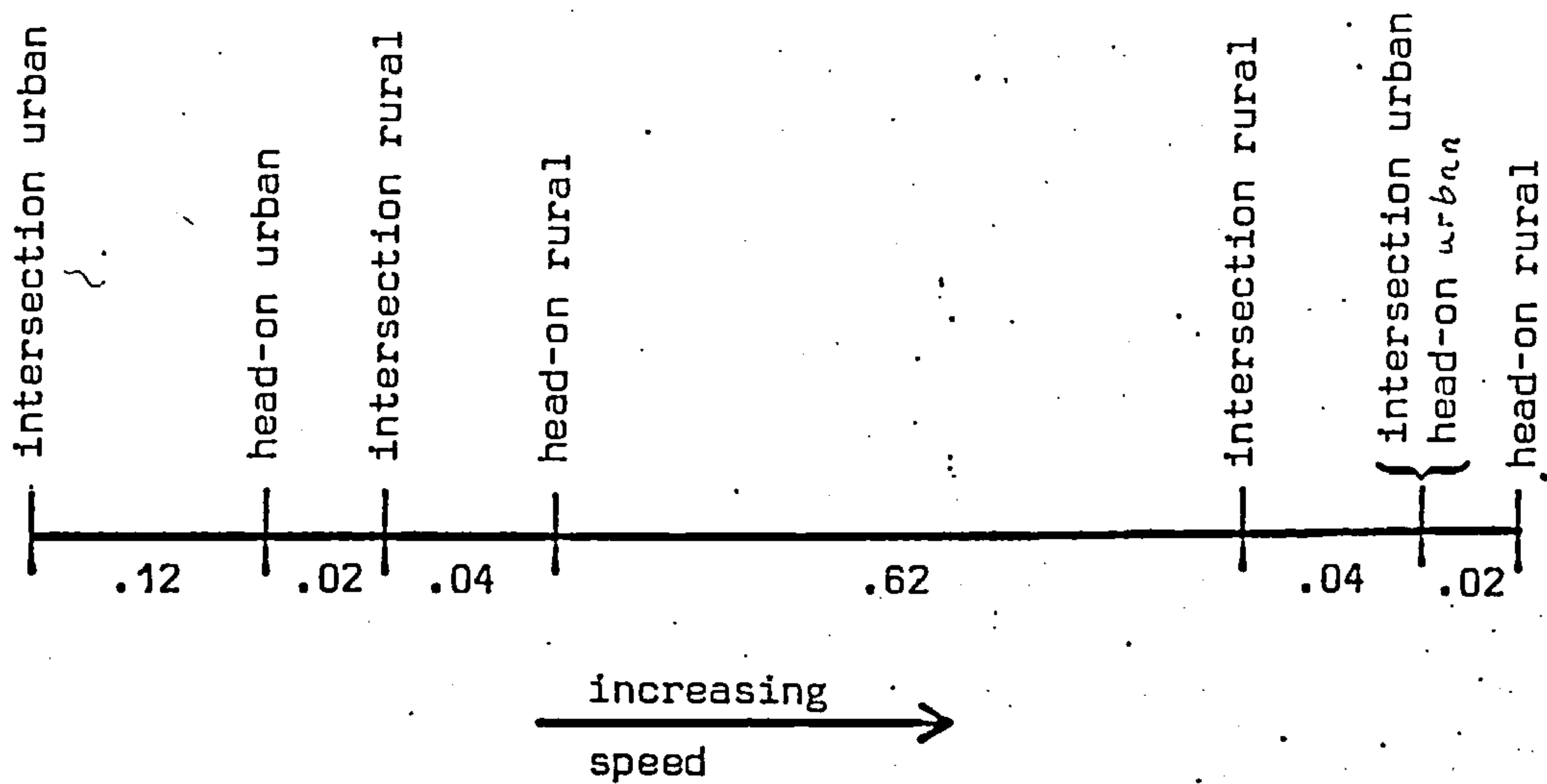


Figure 8.13: As figure 8.3, but including all types of accident.



Variabilities of speeds within accident-types

head-on rural:

83.6% ← → 16.4%

head-on urban:

89.4% ← → 10.6%

intersection rural:

89.0% ← → 11.0%

intersection urban:

98.1% ← → 1.9%

Figure 8.14: Comparison of speeds.

	$\underline{V_1}$	$\underline{V_2}$	$\underline{\% \text{ at } V_1}$	$\underline{\bar{V}}$	$\underline{\sigma}$	$\underline{\kappa}$
Head-on rural	.62	2.95	83.6	1.00	.86	.86
Head-on urban	.59	2.57	89.4	.80	.37	.46
Intersection rural	.54	2.69	89.0	.78	.45	.58
Intersection urban	.59	1.95	98.1	.62	.19	.31

Table 8.16: Estimated speeds relative to the average speed of rural head-on accidents. σ is the estimated standard deviation, and κ is the coefficient of variation (σ/\bar{V}).

Figure 8.15 is another illustration of the differences between the four distributions of impact speeds. What has been done here is to assume (i) that the estimates of the relative means and variances of the distributions have not been biased by the estimation method used, and (ii) that speeds have the Gamma distribution (for which the probability density function is $x^{c-1} e^{-x/b} / (b^c \Gamma(c))$ where $\Gamma(c)$ is the gamma function defined by $\Gamma(c) = \int_0^\infty e^{-u} u^{c-1} du$), this being preferred to the Normal distribution because the coefficients of variation given in table 8.16 are not negligibly small. The scale and shape parameters b and c were fixed so that the means and standard deviations of the distributions were as given in table 8.16, and the probability density functions calculated and plotted in figure 8.15. Their shapes can only very roughly be estimated from this sort of procedure, since they depend crucially on the assumption of the form of the distributions, and it is unlikely that the distribution with the highest mean also has the lowest mode.

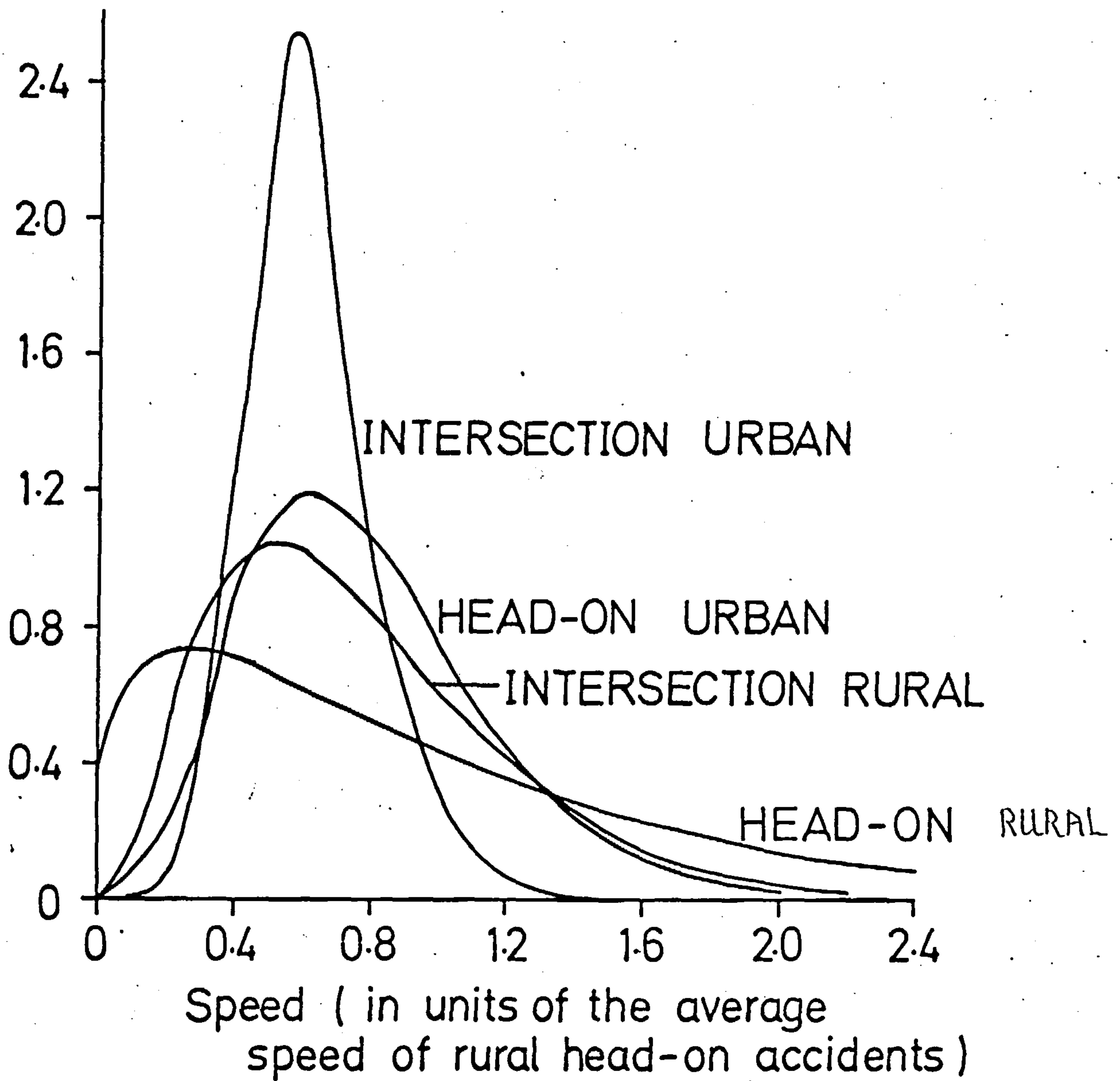


Figure 8.15: estimated distributions of speeds of the four types of accidents, assuming they are Gamma-distributed.

8.6 Discussion

The two key features of the model used in this Chapter are (i) the approximation of the true distribution of impact speeds by two points, and (ii) the exponential connexion between the four degrees of injury.

As to the second of these, we could have assumed a Normal connexion, or one of the other forms discussed in the previous Chapter, but these are mathematically more difficult to work with than the exponential. It would be of interest to know whether the form of the severity versus velocity change graph was altered by this choice. It may also be noted that whereas in the previous Chapter we confined the degrees of injury to three - fatal, serious, and slight - we have here added the uninjured category. Although we cannot necessarily assume that the exponential model can be extended in this way, the theoretical arguments of Chapter 7 still apply - the proportions of the other types of injury ought to be predictable from the proportion of deaths, and the RIF curve ought to be smoothly increasing. Consequently, even if we did not have the evidence from Chapter 7 that the exponential connexion had empirical support, it would still be a natural choice for use in this Chapter.

The results of this Chapter permit the calculation of the chance of both drivers escaping injury at a given mass ratio - for table 8.8 (rural head-on accidents) this is about .2 to .3. Results reported by Bohlin (1967), Faulkner (1968), and Dawson (1967, p.23-26) suggest there may be ten times as many damage-only accidents as ones involving personal injury. Although this figure would almost certainly be lower for such severe accidents as head-on ones in rural areas, it almost certainly should be higher than that found here. It is likely that this

discrepancy arises because of the use of only two speeds in our calculations. Our figure of .2 to .3 may be described as estimating the proportion of potentially-injurious accidents that result in no injury to either driver.

Some possible extensions of the methods described here are included in the following table.

TABLE OF MAJOR RESULTS AND POSSIBLE FUTURE DEVELOPMENTS

<u>Chapter or section</u>	<u>Major results and conclusions</u>	<u>Suggestions for further research</u>	
		<u>(a) Fundamental</u>	<u>(b) Extensions and technical improvements</u>
1.3.1 (Trends in causes of death)	<p>The proportion of RTA deaths ascribed to internal injuries is increasing.</p> <p>For some categories of road user, there has been a decline in the proportion of deaths ascribed to skull fracture.</p>	<p>Why are the causes of death changing? (This should be answered for each category of road user separately.)</p>	<p>Are there regional differences?</p> <p>Are these changes occurring in other countries?</p> <p>Examine data coded according to ICD 3-digit category.</p>
1.3.2 (Trends in injury)	<p>There is a gradual trend towards a greater numerical importance of intracranial injuries.</p>	<p>Why?</p>	<p>Regional differences.</p> <p>Other countries.</p>
Ch.2 (Severity of pedestrian injury)	<p>Among the factors having the largest effect on the proportion of pedestrians severely injured are the casualty's age and the speed limit in force at the site of the accident.</p> <p>Pedestrians struck by heavy goods vehicles tend to be more severely injured than those struck by other types of vehicle.</p> <p>Model of car has a small but statistically significant effect.</p>	<p>How do vehicle design factors affect the nature and severity of pedestrian injury? (More experimental and simulation studies needed.)</p>	<p>Link RGEW and HIPE data with Stats 19 records and examine effect of model of car on nature of injury.</p>

Chapter or section	Major results and conclusions	Suggestions for further research	
		(a) Fundamental	(b) Extensions and technical improvements
3.1 (Estimating the speed of accidents)	<p>There are high correlations (around 0.6) between the estimates of the speeds of pedestrian accidents made by the different people involved.</p> <p>Independent witnesses tend to give a slightly higher (by about 3 mph) estimate of the vehicle's speed than do its driver or passengers.</p> <p>Estimated speed of impact is correlated with vehicle damage.</p>	<p>It is desirable to develop some experimental method of approximating the degree of attention given by an average witness to a road accident, and investigate the reliability of speed estimates (and other details of the accident) made by members of the general public.</p>	<p>Replication is needed - for the records of other police forces, read by other investigators, studying other types of accident.</p>
3.2 (Times till death)	<p>The survival time of pedestrians killed in RTA's may be approximated by the Weibull distribution.</p> <p>The higher the impact speed, the shorter the survival time.</p> <p>When the effect of impact speed is eliminated, the age of the casualty does not affect survival time.</p> <p>But because the elderly are predominately killed by relatively low speed impacts, when all speed groups are combined it appears that the elderly tend to survive longer.</p>	<p>To what extent do immediate first aid and time to hospitalisation affect the likelihood of dying?</p> <p>Can survival time be regarded as a measure of severity of injury in the same way as the conventional division (fatal, serious, slight), and are the models discussed in Chapter 7 applicable?</p>	<p>It would be of interest to examine the effects of velocity change, age of casualty, type of vehicle, and type of accident on the survival times of fatally-injured vehicle occupants.</p>

Chapter or section	Major results and conclusions	Suggestions for further research	
		(a) Fundamental	(b) Extensions and technical improvements
3.2 (Cont.)	Compared to when struck by a car, survival tends to be longer when struck by a motor-cycle and shorter when struck by a heavy goods vehicle.		
4.1 (CATLIN)	CATLIN is extremely useful in extending the types of hypotheses which can be tested about tables of frequencies.	Chapter 7 suggests the extension of CATLIN to deal with functions of the π 's of the forms $\sum_{k=1}^K z(A\pi)$ (where $z(p)$ is the Normal deviate corresponding to a probability p) and $Q \log_e \left\{ \frac{K}{\log_e (A\pi)^e} \right\}$.	Investigate the small-sample behaviour of the χ^2 statistics calculated by CATLIN.
4.2 (Nonparametric tests)	A FORTRAN program to carry out a two-way analysis of variance by a technique due to Benard and van Elteren (1953) is given.		Investigate the adequacy of the χ^2 approximation to the distribution of the test statistic when the number of rows is small.
5.2 (Leg injuries)	Both type of accident and model of car affect the probability of the driver sustaining a leg injury. This probability is about $2\frac{1}{2}$ times as large in head-on accidents as in rear-end ones, and about $2\frac{1}{2}$ times as large in the most injury-producing of the five models considered as in the least.	Relate leg injury to interior design of car.	Better data on injury needed.

Chapter or section	Major results and conclusions	Suggestions for further research	
		(a) Fundamental	(b) Extensions and technical improvements
6.3 (Relative numbers of single- and two-car accidents)	<p>The ratio of single- to two-car accidents is higher for young drivers than for old (a factor of 3 difference between those under 25 and those over 35 years old), and higher in rural areas than in towns (by a factor of nearly 3).</p> <p>Model of car also affects this ratio - the range of variation was a factor of $1\frac{1}{2}$ in urban areas and 2 in rural areas among 12 common models of car.</p>	<p>What are the vehicle design and handling parameters which influence accident involvement?</p> <p>Can the number of collision accidents (or of some subset of them) be taken as a proxy for mileage travelled?</p>	<p>National Travel Survey data may provide sufficient information on mileage travelled by some models of car for accident rates to be calculated.</p>
6.4 (Proportion of overturning in single-car accidents)	<p>The proportion of overturning in single-car accidents is higher for young drivers than for old (a factor of about $1\frac{1}{2}$ difference between those under 25 and those over 35 years old), and higher in rural areas than in towns (by a factor of over 3).</p> <p>Model of car also affects this proportion - the range of variation was a factor of $2\frac{1}{2}$ in urban areas and $1\frac{1}{2}$ in rural areas among 12 common models of car.</p>	<p>What vehicle factors predispose to overturning?</p>	

<u>Chapter or section</u>	<u>Major results and conclusions</u>	<u>Suggestions for further research</u>	
		(a) Fundamental	(b) Extensions and technical improvements
Ch.7 (Statistical aspects of injury severity)	<p>In many situations, the relative proportions of casualties killed, seriously and slightly injured are p, $p^m - p$, and $1 - p^m$, where p measures the severity of the circumstances and m is a constant determined by the definitions of the degrees of injury, and is about 0.33 in Britain.</p> <p>It is suggested that this model should be generally used when studying causes of differences in degrees of injury in preference to taking the average value of arbitrary scores assigned to the several levels of injury.</p> <p>Certain alternative models are also suggested.</p>		<p>It needs to be established under what conditions the distribution of injury severity behaves according to one or other of the suggested models, and the degree of filtering of the data necessary to prevent the true curve of proportion seriously injured versus proportion killed being obscured by the combining of accidents of very different degrees of severity (as illustrated in figure 7.9).</p>

Chapter or section	Major results and conclusions	Suggestions for further research	
		(a) Fundamental	(b) Extensions and technical improvements
Ch.8 (Driver injury correlations)	<p>In two-vehicle accidents of a given mass ratio, there is a positive correlation between the severities of injury to the two drivers. This arises because the relative velocity before impact is the same for both drivers in the one accident, but varies considerably between different accidents. The correlation may be satisfactorily modelled by approximating the relative velocity distribution by two points.</p> <p>Mass ratio has a very strong influence on injury severity, and the shape of the relation between these factors seems to be similar for several types of accident. This has enabled estimates of their relative speeds to be made - for instance, that rural accidents occur at speeds about 25% faster than urban ones.</p>	<p>Instead of effectively assuming a two-point distribution of severity, a continuous distribution of impact speed could be used (such as the Beta distribution) with two or three unknown parameters, together with a function relating speed to the probability of death. It is easy to get the relative proportions in each cell of the table at any one speed, and then numerical integration over the distribution of speeds would give the overall proportions. However, in view of the necessity for numerical integration, it is likely that such a model would use a great deal of computer time.</p> <p>Even more heavy on computation would be using such a model to distinguish between different models of car. Different speed distributions could be assumed for different models, and so could different functions relating speed to severity. The results would enable a distinction to be made between a car whose occupants were more-often-than-usual seriously injured because it</p>	<p>Reanalyse the data using first the Normal and then the Gamma models of Chapter 7 instead of the exponential connexion between the different levels of injury.</p> <p>Classify the drivers according to their ages, and include this in the analysis.</p> <p>It would be useful to assume a distribution of speeds and a relation between speed and probability of death, calculate simulated data of the degrees of injury to the two drivers, and analyse this artificial data according to the two-speed model in order to see how accurately this model recovers the original speed distribution and speed versus severity relation.</p> <p>There is a need for the statistical development of the concepts of Chapter 8. In particular, it would be useful to have estimates of the variances of the parameters. A possible way to approach this might be to consider the partial derivatives of χ^2 with respect to the parameters.</p>

<u>Chapter or section</u>	<u>Major results and conclusions</u>	<u>Suggestions for further research</u>	
		(a) Fundamental	(b) Extensions and technical improvements

Ch.8 (Cont.)

was usually in high speed crashes, and one for which the same result was due to it having a more dangerous interior: in the first case the other vehicle's occupants would be more seriously injured than usual, but not in the second. However, there could be compounding of the effect of high speed and of "aggressiveness" (the tendency to be dangerous towards other vehicles). The age of the person injured should also be included in such a model, if possible.

It would be of interest to study the correlation between injuries to occupants of the same vehicle in single-vehicle accidents. This might be of special use when accident statistics do not distinguish between an uninjured passenger and no passenger being present. Another possible use would be to the effect of wearing seat belts, or some other device that does not necessarily apply to both the driver and a front seat passenger, such as collapsible steering columns.

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ADDENDA AND CORRIGENDA

None known at 1 Jan. 1976.